A Partial Solution of MoM Matrices Based on Characteristic Basis Functions and its Application to On-Board Antennas Positioning

Jaime Laviada, Javier Gutiérrez-Meana, Marcos R. Pino, and Fernando Las-Heras

Área de Teoría de la Señal y Comunicaciones (TSC-UNIOVI)
University of Oviedo, Campus de Viesques, 33203 Gijón, Asturias, Spain
jlaviada@tsc.uniovi.es, jmeana@tsc.uniovi.es, mpino@tsc.uniovi.es, flasheras@tsc.uniovi.es

Abstract **—** A new technique, called incomplete Gauss-Jordan elimination (IGJE), is presented and hybridized with the characteristic basis function method (CBFM) to enable the partial solution of the method of moments (MoM) matrix. As a consequence, the goal of this technique is its application to optimization problems in electrically large scenarios where multiple but similar configurations need to be analyzed, since our method performs these analyses with a considerable reduction in the computational time and also memory. The term “similar” refers to the fact that the original structure is split into different blocks and modifications in the geometry (inserting, eliminating, or changing elements) of only a specific set of these blocks are allowed throughout the optimization process. In particular, we take advantage of this technique to analyze the optimum emplacement of an antenna on a given structure (e.g. a ship or airplane) with just one analysis. An example of an airplane antenna positioning is shown to illustrate the procedure.

Index Terms **—** Characteristic basis functions, method of moments, on-board antennas.

I. INTRODUCTION

Many usual simulations in the electromagnetic engineering involve the evaluation of small modifications on certain parts of a given structure. In this paper, this fixed part of the total structure will be referred to as *mother structure*. Among other examples, we can cite the tuning of antennas by systematically changing the dimensions of certain small metallic additions (e. g. a stub or a parasitic element). Another example of the previous situation is the study of the radiation pattern of an on-board antenna. In the latter case, the antenna is placed on multiple positions in order to study if the elements of the environment, such as the fuselage of an airplane or the tower of a ship, affect the radiation characteristics of the antenna. This results in multiple analyses of structures that share most of the geometry. In the method of moments context, several techniques have been proposed to enable the partial solution of the *mother structure* so that the analysis, after changing a part in the rest of the geometry, can be resumed.

The work on this kind of analysis has been dispersed along the years. However, a common step has been to speed up the computation of the entries of the matrix. For this purpose, the matrix with all the possible metallic parts is calculated and stored. Thus, if a substructure with some eliminated parts has to be analyzed, the related matrix is easily computed by removing the rows and columns associated to the removed metallic parts. After this stage, several techniques have been proposed in order to avoid solving the equations system from the scratch for each analysis. Although there is not an exhaustive comparison among these methods in the literature, their performance is expected to be close to each other since they are based on similar foundations.

Among the pioneering works on this field, we can cite the “add-on” method [1–3] that produces an incremental computation of the inverse of the impedance matrix based on the Sherman-Morrison formula and, therefore, enabling the access to partial solutions.

The research on this field has also been focused on modifying common solving techniques for equations systems such as the LU factorization

[4] or the Gaussian elimination [5] yielding schemes very appropriate for the optimization of microwave circuits and antennas [6]. Another efficient partial solver was proposed in [7] for the optimization via genetic algorithms. The work on partial solving has also been continued more recently for the optimization of non-intuitive planar structures [8, 9].

As previously mentioned, an extreme case of small modifications on a large structure is the positioning of on-board antennas. In this problem, the *mother structure* is typically an aircraft or ship where the antenna must be placed. The derived structures to be analyzed would be composed by the airplane or the ship and the antenna located in multiple positions. Since the aforementioned partial solving techniques must deal with the entire MoM matrix, they are typically limited in the electrical size of the *mother structure* and, therefore, their direct application to electrically large structures is not possible.

The analysis of on-board antennas has been traditionally tackled by hybrid methods that combine asymptotic techniques to analyze the *mother structures* with full-wave methods to analyze the antenna and its nearest environment. Among these techniques, we can cite hybridization of the method of moments with the physical optics [10] or with the uniform theory of diffraction [11].

Current tendencies are oriented towards the application of acceleration schemes such as the fast multipole method [12] since they do not require the approximations introduced by the asymptotic methods. These techniques are very powerful and enable the full-wave analysis of antennas on large electromagnetic structures [13–15]. However, they are based on iterative schemes rather than on direct solutions complicating the hybridization with the aforementioned partial-solving techniques.

A remarkable technique to analyze on-board antennas has been proposed in [16]. Authors decompose the geometry into multiple domains, one for the large and fixed structure and the remainder for the on-board antennas. Each domain is analyzed with full-wave methods in order to compute a scattering matrix relating the incident field on its boundary to the radiated field. Thus, if the antenna is changed, the method only has to recompute the scattering matrix of a small domain. However, if the antenna is moved, then the scattering matrix of the large domain must be also recomputed which can be very time-consuming.

Next sections are arranged as follows. Firstly, we present the incomplete Gauss-Jordan elimination (IGJE) that enables a partial solution compatible with the compression techniques that will be treated later. Afterwards, the compression of the matrix with the characteristic basis function method (CBFM) is detailed and its integration for the efficient evaluation of multiple antenna positioning is considered. In the results section, the application of the IGJE to antenna design is illustrated by means of the optimization of a reconfigurable antenna. The study of a VHF dipole at multiple positions on an airplane is considered to illustrate the capabilities of the inclusion of the IGJE into a locally modified CBFM method for dealing with electrically large structures. Finally, the conclusions are summarized and discussed.

II. INCOMPLETE GAUSS-JORDAN ELIMINATION SCHEME

A. Description of the method

The Gauss-Jordan elimination is a simple and well-known scheme to calculate the inverse of matrices. Although its application to solve equations systems is also possible, it is not usual because it requires a higher number of operations than other schemes (e.g., approximately three times more operations than a LU factorization).

This scheme pursues the reduction of the matrix into a row echelon form by means of basic operations row by row. Thus, in the n-th step, the Gauss-Jordan elimination seeks the first non-zero element (pivot element) in the n-th row, normalizes the row with this element and adds multiples of that row to the rest of rows in order to obtain zeros in the column of the pivot element. Equivalent operations are performed on the right hand side (RHS). At the end of the algorithm, the matrix is reduced to a row echelon form and, therefore, the solution of the equations system is straightforward.

In the case of MoM matrices, it will be proved later that the pivot element is always located in the diagonal so that the matrix is progressively transformed into the identity matrix. The Gauss-Jordan elimination without pivoting can be expressed using Matlab notation as:

```
for n = 1 : N
  Z(n,n:end) = Z(n,n:end) / Z(n,n);
  V(n,:) = V(n,:) / Z(n,n);
  for m = 1 : N
     if n == m
       continue;
     end
     Z(m,n:end) = Z(m,n:end) -…
       …Z(m,n)*Z(n,n:end);
     V(m,:) = V(m,:) - Z(m,n)*V(n,:);
  end
end
```

In the above description, `Z` is the matrix of the equations system with `N` unknowns and `V` is a matrix containing the right hand terms. At the end of the algorithm, the matrix `V` contains the solution to the equations system. After the step n, all the entries of the first n columns of matrix `Z` are zeros except the entries of the diagonal that are equal to one; so, it is not necessary to operate with these columns when adding row multiples and, therefore, it saves some CPU cycles.

Next, we detail how the conventional Gauss-Jordan elimination can be modified to obtain the incremental solution of a MoM problem. Let us split the geometry under analysis into two parts, one containing the *mother structure*, and the other one containing the possible metallic additions (e.g. parasitic elements) to the *mother structure*, with N_m and N_f unknowns respectively (typically $N_m >> N_f$). The total number of unknowns is given by $N = N_m + N_f$.

We will assume that the matrix is rearranged to place the unknowns related to the *mother structure* in the first rows and columns and the unknowns belonging to each metallic additions are, also, arranged consecutively. After N_m iterations, the matrix will reach the form shown in Fig.1a, i.e., the identity matrix is placed in the first $N_m \times N_m$ entries and the elements under this submatrix are zeros. Since we have carried out exactly the same operations on the initial $N_m \times N_m$ submatrix as if we consider the isolated *mother structure*, the first N_m coefficients of the RHS contain the solution of the isolated *mother structure*. If we continue the elimination until solving the unknowns associated to the next metallic addition, we will obtain the solution for the *mother structure* plus that addition. On the other hand, if we had swapped the rows and columns to place the rows and columns related to a different addition at the position of the submatrix related to the first addition (see Fig. 1b), we would have obtained the solution of the *mother structure* plus that different addition. Thus, it is very efficient to analyze the impact of different extensions of the *mother structure* just by resuming from the point of Fig.1a.

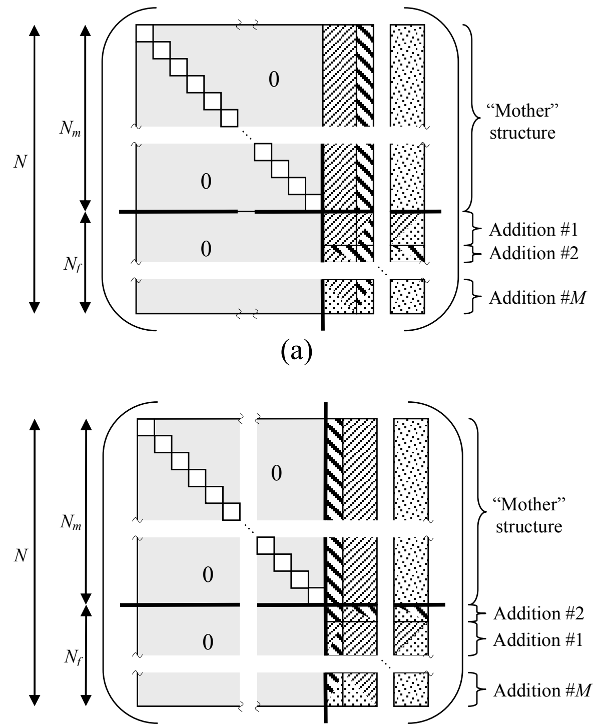


Fig. 1. Equations system matrix after solving the *mother structure*: (a) without any change; (b) after swapping rows and columns of the entries associated to the first and second additions.

Once the previous concepts have been detailed, it is straightforward to prove that the pivot element must be located in the diagonal. After k steps, the first $k \times k$ entries in the MoM matrix must be equal to the identity matrix (solution to the problem considering the k first basis functions). When another step is performed, a solution must exist as it corresponds to the problem of $k+1$ basis functions. Since the first $k \times k$ entries are the identity matrix, there is only one choice for the pivot element: its location in the

$k+1$ position of the diagonal. The reasoning can be extended to any arbitrary number of steps.

B. Complexity of the incomplete Gauss-Jordan elimination

The computational time cost to solve the *mother structure* can be approximated by the cost of solving the entire structure and, therefore, it is $O(N^3)$, as the usual direct solution schemes. On the other hand, the computational cost for resuming the analysis is only $O(N_f^2 N)$. Hence, this strategy involves a first analysis that is time-consuming but the penalty to analyze the rest of the combinations is very low.

The storage of the initial matrix and the time for the first analysis limit the size of the structures to be studied. In the next section, this problem will be mitigated by including recent developments related to reductions in the number of unknowns using efficient sets of macro basis functions to model parts of the geometry.

It is important to remark that after solving the *mother structure* (first N_m steps), it is only necessary to store the last N_f columns of the matrix, saving a large amount of memory in case we need to store several of these matrices (e.g. to carry out frequency sweeps).

III. ANTENNA POSITIONING WITH CHARACTERISTIC BASIS FUNCTIONS

In the previous section, we have seen how the MoM matrix can be partially solved. However, this methodology requires initially to store the entire MoM matrix so it is limited to electrically small geometries. In this section, we will show how the previous method in combination with the CBFM can be applied to the positioning of on-board antennas.

It is important to remark that in this case we must deal with parts of the geometry where two configurations are possible (with or without antenna) rather than analyzing the effect of adding metallic regions.

In particular, the problem under analysis consists in the study of one antenna for S positions on an electrically large structure. This situation is depicted in Fig. 2a for a cactus antenna placed on a ship for $S=5$. In other words, we pursue to analyze a structure where a given set of blocks can potentially contain the antenna yielding an *inter-block* strategy.

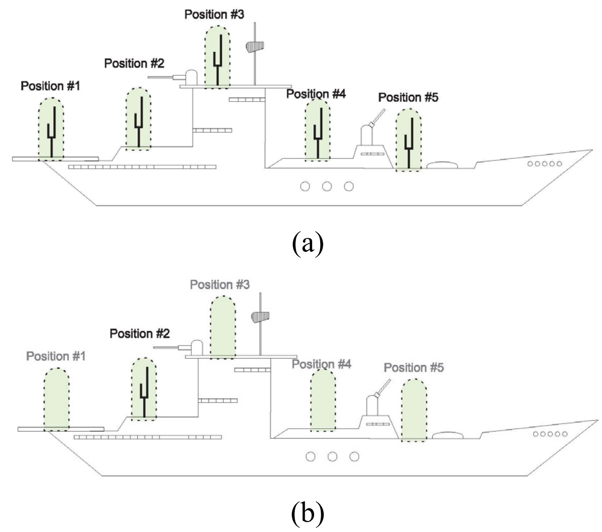


Fig. 2. Antenna positioning: a) all possible locations; b) antenna placed at position #2.

For this purpose, we will combine the aforementioned IGJE with the characteristic basis function method (CBFM) that has shown very desirable properties for the analysis of locally modified structures in the past [17].

The CBFM was developed with the aim of reducing the number of degrees of freedom when analyzing electromagnetic problems. The method is based on the use of the so-called "characteristic basis functions" (CBFs) which are defined on non-overlapped fragments of the geometry that are referred to as blocks.

These new basis functions are usually much less than the conventional low-level basis functions. Thus, this reduction enables the *direct solution* of problems much larger than the ones that can be analyzed with the conventional MoM. In addition, the direct solution avoids any potential convergence issue that could happen in the conventional fast algorithms which are based on iterative schemes. On the other hand, the current direct approaches can deal with a smaller number of unknowns if compared with the most powerful iterative schemes such as the FMM [12]. The CBFM is widely described in the literature and, therefore, we will only explain briefly the parts related to the current work. The reader is referred to [18-21] and the references therein for further details about the method.

Once the CBFs have been defined in terms of the low-level basis functions as in [20] and their coefficients are arranged by columns, then the matrix containing the interactions between the CBFs in the *m*-th observation block and the *n*-th source block is computed as:

$$\mathbf{Z}_{mn}^{(1)} = \mathbf{J}_m^t \mathbf{Z}_{mn}^{(0)} \mathbf{J}_n \ , \qquad (1)$$

where $\mathbf{Z}_{mn}^{(0)}$ contains the reaction terms among the low-level basis functions of the *m*-th and *n*-th blocks. Thus, the original submatrix $\mathbf{Z}_{mn}^{(0)}$ is converted to a $K_m \times K_n$ block whose dimensions are typically around one order of magnitude less than the dimensions of the original matrix. It is also remarkable that the CBFM can be generalized through a multilevel formulation that enables to achieve higher compression rates for electrically large structures [22]. This compression together with the block partitioning is the keys to efficiently modify the geometry. In order to accomplish our goal, we split the geometry into T blocks. These blocks are classified into two types depending if they contain a possible location of the antenna, that will be referred to as antenna positioning block (AP block), or not. The other blocks contain the *mother structure* and, therefore, they will be referred to as *mother structure* blocks (MS blocks). Hence, there will be S AP blocks containing possible locations of the antenna and R blocks containing regular pieces of geometry, where $T = R + S$.

From the previous discussion, it can be inferred that both MS blocks and AP blocks can be created as in the conventional CBFM [18-21]). The only special rule that we have followed in this paper is that the volume enclosed by the antenna must be contained in one single CBFM block. It can be easily carried out by grouping all the blocks, in which the antenna spans, into one single block.

The AP blocks can be made of two possible geometries: i) with the antenna; ii) without the antenna. Since the regular CBFM enables the analysis of only one location in each simulation, an appropriate setup must be carried out to avoid unnecessary computation.

The equations system is built considering both configurations for the AP blocks at the same time yielding an augmented impedance matrix. It is important to remark that as a consequence of considering both configurations for the AP blocks, the CBFs for those blocks must be also computed for both block geometries. For the sake of clarity, the MS blocks are numbered from 1 to M while the AP blocks ranges from $R+1$ to $R+S$ for cases without antenna and from $R+S+1$ to $R+2S$ for configurations with antenna.

The augmented matrix contains the interaction among the MS blocks (subindex M), the AP blocks without the antenna (subindex P), and the AP blocks with the antenna (subindex P'):

$$\mathbf{Z}_a^{(1)} = \begin{pmatrix} \mathbf{Z}_{MM} & \mathbf{Z}_{MP} & \mathbf{Z}_{MP'} \\ \mathbf{Z}_{PM} & \mathbf{Z}_{PP} & \mathbf{Z}_{PP'} \\ \mathbf{Z}_{P'M} & \mathbf{Z}_{P'P} & \mathbf{Z}_{P'P'} \end{pmatrix}, \qquad (2)$$

where the subscript a stands for "augmented" and the submatrices contain the interactions due to the corresponding blocks, i. e.:

$$\mathbf{Z}_{\alpha\beta} = \begin{pmatrix} \mathbf{Z}_{a,b}^{(1)} & \mathbf{Z}_{a,b+1}^{(1)} & \cdots & \mathbf{Z}_{a,b+d}^{(1)} \\ \mathbf{Z}_{a+1,b}^{(1)} & \mathbf{Z}_{a+1,b+1}^{(1)} & \cdots & \mathbf{Z}_{a+1,b+d}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_{a+c,b}^{(1)} & \mathbf{Z}_{a+c,b+1}^{(1)} & \cdots & \mathbf{Z}_{a+c,b+d}^{(1)} \end{pmatrix}, \qquad (3)$$

where α and β can be equal to M, P or P'. The blocks involved in each submatrix, can be easily computed considering the aforementioned scheme numbering for the blocks. In addition, we provide the Tables 1a and 1b that present the values for the indexes a, c and b, d for each possible combination of α and β.

Contrary to the common uses of the partial solving techniques, the augmented matrix contains blocks that are overlapped, i.e., a block corresponding to a piece of geometry and another one corresponding to the same geometry but with the antenna, and, therefore, it is expected to be singular (or at least with a high condition number) yielding a meaningless solution. Nevertheless, as we will see, we never employ the entire matrix to acquire the final solution.

Table 1a: Values of a and c for (3)

	a	c
$\alpha = M$	1	$R-1$
$\alpha = P$	$R+1$	$S-1$
$\alpha = P'$	$R+S+1$	$S-1$

Table 1b: Values of b and d for (3)

	b	d
$\beta = M$	1	$R-1$
$\beta = P$	$R+1$	$S-1$
$\beta = P'$	$R+S+1$	$S-1$

After computing (2), the IGJE is applied until finishing the first M blocks, which corresponds to the MS blocks, so that the augmented matrix becomes:

$$\tilde{\mathbf{Z}}_a^{(1)} = \begin{pmatrix} \mathbf{I} & \tilde{\mathbf{Z}}_{MP} & \tilde{\mathbf{Z}}_{MP'} \\ 0 & \mathbf{Z}_{PP} & \mathbf{Z}_{PP'} \\ 0 & \mathbf{Z}_{P'P} & \mathbf{Z}_{P'P'} \end{pmatrix}, \qquad (4)$$

where $\mathbf{I}$ is the identity matrix and the tilde symbol ~ marks that the submatrix has been modified by the application of the IGJE.

Once the previous stage has been finished, the rows and columns related to the entries of the block with and without antenna (labelled with a prime or not, respectively, in Fig. 3) can be swapped to consider the solution for a particular position. For example, if we want to solve the configuration where the second AP block contains the antenna and the other AP blocks are antenna-free, we swap the rows and columns of the corresponding positions (see Fig. 3b). After that, the IGJE can be resumed in order to solve the following S blocks and, thus, to obtain the solution for the given position. This step can be repeated once and again for each block in order to obtain the solution for each position. It is important to notice that we are only solving the first $R+S$ blocks (possibly reordered) of the augmented matrix and, therefore, the solution is not expected to be singular as for the entire augmented matrix.

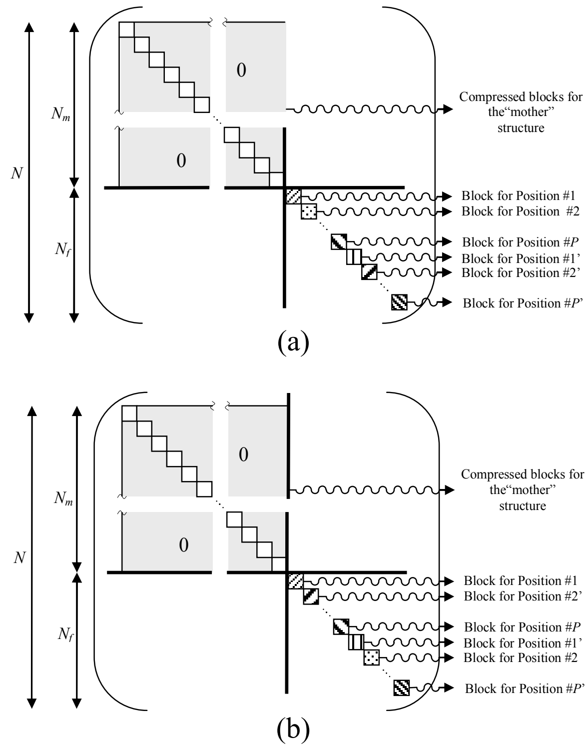


Fig. 3. IGJE application to the positioning of antennas. For the sake of clarity, off-diagonal blocks have been omitted. The primed positions indicate that the block contains the antenna:
a) block status after the first stage of the IGJE;
b) block status before resuming the IGJE for analyzing the antenna in position #2.

Regarding the overall accuracy of the method, it is important to observe that the method will yield the same results as the conventional CBFM as it can be inferred from the previous description. Hence, the accuracy of the method is only limited by the accuracy of the CBFM which has been widely demonstrated in the literature [18-22].

IV. NUMERICAL RESULTS

In this section, we firstly validate the IGJE algorithm in order to study its capacities for the design of antennas by optimizing a reconfigurable antenna. Next, the application of the partial solution of MoM matrices to electrically large structures is illustrated through the analysis of

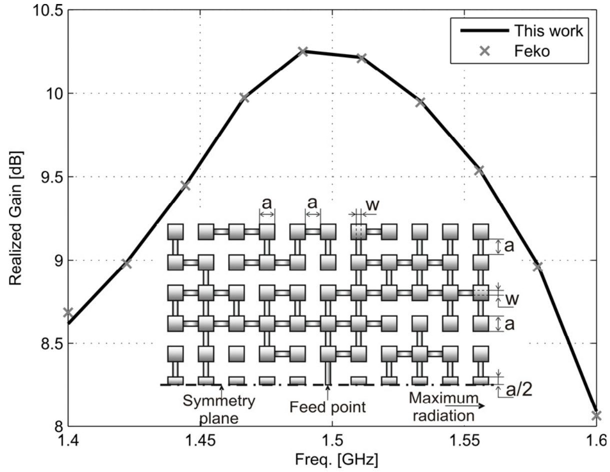


Fig. 4. Realized gain on the design band for the reconfigurable antenna.

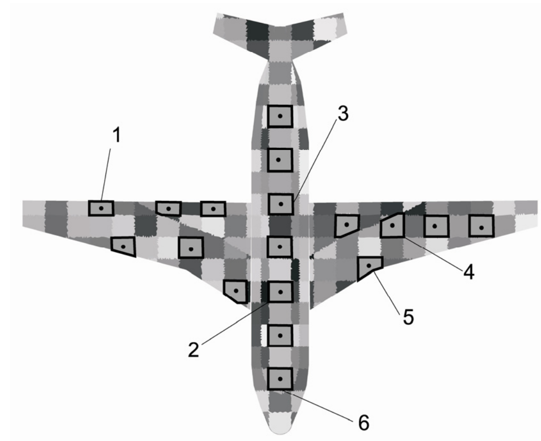


Fig. 5. Block partitioning of the airplane. The border of the blocks containing the possible location of the monopole has been highlighted.

different emplacements of a VHF monopole antenna on an airplane with the objective of finding the optimum positioning.

The times shown in this section correspond to the execution of the code on a CPU AMD Opteron® at 2.4 GHz. Rao-Wilton-Glisson (RWG) basis functions are used to expand the currents.

A. Antenna optimization with the incomplete Gauss-Jordan elimination

This example deals with the design of a reconfigurable antenna to radiate in a given direction. The antenna is based on the model presented in [23] and it consists of a free-standing symmetric array of square patches (see Fig. 4) that can be connected by strips ($a = 90$ mm; $w = 30$ mm) yielding 2^{104} possible combinations. The existence or absence of connections between patches affects the maximum radiation direction enabling multiple radiation pattern configurations.

The *mother structure* consists of the patches ($N_m = 2429$) and the feeding strip, while the additions region is formed by the rest of the strips ($N_f = 624$). The incomplete solution for the *mother structure* consumes 209 s. After that, the analysis of a certain configuration takes 1.58 s in the average case (calculated over 1000 runs) and 13.95 s in the worst case (all patches connected). The LU decomposition for the best case (no strips) spends 16.23 s. The antenna is optimized by using a genetic algorithm to radiate in the endfire direction in the band 1.4-1.6 GHz. Figure 4 shows the obtained configuration and the realized gain (the gain including the mismatch [23]) with a very good agreement with the commercial software Feko [24].

B. Monopole positioning on an aircraft

In order to illustrate the capabilities of the combination of the CBFM with the partial solving scheme in the context of electrically large structures, we will consider the positioning of a VHF $\lambda/4$ monopole at 120MHz on an airplane-like geometry (length of 50.8 m and wingspan of 61 m). The model is discretized using 66,476 RWG basis functions, so the regular MoM application is not feasible.

In order to apply the CBFM, the airplane is fragmented into 247 blocks and we choose 18 possible blocks to contain the monopole antenna. Seven of the 18 blocks are chosen along the highest part of the fuselage of the airplane since more equilibrated radiation patterns are expected on these positions. The remainder eleven blocks are chosen on the wings in order to also check the performance of the monopole on them. This partitioning as well as the antenna positions are depicted in Fig. 5. The CBFs generation is carried out illuminating each block with 400 plane waves and applying a SVD threshold of 10^{-2}.

If we consider the analysis of the CBFM for a single antenna, e.g. position #2 in Fig. 5, the

CBFM reduces the problem to 9145 unknowns and it is solved in 6522 s. Thus, the study for all the locations with the CBFM would require approximately 117,396 s.

On the other hand, if the problem is solved for all the 18 locations of the monopole in a single analysis with the locally modified CBFM plus the IGJE, it results in a total number of unknowns of 10,175 (the increment with respect to 9145 is due to the duplication of the AP blocks). In this case, the number of unknowns belonging to the *mother structure* (the airplane without the blocks with a possible location of the monopole) is $N_m = 8383$.

The total time until reaching the partial solution corresponding to the *mother structure* – i.e., CBFs generation, matrix filling and first N_m steps of the IGJE– is 16,430 s. After that, each position can be analyzed in only 30 s and, therefore, the total time to analyze the 18 positions is 16,970 s. The computational times for both strategies as well as the time for analyzing 18 monopole positions with the CBFM are detailed in Table 2.

The radiation patterns are shown in Fig. 6 for the six positions numbered in Fig. 5 together with the results provided by Feko in order to validate the accuracy of the radiation patterns. It is important to remark that all the possible locations of the antenna are on the upper part of the fuselage and wings in order to provide coverage during the taxiing on the ground. Thus, a mask of $\pm 30^o$ has been plotted in the elevation patterns to facilitate the graphical inspection. According to this mask, the most suitable diagram is the one corresponding to placing the antenna on the nose (position #6).

V. CONCLUSIONS AND DISCUSSION

The partial solving techniques available in the literature have been traditionally limited to electrically small structures because they need to deal with the entire MoM matrix. In this paper, we have presented a new partial solving technique based on the incomplete Gauss-Jordan elimination scheme and its extension to face electrically large problems. This extension has been carried out by combining the IGJE with the expansion of the currents by means of characteristic basis functions. Then, the number of unknowns is considerably reduced so the matrix can be efficiently manipulated in order to store and solve the MoM equations system. This fact together with the CBFM block partitioning have been exploited to efficiently analyze multiple given configurations of a certain set of blocks (*inter-block* strategy). The applicability of the IGJE plus the CBFM has been illustrated by considering the evaluation of positioning an antenna at multiple locations of an airplane, and has proven to be a highly efficient technique for optimization problems involving electrically large structures.

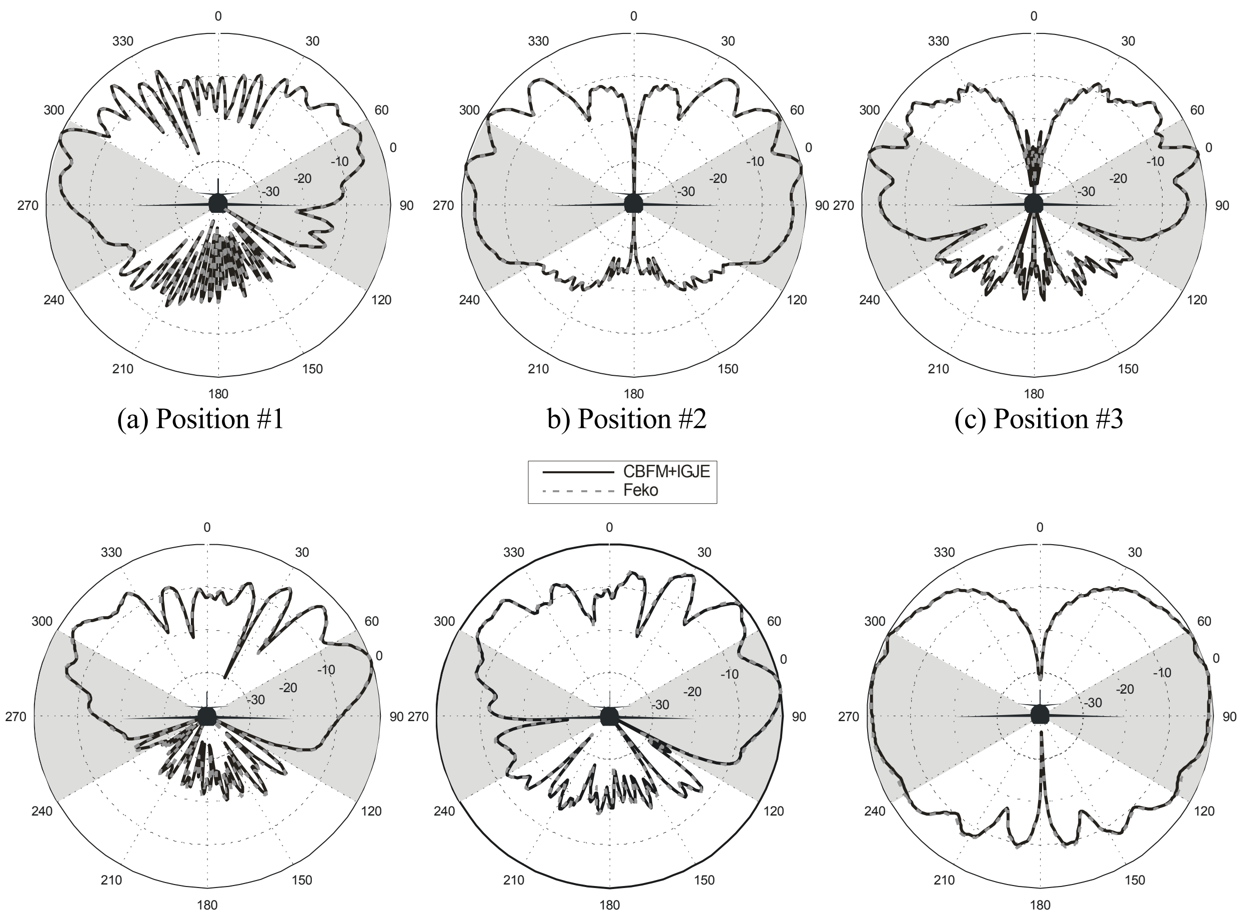
Table 2: Computational times for the analysis of the on-board VHF monopole on an airplane using the conventional CBFM and the modified CBFM

	CBFM 1 monopole	CBFM 18 monopoles	CBFM+IGJE 18 monopoles
CBFs generation	2635 s	2635 s × 18	2828 s
Matrix filling	3265 s	3265 s × 18	3863 s
LU factorization	622 s	622 s × 18	-
IGJE 1st stage	-	-	9739 s
IGJE 2nd stage	-	-	30 s × 18= 540s
Total time	6522 s	6522 s × 18 = 117,396 s	16,970 s

The future research lines are focused on including the multilevel formulation with the aim of dealing with even larger problems. The authors are also working in the effective modification of the content inside a block (intra-block strategy) in order to make possible the fine-tuning of the position of the antenna.

ACKNOWLEDGMENT

This work has been supported by Ministerio de Ciencia e Innovación of Spain /FEDER under projects TEC2008-01638/TEC (INVEMTA) and CONSOLIDER-INGENIO CSD2008-00068 (TERASENSE); by Unión Europea-Fondo Europeo de Desarrollo Regional under project EQP06-015; and by Cátedra Telefónica-Universidad de Oviedo.



(a) Position #1 b) Position #2 (c) Position #3

(a) Position #4 b) Position #5 (c) Position #6

Fig. 6. Normalized radiation pattern in the roll-plane for different antenna locations. The position number corresponds to patterns of Fig. 5.

REFERENCES

[1] R. Kastner, "On matrix partitioning, the Sherman-Morrison-Woodbury technique and the add-on method," *Antennas and Propagation Society International Symposium*, 1989. AP-S. Digest, pp. 15–18, vol.1, Jun. 1989.

[2] R. Kastner, "An 'add-on' method for the analysis of scattering from large planar structures," *IEEE Trans. Antennas and Propagation*, vol. 37, no. 3, pp. 353-361, Mar. 1989.

[3] R. Kastner, "Analysis of microstrip antenna structures using the 'add-on' technique," *IEEE Trans. Antennas and Propagation*, vol. 38, no. 1, pp. 114-117, Jan. 1990.

[4] K. Naishadham and P. Misra, "Order recursive method of moments (ORMoM) for iterative design applications," *IEEE Trans. Microwave Theory Tech.*, vol. 44, no. 12, pp. 2595-2604, Dec. 1996.

[5] P. Misra and K. Naishadham, "Order-recursive Gaussian elimination (ORGE) and efficient CAD of microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 44, no. 12, pp. 2166-2173, Dec. 1996.

[6] L. Alatan, M. Aksun, K. Lablebicioglu, and M. Birand, "Use of computationally efficient method of moments in the optimization of printed antennas," *IEEE Trans. Antennas and Propagation*, vol. 47, no. 4, pp. 725-732, Apr. 1999.

[7] J. M. Johnson and Y. Rahmat-Samii, "Genetic algorithms and method of moments (GA/MoM) for the design of printed antennas," *IEEE Trans. Antennas and Propagation*, vol. 47, no. 10, pp. 1606-1614, Oct. 1999.

[8] D. Cormos, R. Loison, and R. Gillard, "A multistructure method of moments for EM optimizations," *Microwave Opt. Technol. Letters*, vol. 40, no. 2, pp. 114-117, Jan. 2004.

[9] D. Cormos, R. Loison, and R. Gillard, "Fast optimization and sensitivity analysis of nonintuitive planar structures," *IEEE Trans. Microwave Theory Tech.*, vol. 53, no. 6, pp. 2019-2025, Jun. 2005.

[10] F. Obelleiro, J. M. Taboada, J. L. Rodríguez, and J. M. Bertolo, "HEMCUVI: a software package for the electromagnetic analysis and design of radiating systems on board real platforms," *IEEE Antennas and Propagation Mag.*, vol. 44, no. 5, pp. 44-61, Oct. 2002.

[11] E. Koper, W. Wood, and S. Schneider, "Aircraft antenna coupling minimization using genetic algorithms and approximations," *IEEE Trans. Aerospace and Electronic Systems*, vol. 40, no. 2, pp. 742–751, 2004.

[12] R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas and Propagation Mag.*, vol. 53, no. 3, pp. 7–12, Jun. 1993.

[13] A. Thain, G. Peres, J.-P. Estienne, G. Sylvand, P. Cailleu, P. Benjamin, I. Terrasse, G.-P. Piau, and G. Sabanowski, "Numerical modelling of aircraft antenna installations," *EuCAP 2007. The Second European Conference on, Antennas and Propagatio,n* pp. 1–6, Nov. 2007.

[14] M. Heckler and A. Dreher, "Analysis of planar 4x4 microstrip antenna arrays installed on airframes," *EuCAP 2007. The Second European Conference on, Antennas and Propagation,* pp. 1–5, Nov. 2007.

[15] C. Penwell, "Prediction of VHF radiation patterns on gulfstream aircraft," *Wireless Communications and Applied Computational Electromagnetics, 2005. IEEE/ACES International Conference on*, pp. 519–522, Apr. 2005.

[16] A. Barka and P. Caudrillier, "Domain decomposition method based on generalized scattering for installed performance of antennas on aircraft," *IEEE Trans. Antennas and Propagation*, vol. 55, no. 6, pp. 1833–1842, Jun. 2007.

[17] E. Lucente, R. Mittra, and A. Monorchio, "An efficient procedure to analyze locally-modified scattering problems by using the characteristic basis function method," *Antennas and Propagation Society International Symposium, AP-S 2008. IEEE*, pp. 1–4, Jul. 2008.

[18] R. Mittra, "Characteristic Basis Function Method (CBFM) – An Iteration-free Domain Decomposition Approach in Computational Electromagnetics," *Applied Computational Electromagnetic Society* (*ACES) Journal*, vol. 24, no. 2, pp. 204-223, 2009.

[19] R. Maaskant, R. Mittra, and A. Tijhuis, "Fast Solution of Multi-Scale Antenna Problems for the Square Kilometre Array (SKA) Radio Telescope using the Characteristic Basis Function Method (CBFM)," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 24, no. 2, pp. 174-188, 2009.

[20] E. Lucente, A. Monorchio, and R. Mittra, "An iteration-free MoM approach based on excitation independent characteristic basis functions for solving large multiscale electromagnetic scattering problems," *IEEE Antennas and Propagation Mag.*, vol. 56, no. 4, pp. 999–1007, Apr. 2008.

[21] C. Delgado, M. F. Cátedra, and R. Mittra, "Application of the characteristic basis function method utilizing a class of basis and testing functions defined on NURBS patches," *IEEE Trans. Antennas and Propagation*, vol. 56, no. 3, pp. 784–791, Mar. 2008.

[22] J. Laviada, M. R. Pino, F. Las-Heras, and R. Mittra, "Solution of electrically large problems with multilevel characteristic basis functions," *IEEE Trans. Antennas and Propagation*, vol. 57, no. 10, pp. 3189–3198, Oct. 2009.

[23] L. Pringle, P. H. Harms, S. P. Blalock, G. N. Kiesel, E. J. Kuster, P. G. Friederich, R. J. Prado, J. M. Morris, and G. Smith, "A reconfigurable aperture antenna based on switched links between electrically small metallic patches," *IEEE Trans. Antennas and Propagation*, vol. 52, no. 6, pp. 1434–1445, Jun. 2004.

[24] *Feko User's manual*, EM Software & Systems, 2008, http://www.feko.info/.

Jaime Laviada was born in Gijón, Spain, in 1982. He received the degree in Ingeniería de Telecomunicación and Ph.D. from the Universidad de Oviedo, Spain, in 2005 and 2010, respectively. In 2006, he joined the Área de Teoría de la Señal y Comunicaciones, Universidad de Oviedo. He has been a Visiting Scholar in the Electromagnetics and Communications Laboratory of the Pennsylvania State University, University Park, during 2007 and 2008. His main research interests are in numerical efficient techniques and antenna pattern synthesis.

Javier Gutiérrez-Meana was born in Gijón, Spain, in 1982. He received his M.S. and Ph.D. degrees in electrical engineering from the University of Oviedo (Spain) in 2005 and 2010, respectively. He joined the R and D department of CTIC Foundation in 2005, and since 2006, he is a Research Assistant with

the Area of Theory of Signal and Communications (University of Oviedo). He was a Visiting Researcher at The Gordon CenSSIS – Northeastern University (Boston) in 2008. His interests and research studies are focus on the evaluation of electromagnetic coverage in rural/urban electrically large scenarios by means of high frequency techniques.

Marcos Rodríguez Pino was born in Vigo, Spain, in 1972. He received the M.Sc. and Ph.D. degrees in electrical engineering from the University of Vigo, Vigo, Spain, in 1997 and 2000, respectively. Since 2001, he is with the Electrical Eng. Department, University of Oviedo, where he currently is Associate Professor teaching courses on electromagnetic fields. During 1998, he was a Visiting Scholar at the ElectroScience Laboratory, The Ohio State University, Columbus, OH. From 2000 to 2001, he was Assistant Professor at the University of Vigo. His research areas are radar cross section, rough surface scattering, and applied mathematics for computational electromagnetics.

Fernando Las-Heras was born in Zaragoza, Spain. He received the M.S. degree in 1987 and the Ph.D. degree in 1990, both in telecommunication engineering, from the Universidad Politécnica de Madrid (UPM). From 1988 to 1990, he was a National Graduate Research Fellow. From 1991 to 2000, he held a position of Associate Professor at the Dept. of Señales, Sistemas y Radiocomunicaciones (Dept. of Signal, Systems and Radiocommunications) of the UPM. From 2001 to 2003, he held a position of Associate Professor at the Dept. de Ingeniería Eléctrica (Dept. of Electrical Engineering) of the Universidad de Oviedo, heading the research group Teoría de la Señal y Comunicaciones (Signal Theory and Communications) at that University. From December 2003, he holds a Full-Professor position at the Universidad de Oviedo where he has been Vice-dean for the Telecommunication Engineering degree at the Escuela Politécnica Superior de Ingeniería de Gijón (Technical School of Engineering at Gijón), Spain. He has been Visiting Researcher at Syracuse University, New York, and Visiting Lecturer at the Universidad Nacional de Ingeniería, Peru, and ESIGELEC, France. He has authored over 200 technical journal and conference papers in the areas of antenna design; the inverse electromagnetic problem with application to diagnostic, measurement, and synthesis of antennas; propagation and computational electromagnetics and engineering education.