COUPLING 2D FINITE ELEMENT MODELS AND CIRCUIT EQUATIONS USING A BOTTOM-UP METHODOLOGY

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ABSTRACT. The aim of this paper is to present an approach, able to deal with all possible connections of voltage and current sources and impedances, combining conductors in which the skin effect is taken into account and conductors in which skin effect is neglected. This approach is obtained using a bottom-up methodology. In this way, the meaning of terms in the generalized approach is naturally inherited from some specific problems. This model is presented in a compact form, preserving sparse, symmetric and positive-definiteness matrices. The vectors and matrices, computed during the solution stage, are employed in engineering calculations as current and inductance computations providing compact expressions suitable for efficient algorithms. Finally, the proposed approach, implemented in a FEM software package developed by the authors, is applied to the study of a three-phase transformer, supplied with a balanced three-phase voltage (sinusoidal and nonsinusoidal) and loaded with an unbalanced three-phase RL impedance. The agreement between the computed and experimental results shows the validity of the proposed model and its implementation.

1 INTRODUCTION

The natural excitation for the Finite Element Method (FEM) in electromagnetism is current (current density). This current is supplied by an external electric circuit; and, usually, the value of this current is unknown, as the circuits consist of passive elements and voltage sources. Therefore, FEM and electrical equations are coupled by these unknown currents. There are two different approaches to couple these equations. In the first approach, called direct coupling, the equations obtained from FEM and the electrical circuits are integrated in a single system of equations having the magnetic vector potential as its solution. In the second one, called indirect coupling, two different systems of equations are obtained; one of them takes into account the FEM equations, and the other one, the circuit equations. In this paper, a method, based on the direct coupling of FEM and circuit equations is presented.

Many approaches on coupled 2D Finite Element models and circuit equations have been proposed [1]-[7]. Most of them, take into account the skin effect for all the conductors or, they neglect its influence, without evaluating its magnitude over each conductor. In this paper, a bottom-up methodology is applied, generalizing the meaning of the studied examples, instead of a theoretically obtained approach. So, some common winding connections were studied—one-phase circuit and three-phase circuits with star and delta connections—, being presented here the three-phase star connection handling unbalanced load and voltage conditions. In this paper, the model is applied to the study of a power transformer, although it has been applied to other non-static machines as induction motors, [10].

2 BASIC EQUATIONS

Assuming that the displacement currents are neglected, Maxwell's equations using a magnetic vector potential formulation can be written in 2D as:

$$\frac{\partial}{\partial x}\left(\nu(\mathbf{B})\frac{\partial A(x,y)}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu(\mathbf{B})\frac{\partial A(x,y)}{\partial y}\right) = -J(x,y) \quad (1)$$

Where $A(x,y)$ is the magnetic vector potential, $J(x,y)$ the current density (z component) and ν the reluctivity that depends on the magnetic flux density vector ($\boldsymbol{B}$). In anisotropic materials, reluctivity depends on $|\boldsymbol{B}|$ and the angle (θ) between $\boldsymbol{B}$ and the rolling direction of the material $\nu=\nu(A,\theta)$.

The expression of current density depends on how the conductors are defined. So, the conductors can be modelled as solid conductors, in which case the skin effect is taken into account, or stranded conductors, in which case the skin effect is neglected. Stranded conductors are particularly useful to model windings in which an individual mesh considering their relations would exceed in many cases the requirements of actual computers without providing a significantly greater accuracy. If all the conductors in the problem are solid conductors, the following equation is employed, [8] [9]:

$$\nabla\times\nu\nabla\times\mathbf{A} = \sigma\mathbf{E}_{\mathbf{ext}} - \sigma\frac{\partial\mathbf{A}}{\partial t} - \sigma v_e\times\nabla\times\mathbf{A} \quad (2)$$

The first term on the right side in (2), $\boldsymbol{E}_{ext}$ being the external electric field and σ the electrical conductivity, represents the current density due to the voltage source. The second term takes into account the current density due to the temporal variation of $\boldsymbol{A}$. The third one represents the current density due to the movement, being $\boldsymbol{v_e}$ the velocity of the conductor with respect to **B**. By using a frame of reference, such that the relative velocity becomes zero, (2) can be written as, [8]:

$$\nabla \times \nu \nabla \times \mathbf{A} = \sigma \mathbf{E_{ext}} - \sigma \frac{d\mathbf{A}}{dt} \tag{3}$$

The total current of a conductor can be obtained by integrating the right side of (3):

$$I_c = \iint_{S_c} \left(\sigma \mathbf{E_{ext}} - \sigma \frac{d\mathbf{A}}{dt} \right) dS \tag{4}$$

In 2D problems (3) becomes:

$$-\nabla(\nu \nabla A) = \sigma \frac{u_s}{l} - \sigma \frac{dA}{dt} \tag{5}$$

Where u_s is the voltage between the terminals of the conductor and l the length of the conductor (thickness of the considered domain along the z-axis). Equation (5) takes into account the skin effect.

When the conductors are considered stranded, it is supposed that the current (i_f) is the same in all the conductors connected in series (winding); then, their current density can be obtained by $J=Ni_f/S$. Where S is the entire section of all the conductors (N turns) integrating the winding, σ the electrical conductivity and R_F the DC resistance. The current can be calculated in 2D problems, by considering the voltage u_F between the terminals of the winding as the sum of the voltages of its conductors:

$$i_f = \frac{u_F}{R_F} - \frac{l\,N}{R_F\,S} \iint_S \frac{dA}{dt} dS \tag{6}$$

Using (1) and (6):

$$-\nabla(\nu \nabla A) = \frac{N u_F}{S R_F} - \frac{\sigma N}{S} \iint_S \frac{dA}{dt} dS \tag{7}$$

Summarizing (3) and (7) into a single equation for 2D problems:

$$-\nabla(\nu \nabla A) = \begin{cases} \dfrac{N u_F}{S R_f} - \dfrac{\sigma N}{S} \displaystyle\iint_S \dfrac{dA}{dt} dS & \textit{Stranded} \\ \sigma \dfrac{u_s}{l} - \sigma \dfrac{dA}{dt} & \textit{Solid} \end{cases} \tag{8}$$

3 EXAMPLE. THE WINDINGS ARE STAR CONNECTED

The authors have solved some complex winding connections. In this case, three windings of stranded conductors are star connected (figure 1). They are modelled with FEM, and indeed can or can not share the same 2D magnetic circuit. The total DC resistances, including the resistances corresponding to the windings and the load, are called R_R, R_S, R_T. In the same way L_R, L_S, L_T take into account the inductance corresponding to the load and the part of leakage flux not considered in the two-dimensional model.

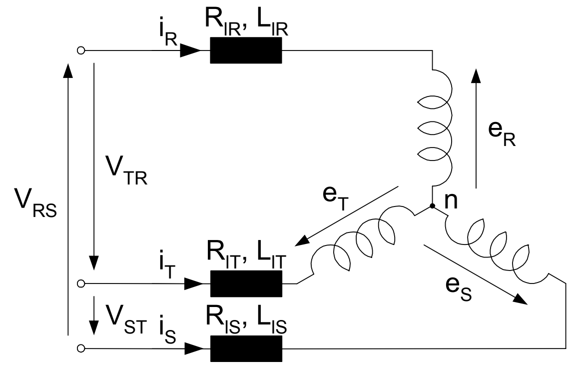

Figure 1. The windings are star connected.

From figure1 the following equations can be obtained:

$$V_{RS} = e_R - e_S + \left(R_R i_R + L_R \frac{di_R}{dt} \right) - \left(R_S i_S + L_S \frac{di_S}{dt} \right) \tag{9}$$

$$V_{ST} = e_S - e_T + \left(R_S i_S + L_S \frac{di_S}{dt} \right) - \left(R_T i_T + L_T \frac{di_T}{dt} \right) \tag{10}$$

$$V_{TR} = e_T - e_R + \left(R_T i_T + L_T \frac{di_T}{dt} \right) - \left(R_R i_R + L_R \frac{di_R}{dt} \right) \tag{11}$$

$$i_R + i_S + i_T = 0 \tag{12}$$

values of e_R, e_S and e_T can be obtained using:

$$e_j = -l \quad N_j \left(\frac{dA_{m,j}}{dt} \right) \qquad j = R, S, T \tag{13}$$

where A_m is the mean magnetic vector potential on j winding, defined by:

$$A_{m,j}(t) = \frac{\iint_{S_j} \{\varphi(x,y)\}^T \{A(t)\} dS}{\iint_{S_j} dS} = \frac{\iint_{\Omega} \beta_j(x,y) \{\varphi(x,y)\}^T d\Omega}{\iint_{S_j} dS} \{A(t)\} = \{w\}^T \{A(t)\} \tag{14}$$

Being $\{w\}^T$ the phase vector, which provides information about the topology of each winding, $\varphi(x,y)$ the shape or interpolation functions, whose formulas are well-known for classical elements, and Ω the entire domain where the integral is extended by using the phase functions β_j:

$$\beta_j(x,y)=\begin{cases}1 & \text{If} \quad (x,y)\in S_j^+\\ -1 & \text{If} \quad (x,y)\in S_j^-\\ 0 & \text{Otherwise}\end{cases} \tag{15}$$

Using the Crank-Nicholson time stepping method, the following equations are obtained:

$$\frac{1}{2}\left(\frac{di}{dt}\right)^{t+\Delta t}+\left(1-\frac{1}{2}\right)\left(\frac{di}{dt}\right)^{t}=\frac{i^{t+\Delta t}-i^{t}}{\Delta t} \tag{16}$$

$$\frac{1}{2}\left(\frac{dA_m}{dt}\right)^{t+\Delta t}+\left(1-\frac{1}{2}\right)\left(\frac{dA_m}{dt}\right)^{t}=\frac{A_m^{t+\Delta t}-A_m^{t}}{\Delta t} \tag{17}$$

Using (9), (10), (11), (12), (13), (16) and (17):

$$\begin{aligned}\begin{Bmatrix} i_R^{t+\Delta t}\\ i_S^{t+\Delta t}\\ i_T^{t+\Delta t}\end{Bmatrix} &= \frac{\Delta t}{M_E^2}\begin{bmatrix} M_T & 0 & -M_S\\ -M_T & M_R & 0\\ 0 & -M_R & M_S\end{bmatrix}\begin{Bmatrix} V_{RS}^{t+\Delta t}+V_{RS}^{t}\\ V_{ST}^{t+\Delta t}+V_{ST}^{t}\\ V_{TR}^{t+\Delta t}+V_{TR}^{t}\end{Bmatrix}-\frac{2l}{M_E^2}\\
&\begin{bmatrix} M_S+M_T & -M_T & -M_S\\ M_T & M_T+M_R & M_R\\ M_S & M_R & M_S+M_R\end{bmatrix}\begin{bmatrix} N_R & 0 & 0\\ 0 & N_S & 0\\ 0 & 0 & N_T\end{bmatrix}\begin{Bmatrix} A_{m,R}^{t+\Delta t}-A_{m,R}^{t}\\ A_{m,S}^{t+\Delta t}-A_{m,S}^{t}\\ A_{m,T}^{t+\Delta t}-A_{m,T}^{t}\end{Bmatrix}\\
&-\frac{1}{M_E^2}\begin{bmatrix} M'_R(M_S+M_T) & M'_S M_T & -M'_T M_S\\ M'_R M_T & M'_S(M_T+M_R) & M'_T M_R\\ M'_R M_S & M'_S M_R & M'_T(M_S+M_R)\end{bmatrix}\begin{Bmatrix} i_R^{t}\\ i_S^{t}\\ i_T^{t}\end{Bmatrix}\end{aligned} \tag{18}$$

where:

$$\begin{aligned} M_j &= R_j\Delta t+2L_j \qquad j=R,S,T\\ M'_j &= R_j\Delta t-2L_j \qquad j=R,S,T\\ M_E^2 &= M_R M_S+M_S M_T+M_T M_R\end{aligned} \tag{19}$$

Equation (18) can be written in a packed form as:

$$\begin{aligned}\left\{i^{t+\Delta t}\right\} &= \Delta t\left[G_a\right]\left\{u_{ext}^{(t+\Delta t)+t}\right\}\\ -l\ &\left[G_b\right]\left[N\right]\left(\left\{A_m^{t+\Delta t}\right\}-\left\{A_m^{t}\right\}\right)-\left[G_a\right]\left[G_c\right]\left\{i^{t}\right\}\end{aligned} \tag{20}$$

The current density can be obtained using:

$$\left\{J^{t+\Delta t}\right\}=\begin{Bmatrix} J_R^{t+\Delta t}\\ J_S^{t+\Delta t}\\ J_T^{t+\Delta t}\end{Bmatrix}=\begin{bmatrix}\frac{N_R}{S_R} & 0 & 0\\ 0 & \frac{N_S}{S_S} & 0\\ 0 & 0 & \frac{N_T}{S_T}\end{bmatrix}\begin{Bmatrix} i_R^{t+\Delta t}\\ i_S^{t+\Delta t}\\ i_T^{t+\Delta t}\end{Bmatrix} \tag{21}$$

Using (20), (21) and the definition of A_m in (14) the following equation is obtained:

$$\begin{aligned}\left\{J^{t+\Delta t}\right\} &= \Delta t\left[W\right]\left[N\right]\left[G_a\right]\left\{u_{ext}^{(t+\Delta t)+t}\right\}\\ &-l\left[W\right]\left[N\right]\left[G_b\right]\left[N\right]\left[W\right]^T\left(\left\{A^{t+\Delta t}\right\}-\left\{A^{t}\right\}\right)\\ &-\left[W\right]\left[N\right]\left[G_a\right]\left[G_c\right]\left\{i^{t}\right\}\end{aligned} \tag{22}$$

Where the term $[W]$ is called the phase matrix defined as the matrix containing the phase vectors presented in the problem (three in this case).

Using (1) and (22):

$$\begin{aligned}&\left(\left[S^{t+\Delta t}\right]+l\left[W\right]\left[N\right]\left[G_b\right]\left[N\right]\left[W\right]^T\right)\left\{A^{t+\Delta t}\right\}=\\ &\Delta t\left[W\right]\left[N\right]\left[G_a\right]\left\{u_{ext}^{(t+\Delta t)+t}\right\}+l\left[W\right]\left[N\right]\left[G_b\right]\left[N\right]\left[W\right]^T\left\{A^{t}\right\}\\ &-\left[W\right]\left[N\right]\left[G_a\right]\left[G_c\right]\left\{i^{t}\right\}\end{aligned} \tag{23}$$

where $[S^{t+\Delta t}]$ is the stiffness matrix. Therefore, equation (23) models 2D magnetic circuits with a three star connected windings of stranded conductors, the magnetic vector potential in the nodes at $t+\Delta t$ being the unknown values. Besides, current in the windings at t must be computed using efficient algorithms.

4 THE GENERALIZED SYSTEM OF EQUATIONS

The example analyzed above can be generalized to different types of connections and conductors (solid and stranded) [11].

$$\begin{aligned}&\left(\left[S^{t+\Delta t}\right]+\frac{2\sigma}{\Delta t}\left[T\right]+l\left[W\right]\left[N\right]\left[G_b\right]\left[N\right]\left[W\right]^T\right)\left\{A^{t+\Delta t}\right\}=\\ &\left(-\left[S^{t}\right]+\frac{2\sigma}{\Delta t}\left[T\right]+l\left[W\right]\left[N\right]\left[G_b\right]\left[N\right]\left[W\right]^T\right)\left\{A^{t}\right\}\\ &+\left[T\right]\left[\tfrac{\sigma}{l}Q\right]\left\{u_{mas,ext}^{(t+\Delta t)+t}\right\}+\Delta t\left[W\right]\left[N\right]\left[G_a\right]\left\{u_{ext}^{(t+\Delta t)+t}\right\}\\ &-\left(\left[W\right]\left[N\right]\left[G_a\right]\left[G_c\right]-\left[W\right]\left[N\right]\right)\left\{i_{cab}^{t}\right\}\end{aligned} \tag{24}$$

So, by example, if a specific problem includes windings of stranded conductors with star connection and other windings with whatever connections, the contribution of the star connected windings to the terms of (24) is obtained from (23).

The matrices in (24) maintain the sparsity, symmetry and positive-definiteness of the traditional FEM. Obviously, sparsity depends on geometry and the number of nodes since the mean magnetic vector potential introduces connections among the nodes of the windings modelled with stranded conductors (14).

Defining n, m and p as the number of nodes, and windings made of stranded and solid conductors respectively, the meaning and dimensions of the terms in (24) can be expressed as:

- $[S]_{nxn}$ is the stiffness matrix.
- $[T]_{nxn}$ is called the mass matrix, containing information about the current distribution.
- $[W]_{nxm}$ is the phase matrix showing information about the distribution of windings made of stranded conductors. It is composed of the phase vectors defined in (14).
- $[G_a]_{mxm}$, $[G_b]_{mxm}$ and $[G_c]_{mxm}$ take into account how the windings defined in [W] are connected. In (18) and (20) they are defined to star connections.
- $\{A^{t+\Delta t}\}_n$ is the vector of unknown nodal values representing the value of the magnetic vector potential at the current time step $t+\Delta t$
- $\{A^t\}_n$ represents the value of the magnetic vector potential at the previous time step t.
- [Q] has the same meaning in solid conductors as [W] in stranded conductors.
- $\left\{u_{mas,ext}^{(t+\Delta t)+t}\right\}_p$ represents the addition of voltages applied to the solid conductors at the current time step $t+\Delta t$ and at the previous time step t.
- $\left\{u_{ext}^{(t+\Delta t)+t}\right\}_m$ represents the addition of voltages applied to the stranded conductors at the current time step $t+\Delta t$ and at the previous time step t.
- $\left\{i_{cab}^{t}\right\}_m$ represents the value of current at the previous time step in conductors defined as stranded.

5 COMPUTATION OF ENGINEERING RESULTS

As shown in (23) and (24), currents in windings must be calculated during the solution stage. Besides, this computational cost must be kept as low as possible. Other engineering results such as inductance, magnetic flux and electromotive force can be obtained by making use of the terms defined in (24), computed at each time step avoiding time consuming calculations in the postprocessing stage.

5.1 Current Calculations

The winding currents can be calculated using:

$$i_f(t) = \int_{\Omega_f} \mathbf{J}(t)\ d\Omega \tag{25}$$

In solid conductors (25) can be expressed as:

$$I_c = \iint_{S_c} \left(\sigma \frac{u_s}{l} - \sigma \frac{\partial \mathbf{A}}{\partial t} \right) dS \tag{26}$$

Current in windings made of conductors considered as stranded ones can be calculated using (1), (5) and the definition of phase vector:

$$\left[S^{t+\Delta t}\right]\left\{A^{t+\Delta t}\right\} = \left\{I^{t+\Delta t}\right\} = \sum_{k=1}^{m} N_k \left\{\omega_k\right\}\ i_k^{\,t+\Delta t} \tag{27}$$

Where m is the number of windings. The current of a winding (i) can be computed multiplying both terms in (27) by the transposed i-phase vector $\{\omega_i\}^T$, taking into account that the product of $\{\omega_i\}^T$ and $\{\omega_k\}$ is null when k is not the same as i:

$$i_i^{\,t+\Delta t} = \frac{\left\{\omega_i\right\}^T \left[S^{t+\Delta t}\right]\left\{A^{t+\Delta t}\right\}}{N_i \left\{\omega_i\right\}^T \left\{\omega_i\right\}} \tag{28}$$

This method computes the phase current using as data, the phase vector, the number of turns, together with the stiffness matrix and the vector of the magnetic vector potential nodal values at the current time step $t+\Delta t$. Therefore, all the terms in (28) are computed during the solution stage of FEM.

5.2 Flux and Electromotive Force Calculations

These results can be easily computed using (13) and (14).

5.3 Inductance Calculations

Inductance has a geometrical meaning when all the materials exhibit a linear behaviour. In other case, several non-coincident definitions can be taken into account [12]. Using the definition based on flux linkages, self-inductance can be calculated in windings made of stranded conductors as:

$$\begin{aligned} L_i^{\,t+\Delta t} &= \frac{N_i\, \phi^{t+\Delta t}}{i_i^{\,t+\Delta t}} = \frac{l\, N_i \left\{\omega_i\right\}^T \left\{A^{t+\Delta t}\right\}}{\dfrac{\left\{\omega_i\right\}^T \left[S^{t+\Delta t}\right] \left\{A^{t+\Delta t}\right\}}{N_i \left\{\omega_i\right\}^T \left\{\omega_i\right\}}} \\ &= l\, N_i^2 \left\{\omega_i\right\}^T \left[S^{t+\Delta t}\right]^{-1} \left\{\omega_i\right\} \end{aligned} \tag{29}$$

In the same way mutual inductance can be computed using the flux linkage definition:

$$\begin{aligned} M_{ij}^{\,t+\Delta t} &= \frac{N_i\, \phi_i^{\,t+\Delta t}}{i_j^{\,t+\Delta t}} = \frac{l\ N_i \left\{\omega_i\right\}^T \left\{A^{t+\Delta t}\right\}}{\dfrac{\left\{\omega_j\right\}^T \left[S^{t+\Delta t}\right] \left\{A^{t+\Delta t}\right\}}{N_j \left\{\omega_j\right\}^T \left\{\omega_j\right\}}} \\ &= l\ N_i\, N_j \left\{\omega_i\right\}^T \left[S^{t+\Delta t}\right]^{-1} \left\{\omega_j\right\} \end{aligned} \tag{30}$$

As the current is computed in (28), self and mutual inductances are calculated using the phase vectors and the stiffness matrix, that can be dependent on time. Stiffness matrix is affected by the B-H curve when it exhibits a nonlinear behaviour.

In [13] the inductance computation in case of solid conductors is stated.

6 EXAMPLES

A software package has been developed implementing (24). It has been tested with many examples of increasing complexity over transformers, considering several balanced and unbalanced connections in primary and secondary –one phase and three phase transformers–, steady and unsteady states, sinusoidal and nonsinusoidal voltages and current sources (balanced and unbalanced); two of these examples are presented below.

6.1 Steady-state

The model is applied to a 3 kVA three-phase ΔY transformer (figure 2). The transformer is supplied with a balanced nonsinusoidal three-phase voltage (figure 3), generated with a programmable AC power source. These waveforms are applied to the software package using files obtained from an oscilloscope. The secondary is loaded with an unbalanced three-phase RL impedance. Figure 4 shows the simulated currents in steady state, and figure 5 shows the computed and measured currents.

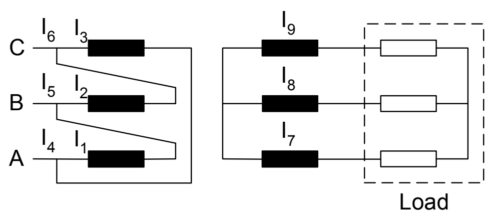

Figure 2. The transformer is delta-star connected.

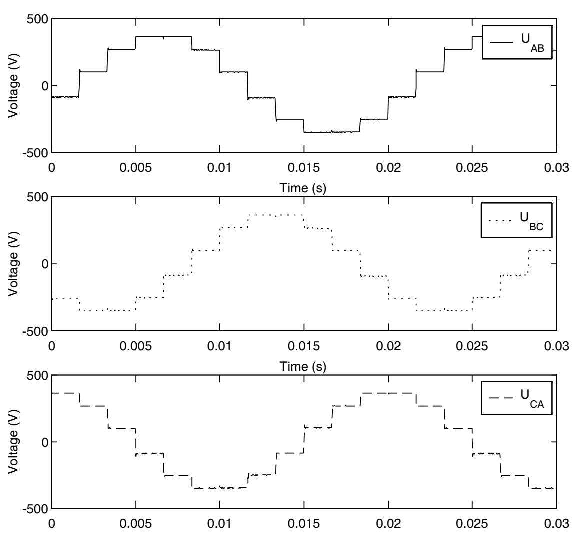

Figure 3. Line-to-line supplied voltages.

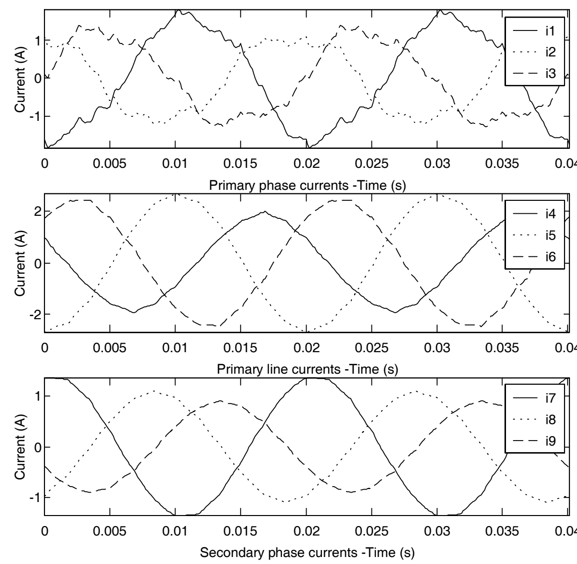

Figure 4. Primary and secondary computed currents.

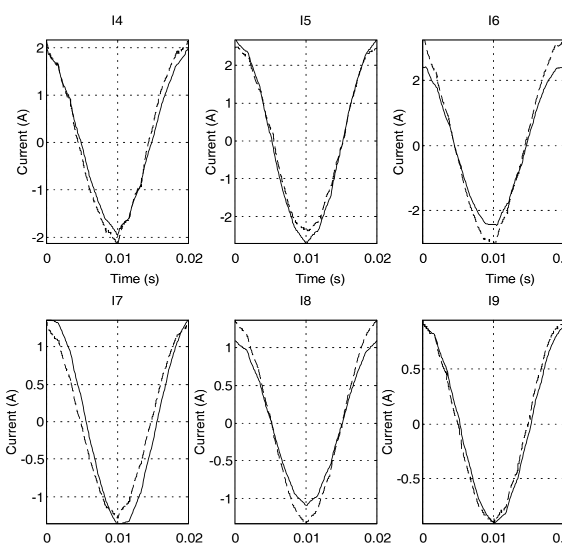

Figure 5. Computed (solid) and measured (dashed) currents.

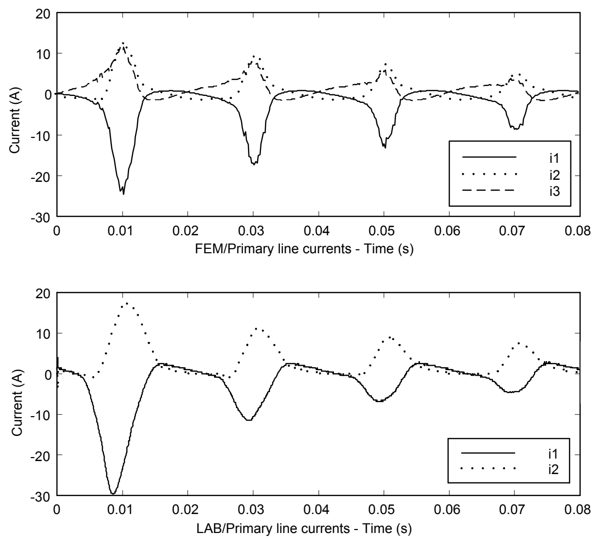

Figure 6. Computed (FEM) and measured (LAB) currents.

6.2 Non steady-state

The transformer is now YY connected, and loaded with the same unbalanced three-phase RL impedance. Figure 6 shows the computed and measured primary currents; the discrepancies between them are due to the unfavourable conditions of the high supply voltage and the instant chosen for the switching-on of the transformer.

7 CONCLUSIONS

A generalized approach for FEM and external circuits coupling has been presented considering both types of conductors simultaneously, stranded and solid respectively. This model has been established using a bottom-up methodology. The meanings of the terms of the particularized equation (23) have been generalized to obtain the terms in general equation (24). Indeed, when a problem containing a particularized example is selected, the expressions of these are included in the terms of (24).

Current, magnetic flux, electromotive force and inductance definitions have been obtained using some terms of (24), simplifying their notation and speeding-up their computation during the solution stage rather than during the postprocessing stage.

Finally, the approach, implemented in a software package developed by the authors, has been applied to several cases. The results have shown a good agreement with those measured in the lab.

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