

Parallelization of MLFMA with Composite Load Partition Criteria and Asynchronous Communication

Huapeng Zhao^{1,2}, Jun Hu¹, and Zaiping Nie¹

¹Department of Microwave Engineering
University of Electronic Science and Technology of China, Chengdu 610054, China

²School of Electrical and Electronic Engineering
Nanyang Technological University, 50 Nanyang Avenue, Singapore, 639798
huapengzhao@uestc.edu.cn

Abstract— This paper describes an efficient parallelization of the multi-level fast multi-pole algorithm (MLFMA) for fast solution of very large scale electromagnetic scattering problems. Computation in the MLFMA can be divided into several stages. To accomplish load balance at any time, load partition criteria are adjusted according to different features of every phase. Meanwhile, an asynchronous communication method is designed to overlap the communication with computation and thus the communication cost in parallelization is reduced. Numerical results show that good parallel efficiency is obtained in the presented parallelization of MLFMA. With our parallel MLFMA, a scattering problem with nearly 5, 300, 000 unknowns is solved in about six hours using 8 CPUs on SGI O350 server.

Index Terms—Large scale scattering problem, Parallel multi-level fast multi-pole algorithm.

I. INTRODUCTION

The multi-level fast multi-pole algorithm (MLFMA) [1] obtains computational and storing complexity of $O(N \log(N))$, where N is the number of unknowns. The low complexity of MLFMA makes it possible to solve large scale scattering problems on a single PC. The challenges in integral equation methods are to further improve the efficiency and solve very large scale problems quickly. Therefore, optimizing techniques for MLFMA were designed, and variants of MLFMA were developed as well. These improved the

performance of MLFMA greatly and were well documented in [2]–[6]. However, before the application of parallel technique, the solution of very large scale problems was still a challenge due to its requirement for huge computing resource. Offering strong computing ability and huge storing space, parallel technique is widely used for efficient solution of large scale electromagnetic problems [7]–[11]. With a parallel MLFMA (PMLFMA), Chew et al. [7] solved problems with ten millions unknowns, and they reported performance analysis of their parallelization in [8].

As several different computing stages are involved in MLFMA, it's hard to obtain a high parallel efficiency without careful design of load partition. This paper presents a composite load partition method, where load distributing criteria are adjusted according to the features of every computation stage. In that way, the computing and storing load balance is ensured at any time. To reduce the communication cost, an asynchronous communication method is designed so that the communication is overlapped with computation and time spent on communication is reduced. Numerical results show that the proposed parallel method can achieve good parallel efficiency and has strong ability to solve very large scale problems quickly and accurately. With our PMLFMA, a scattering problem with nearly 5, 300, 000 unknowns was solved on SGI O350 supercomputer.

The rest of this paper is organized as follows. A brief review of MLFMA is given in Section II. After that, load partition criteria

and communication management in our parallelization method are described in Section III. Numerical results are given in Section IV and finally conclusions are made in Section V.

II. BRIEF REVIEW OF MLFMA

Consider the electric field integral equation (EFIE) for conducting objects:

$$\hat{\mathbf{t}}(\mathbf{r}) \cdot \int_S \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS' = \frac{4\pi i}{k\eta} \hat{\mathbf{t}}(\mathbf{r}) \cdot \mathbf{E}^{inc}(\mathbf{r}), \quad (1)$$

where S is the surface of the object, and \mathbf{r} and \mathbf{r}' are the observing and source points on S , respectively. $\overline{\mathbf{G}}$ is the dyadic Green's function and \mathbf{J} represents the current density. $\hat{\mathbf{t}}(\mathbf{r})$ is the unit vector tangential to S at \mathbf{r} and η is the free space characteristic impedance. \mathbf{E}^{inc} is the incident electric field. Adopting proper expansion basis and invoking some kind of matching process [12], equation (1) can be discretized into a matrix equation

$$\sum_{l=1}^N A_{jl} a_l = b_j, \quad j = 1, 2, 3, \dots, N, \quad (2)$$

where A_{jl} is the interaction impedance between the j th and the l th bases, a_l is the l th unknown expansion coefficient for the current density, and b_j is computed from \mathbf{E}^{inc} . In our PMLFMA, the curvilinear RWG basis [13] and Galerkin's matching process are used. Applying vector addition theorem, the dyadic Green's function in equation (1) can be expanded as

$$\overline{\mathbf{G}}(\mathbf{r}_j, \mathbf{r}_l) = \frac{ik}{4\pi} \int d^2 \hat{\mathbf{k}} (\overline{\mathbf{I}} - \hat{\mathbf{k}}\hat{\mathbf{k}}) e^{ik \cdot (\mathbf{r}_{jm} - \mathbf{r}_{lm'})}. \quad (3)$$

$$\alpha_{mm'}(\mathbf{r}_{mm'} \cdot \hat{\mathbf{k}}), \quad |\mathbf{r}_{mm'}| > |\mathbf{r}_{jm} - \mathbf{r}_{lm'}|,$$

where \mathbf{r}_j and \mathbf{r}_l are position vectors of the observing point and source point, \mathbf{r}_m and $\mathbf{r}_{m'}$ are position vectors of the field group and source group centers, respectively. \mathbf{r}_{jm} is the vector connecting \mathbf{r}_j and \mathbf{r}_m , and the same definition applies for \mathbf{r}_{jm} , $\mathbf{r}_{lm'}$, and $\mathbf{r}_{mm'}$. $\alpha_{mm'}$ is termed as the translator, whose definition and expression can be found in [1]. In fast multi-pole method (FMM), the left hand side of equation (2) is expressed as the

sum of two terms, which are

$$\sum_{l=1}^N A_{jl} a_l = \sum_{m \in NG} \sum_{l \in G_m} A_{jl} a_l + \frac{ik}{4\pi} \int d^2 \hat{\mathbf{k}} \mathbf{V}_{fmj}(\hat{\mathbf{k}}) \cdot \sum_{m' \in FG} \alpha_{mm'}(\hat{\mathbf{k}} \cdot \mathbf{r}_{mm'}) \sum_{l \in G_m} \mathbf{V}_{sm'l}^*(\hat{\mathbf{k}}) a_l, \quad j \in G_m, \quad (4)$$

where $\mathbf{V}_{sm'l}(\hat{\mathbf{k}})$ and $\mathbf{V}_{fmj}(\hat{\mathbf{k}})$ are named as the aggregation and disaggregation, respectively, whose expressions can be found in [1]. NG and FG represent the near region groups and far region groups, respectively, and G_m is the m th group. For convenience, equation (4) is written in matrix form as

$$\overline{\mathbf{A}} \cdot \mathbf{a} = \overline{\mathbf{A}}_{near} \cdot \mathbf{a} + \overline{\mathbf{U}} \cdot \overline{\mathbf{T}} \cdot \overline{\mathbf{V}} \cdot \mathbf{a}, \quad (5)$$

where $\overline{\mathbf{A}}_{near}$, $\overline{\mathbf{U}}$, $\overline{\mathbf{T}}$ and $\overline{\mathbf{V}}$ represent the near group interaction matrix, aggregation matrix, translation matrix and disaggregation matrix, respectively, and \mathbf{a} is the vector consisting of the unknown current expansion coefficients. The MLFMA is generalization of FMM in a multi-level structure, where equation (5) can be rewritten as

$$\overline{\mathbf{A}} \cdot \mathbf{a} = \overline{\mathbf{A}}_{near} \cdot \mathbf{a} + \overline{\mathbf{U}}_{NL} \cdot \overline{\mathbf{T}}_{NL} \cdot \overline{\mathbf{V}}_{NL} \cdot \mathbf{a} + \sum_{l=2}^{NL-1} \overline{\mathbf{U}}_l \cdot \overline{\mathbf{T}}_l \cdot \overline{\mathbf{V}}_l \cdot \mathbf{a}, \quad (6)$$

where $\overline{\mathbf{V}}_l$, $\overline{\mathbf{T}}_l$, and $\overline{\mathbf{U}}_l$ are the aggregation matrix, translation matrix and disaggregation matrix in the l th level, respectively. NL is the total number of levels used in MLFMA. $\overline{\mathbf{V}}_l$ and $\overline{\mathbf{U}}_l$ are computed by interpolation for $l < NL$. $\overline{\mathbf{T}}_l$ can be obtained either by real-time computation or from the pre-filled matrix [5]. Given a problem with N unknowns, MLFMA obtains the computing and storing complexity of $O(N \log N)$.

III. PARALLEL MLFMA

To solve scattering problems using MLFMA, the following steps are involved: (1). Filling of $\overline{\mathbf{A}}_{near}$; (2). Computation of $\overline{\mathbf{U}}_{NL}$ and $\overline{\mathbf{V}}_{NL}$; (3). Implementation of matrix vector multiplication using equation (6), including upward pass, translation stage, downward pass and direct matrix vector multiplication. For convenience, terms defined in [8] have been

followed in our paper and more details about these terms can be found in [8]. To achieve good load balance at any time, the load partition criteria should be adjusted according to features of the above computing stages. Section III-A describes the load partition criteria at every computation stage. For some stages, load partition will introduce indispensable communication, which is managed with an asynchronous communication method described in section III-B.

A. Load Partition Criteria

This section mainly discusses the problem of load balance. The total number of processes is assumed to be p .

a) Filling of $\bar{\mathbf{A}}_{near}$: As long as numbers of interaction elements are equal on all processes, the computation and storage load will be balanced at this stage. The matrix $\bar{\mathbf{A}}_{near}$ is constituted by the self group and near group interaction elements, which are computed group by group and list by list, respectively. Assume the total number of interaction elements is Z_{count} , the number of self group interaction elements assigned on process l is Z_{self}^l and the number of non-self group interaction elements to be allotted is Z_{nself}^l . Since the non-empty groups are distributed uniformly among the p processes, Z_{self}^l s on all processes are fixed. However, Z_{self}^l s on p process are different from each other, because numbers of basis functions enclosed by different groups are different. To ensure total numbers of interaction elements are equal on all processes, the near group lists are distributed from process 1 to $p-1$ abiding by the criteria that $Z_{self}^l + Z_{nself}^l$ approximately equals Z_{count}/p ($1 \leq l \leq p-1$). Then the rest near group lists are assigned to process p and $Z_{self}^p + Z_{nself}^p$ will approximately equal Z_{count}/p automatically.

b) Computation of $\bar{\mathbf{U}}_{NL}$ and $\bar{\mathbf{V}}_{NL}$: $\bar{\mathbf{U}}_{NL}$ and $\bar{\mathbf{V}}_{NL}$ are the receiving and radio

patterns of the basis functions. A simple and effective partition method is to distribute the basis functions equally among p processes. However, this may make the basis functions belonging to the same group be distributed on different processes, which will introduce a mass of communications in the aggregation and disaggregation phase. Therefore, according to [14], the basis functions are first ordered by their Morton Key values and then distributed equally to the p processes, so that the communication cost is minimized.

c) Upward and Downward phase: A simple parallelizing method for these two stages is to construct an independent tree structure on every process according to distribution of basis functions. This method introduces no communication, but load on the p processes may be unequal, because numbers of boxes on all processes may be different. Another disadvantage of this partition method is that the inner and outer waves of one group may be repeatedly stored on two or more processes, which obviously wastes the memory.

Since computation is conducted group by group in every level during these two stages, groups in all levels should be distributed equally among the p processes, and then the load on all processes will be equal. Since a parent group and its child groups may be distributed on different processes, this partition method will introduce indispensable communication, which will be approached with the asynchronous communication method described in Section III-B.

d) Translation Phase: The computing unit at this phase is the far group list. The load balance is easily implemented by distributing all the far group lists equally among the p processes. However, two boxes belonging to a far list may be stored on different processes. Communication is therefore needed to accomplish the translation phase, which will be discussed in Section III-B.

e) Direct Matrix Vector Multiplication: As long as the numbers of near group interaction elements on all the processes are equal, the computing load in this stage is balanced. Therefore, load balance for this

stage is realized at the stage of filling \bar{A}_{near} .

B. Communication Management

As discussed in Section III-A, communication is required in upward, downward and translation phases. The boxes and far lists which need communication are defined as non-local boxes and lists, while those need no communication are defined as local boxes and lists. To reduce the communication cost, an asynchronous communication method is designed. Asynchronous communication is non-block communication. The term non-block means the communication doesn't block the

computation. Assuming in one communication, process m needs the information on process n , communication in our PMLFMA is conducted

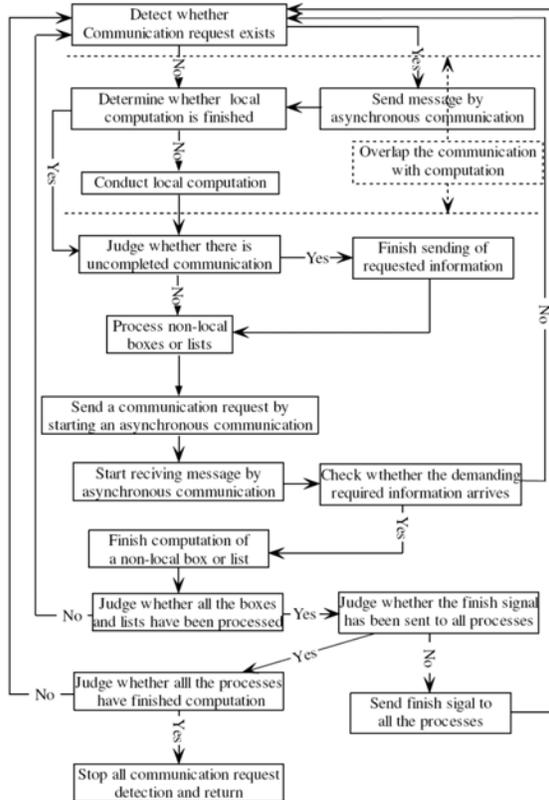


Fig. 1. Flow chart of the asynchronous communication method.

as follows: process m sends a request to process n and then continues the computation requiring no nonlocal information. Once process n detects the request from process m , it sends the requested information by starting

an asynchronous communication and then continues its own computation. The above communication method can be demonstrated using Fig. 1, which shows that the communication is overlapped with computation. This flow chart is applicable to all the stages discussed in section III-A, just by substituting the corresponding computation module into the flow chart. Specific flow chart for every phase is hence omitted here for simplicity. For the detection, send and request actions in Fig. 1, one can implement them with corresponding message passing interface functions.

IV. NUMERICAL RESULTS

In all the simulations of this section, the group size of the finest level is set to be 0.3λ . To investigate the parallel efficiency of the proposed parallel scheme, two simulations are conducted on SGI O350 server. The parallel efficiency η is computed as

$$\eta = T_1/T_p/p \times 100\%, \quad (7)$$

where p is the number of processes, T_1 is the solving time using one process and T_p is the solving time using p processes.

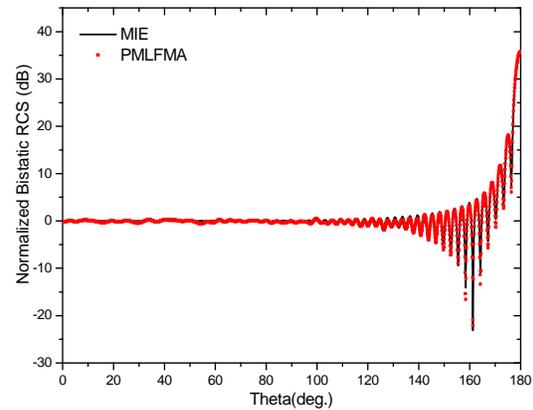


Fig. 2. Normalized bistatic RCS of a 20λ diameter conducting sphere.

First, the bistatic radar cross section (RCS) of a 20λ diameter conducting sphere is solved. The number of unknowns is about 0.2 million. A six-level MLFMA is used. The numerical result and parallel efficiency are shown in Fig. 2 and Fig. 3, respectively. In Fig. 3, it is seen that the parallel efficiency with eight processes is more than 80%. It is also shown that the parallel efficiency decreases as the number of processes increases. This is

because more communication is introduced as the number of processes increases. However, the proposed parallel scheme can still obtain a parallel efficiency of 70% when the number of processes increases to 10.

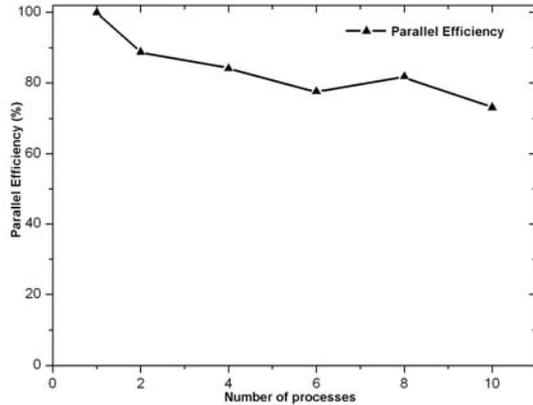


Fig. 3. Parallel efficiency for solving bistatic RCS of a 20λ diameter conducting sphere.

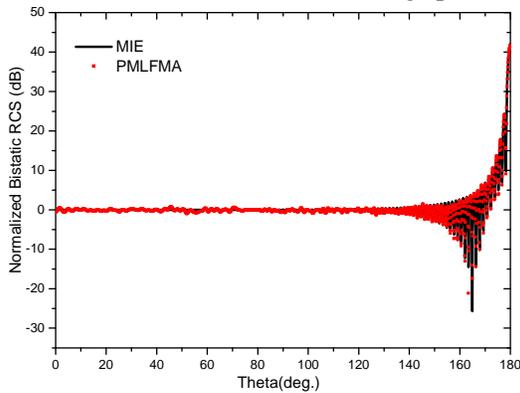


Fig. 4. Normalized bistatic RCS of a 40λ diameter conducting sphere.

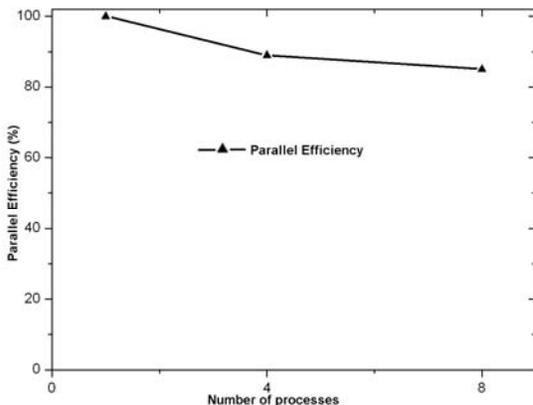


Fig. 5. Parallel efficiency for solving bistatic RCS of 40λ diameter conducting sphere.

Second, the problem scale is enlarged. The

object is a 40λ diameter conducting sphere. The number of unknowns is about 1.8 million, and a six-level MLFMA is used. Fig. 4 and Fig. 5 show the numerical result and parallel efficiency, respectively. Compared with Fig. 3, Fig. 5 shows a higher parallel efficiency, which means that the parallel efficiency increases as the problem scale increases. Therefore, the proposed parallel scheme is scalable.

The reported parallel efficiency is not as high as that of Chew’s group [8] for the following two reasons. First and most important, the translation phase is not well managed in our parallelization. In the translation phase, Chew’s group achieved load balance by partitioning the far-field patterns equally among all processors for coarse levels, communication is therefore totally avoided in the translation phase for coarse levels. However, in our parallelization, mass communication is introduced in the translation phase to ensure load balance, especially for the coarse levels. Nevertheless, with the designed asynchronous communication method, our parallelization still obtains good parallel efficiency, which justifies the

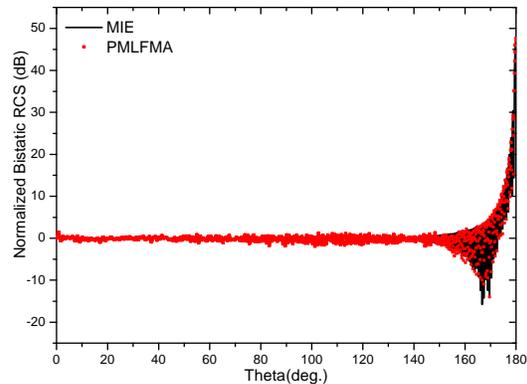


Fig. 6. Normalized bistatic RCS of a 100λ diameter conducting sphere.

effectiveness of our communication method. The second reason might be differences between platforms. In the SGI Origin 2000 supercomputer adopted in [8], each processor has a clock speed of 250 MHz. In our supercomputer, the clock speed of each processor is of 1 GHz. If communication cost is given, parallel efficiency will be higher in systems with lower clock speed.

Finally, using the proposed PMLFMA, the

bistatic RCS of a 100λ diameter conducting sphere is solved with 5,287,752 unknowns on SGI O350 supercomputer. For this simulation, a nine-level MLFMA is used. It takes 8 CPUs 22147 seconds to solve this problem. This simulation demonstrates the ability of our PMLFMA in solving very large scale problems.

V. CONCLUSIONS

This paper presented an efficient parallel scheme for MLFMA. Composite load partition criteria have been designed to achieve load balance at any time and an asynchronous communication method has been developed to reduce the communication cost. Numerical results show the good parallel efficiency achieved by the proposed parallel scheme. With our PMLFMA, a scattering problem with about 5.3 million unknowns has been solved, demonstrating its ability in solving very large scale problems. Our future work is to improve the load partition for the translation phase in coarse levels so that higher parallel efficiency can be achieved.

ACKNOWLEDGEMENT

This work is supported partly by NSFC (No. 60601008), research funding (No. 9140A03010708DZ0235, 9140A07030109DZ02), and the Programme of Introducing Talents of Discipline to Universities under Grant b07046.

REFERENCES

- [1] J. Song and W. Chew, "Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering," *Microwave and Optical Tech. Lett.*, vol. 10, no. 1, pp. 14-19, Sept. 1995.
- [2] W. Chew, T. Cui and J. Song, "A FAFFA-MLFMA algorithm for electromagnetic scattering," *IEEE Transactions on Antennas and Propagation*, vol. 50, no. 11, pp. 1641-1649, Nov. 2002.
- [3] T. Cui, W. Chew and G. Chen et al., "Efficient MLFMA, RPFMA, and FAFFA algorithms for EM scattering by very large structures," *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 3, pp. 759-770, Mar. 2004.
- [4] J. Hu and Z. Nie et al., "Fast solution of scattering from conducting structures by local MLFMA based on improved electric field integral equation," *IEEE Transactions on Electromagnetic Compatibility*, vol. 50, no. 4, pp. 940-945, Nov. 2008.
- [5] S. Velamparambil and W. Chew, "A fast polynomial representation for the translation operators of an MLFMA," *Microwave and Optical Tech. Lett.*, vol. 28, no. 5, pp. 298-303, Mar. 2001.
- [6] H. Zhao, J. Hu and Z. Nie, "Block storing method for efficient storage of near group impedance in MLFMA," *Electronics Letters*, vol. 44, no. 20, pp. 1171-1173, Sept. 2008.
- [7] S. Velamparambil, W. Chew and J. Song, "10 million unknowns: is it that big?" *IEEE Antennas and Propag. Mag.* vol. 45, no. 2, pp. 43-58, Apr. 2003.
- [8] S. Velamparambil and W. Chew, "Analysis and performance of a distributed memory multilevel fast multipole algorithm," *IEEE Transactions on Antennas and Propagation* vol. 53, no. 8, pp. 2719-2727, Aug. 2005.
- [9] I. Gonzalez, E. Garcia, F. S. de Adana and M. F. Catedra, "MONURBS: A parallelized fast multipole multilevel code for analyzing complex bodies modeled by NURBS surfaces," *Applied Computational Electromagnetics Society Journal*, vol. 23, no. 2, pp. 134-142, June 2008.
- [10] T. Iwashita, M. Shimasaki and J. Lu, "Parallel ICCG solvers for a finite element eddy-current analysis on heterogeneous parallel computation environment," *Applied Computational Electromagnetics Society Journal*, vol. 22, no. 2, pp. 195-200, July 2007.
- [11] W. R. Dearholt and S. P. Castillo, "Electromagnetic scattering problems utilizing a direct, parallel solver," *Applied Computational Electromagnetics Society Journal*, vol. 22, no. 3, pp. 395-413, Nov. 2007.
- [12] R. F. Harrington, *Field computation by Moment Methods*, Robert E. Krieger, FL, 1968.
- [13] J. Hu, Z. Nie and X. Gong, "Solving electromagnetic scattering and radiation by FMM with curvilinear RWG basis," *Chinese Journal of Electronics*, vol. 12, no. 3, pp. 457-460, July 2003.
- [14] J. Dull, K. Gallivan, J. Song and W.

Chew, "Parallel fast multipole capacitance solver," IEEE Antennas and Propagation Symp., Atlanta, June 1998.



Huapeng Zhao was born in Hebei province, China, in 1983. He received the B.E. and M. Eng. degrees in electronic engineering from the University of Electronic Science and Technology of China, Chengdu, China, in July 2004 and March 2007, respectively. He is now working towards his Ph.D. degree in the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. His research interests include numerical methods and electromagnetic measurements.



Jun Hu received the B.S., M.S., and Ph.D. degrees in electromagnetic field and microwave technique from the Department of Microwave Engineering, University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 1995, 1998, and 2000, respectively.

During 2001, he was a Research Assistant in the Center of Wireless Communication,

City University of Hong Kong. He is currently a Full Professor in the School of Electronic Engineering, UESTC. He is the author or coauthor of more than 100 technical papers. His current research interests include computational electromagnetics, electromagnetic scattering and radiation, and electromagnetic compatibility.



Zaiping Nie received the B.S. degree in radar engineering and M.S. degree in electromagnetic field and microwave technique from Chengdu Radio Engineering Institute, now University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 1968 and 1981, respectively.

From 1987 to 1989, he was a visiting scholar in the Electromagnetic Laboratory, University of Illinois, Urban Champagne. He is currently a Full Professor in the School of Electronic Engineering, UESTC. He is the author or coauthor of more than 300 technical papers. His current research interests include antenna theory and technique, field and wave in inhomogeneous media, computational electromagnetics, electromagnetic scattering, and inverse scattering.