# Predicting MoM Error Currents by Inverse Application of Residual E-Fields

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*Abstract*— **This paper presents a methodology to predict *a posteriori* the error associated with a Method-of-Moments solution. The discussion is limited to a one dimensional pulse basis function wire-based implementation, but is easily extended. A Formulation for Error Prediction based on the relationship between the error in the boundary conditions and the error in the solution is presented, and validated by an over-segmented problem. The formulation is then used in a normally-segmented solution to predict the error by means of a linear interpolation of the calculated current which results in a smoother boundary condition error. The results show that this normally-segmented methodology predicts the error current within 5% of the "accurate" error current obtained by a 20:1 oversegmentation of the problem. Further work needs to be performed to extend this to the multidimensional case, although no technical difficulties are expected with this.**

## I. Introduction

In a method of moments (MoM) solution to an electromagnetic problem, the structure currents are calculated to ensure that the boundary conditions are satisfied in some sense, usually at specific match points (point matching) or over specific domains in an average, or weighted average, manner.

The boundary condition to be satisfied—when using a MoM for perfectly conducting wires—is that the total tangential **E**-field should be zero at all points on the wire. A current which ensures zero tangential **E**-fields in a continuous sense on all wires would be accurate in accordance with the uniqueness theorem.

Many researchers have recognized that the accuracy of a method must be related to how well the boundary conditions are met by a specific solution (Hsaio & Kleinman 1996, Meyer & Davidson 1996). The exact relationship between the error in the boundary conditions and the error in the observable quantities has not been defined and has also not been used to gain an estimate of solution (or method) accuracy. This study investigates the possibility of getting such a relationship and then formulates approximations which allow its incorporation in MoM (and other methods) with the lowest computational overhead.

## II. Formulation for Error Prediction (FEP)

The obvious relationship between the error in the boundary conditions and the error in the solution is quite clear in the following formulation for perfectly conducting wires (Thiele 1973):

Suppose we have obtained a current solution, $I(s)$, where

$$I(s) = I_a(s) + I_e(s) \qquad (1)$$

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and $I_a(s)$ is the accurate solution and $I_e(s)$ is the error associated with the solution $I(s)$.

If we have the continuous (and assumed accurate for the moment) relationship between the current and the excitation then

$$L_{op}I_a(s) = V(s) \qquad (2)$$

where $L_{op}$ is the linear operator (Pocklington's equation is an example for wire problems) which defines the excitation tangential to the wire, $V(s)$. Pocklington's equation is usually solved using the Method-of-Moments. In the case of a point matching solution, this results in a tangential **E**-field error of zero *only* at the match points. At any other point on the structure there *will* be a tangential **E**-field error.

If pulse weighting functions were used in the MoM solution, then it is likely that there will be a tangential **E**-field error at any point on the structure—the tangential **E**-field error will be zero only in the average sense.

The tangential **E**-field *errors* found on the structure are defined to be the *residuals*. The tangential **E**-field from the inaccurate current, $I(s)$, is:

$$L_{op}I(s) = V(s) + V_e(s) \qquad (3)$$

where $V_e(s)$ is the tangential **E**-field residual (or error) on the wires. Combining equations (1) and (3), and from the linearity of the operator and superposition it follows that

$$L_{op}I_e(s) = V_e(s) \qquad (4)$$

The inverse of equation (4) will allow the error current $I_e(s)$ to be obtained from the **E**-field residual (error), $V_e(s)$.

$$I_e(s) = L_{op}^{-1}V_e(s) \qquad (5)$$

We shall call the above mathematical development the Formulation for Error Prediction (FEP) for later reference. Once the error currents have been found, the errors in secondary parameters such as radiation pattern or input impedance are easily quantified. This may be done, for example, by using the error current to obtain near/far fields at the same points as the full solution values were computed. This will produce "error fields" which places a bound on the error in the computed field values. Errors in other parameters such as input impedance, etc. may be obtained in a similar manner.

This somewhat trivial proof indicates that the error in any solution may, in principle, be calculated exactly provided that:

- The residual **E**-field over the structure can be calculated continuously and accurately.

- An accurate continuous operator $L_{op}$ and its inverse exist.

In practice the following obstacles to using this result are apparent:

- Calculation of the residual **E**-field over the structure can be computationally time intensive.
- The operator is normally discretized as an interaction matrix and hence is neither continuous nor accurate.

## III. Using the FEP to obtain accurate error currents.

The graphs which follow illustrate an investigation using the FEP for a specific problem. A dipole antenna of length $\ell = 0.5\lambda$ and with radius, $a = 0.005\lambda$ was excited with an incident **E**-field of 1 V/m.

A simple MoM program was written which uses pulse basis functions and point matching. The convergence of the method is shown in Figure 1. The method clearly converges to a stable answer when around 100 to 200 segments are used for the dipole. The Formulation for Error Prediction (FEP) is illustrated by considering the current from the 200 segment problem to be the "accurate" current. Two "inaccurate" currents were obtained by solving the same problem with only 10 and 20 segments.

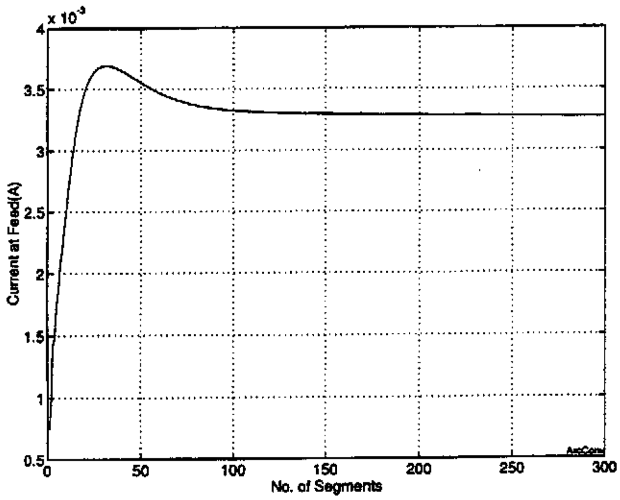


Fig. 1. The feed current magnitude for a variation in number of segments of a dipole with $\ell = 0.5\lambda$ and $a = 0.005\lambda$ using pulse basis functions and point matching.

The residual **E**-fields from these two inaccurate solutions were then obtained at 200 points across the dipole (i.e. at many more points than the match points). Figure 2 shows the residual **E**-field magnitude for a 10 segment solution and a 20 segment solution, which clearly demonstrates that the boundary conditions are exactly met at the match points, but also shows a significant error (50V/m for a 1V/m excitation) at other points. The residual **E**-fields in Figure 2 can be applied as an *excitation vector* on the same geometry divided into 200 segments to yield the error current in accordance with the FEP as stated in equations (4) and (5). Figure 3 shows this graphically. It is clear

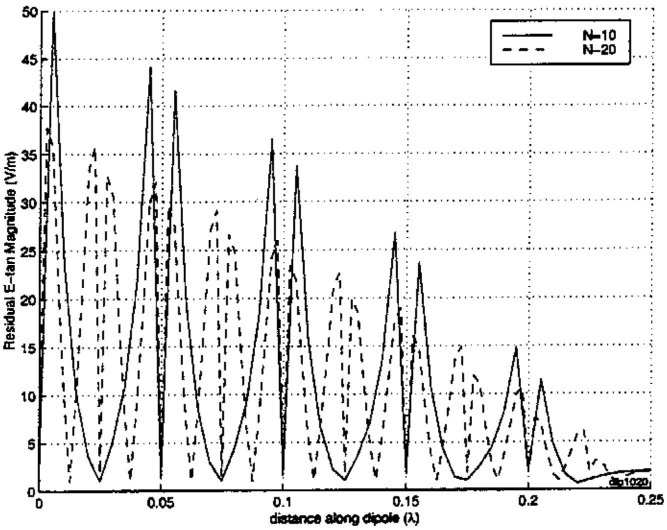


Fig. 2. The magnitude of the residual **E**-fields obtained for a 10 and 20 segment dipole with $\ell = 0.5\lambda$ and $a = 0.005\lambda$ using pulse basis functions and point matching.

from these figures that sum of the inaccurate current (obtained from the 10 segment solution) and the error current (obtained by applying the residual **E**-fields via FEP) is practically equal to the "accurate" current obtained from the 200 segment case. The 200 segment inverse interaction matrix hence represents an accurate and pseudo-continuous inverse operator, $L_{op}$, and the 200 point discretization of the residual **E**-field a pseudo-continuous representation on the error, $E_e(s)$. Figure 3 illustrates the principle behind

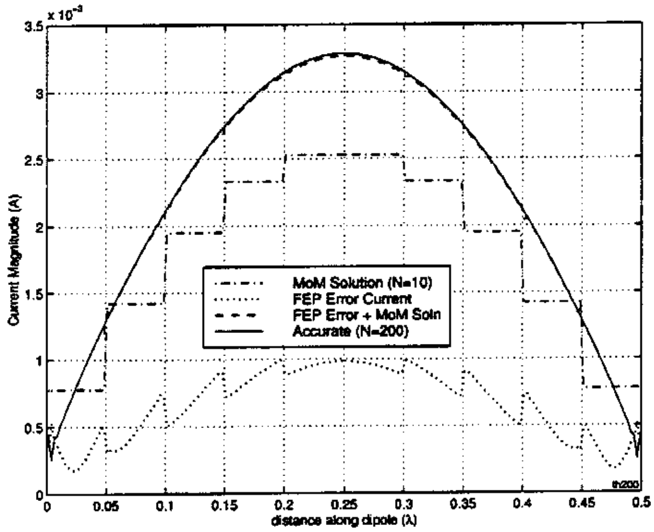


Fig. 3. The current magnitude to prove the FEP for a 10 segment dipole with $\ell = 0.5\lambda$ and $a = 0.005\lambda$ using pulse basis functions and point matching.

error prediction by inverse application of the residual **E**-fields as excitations to produce an error current. The computational effort does not render it suitable as a general tool for error prediction—oversegmenting a problem by a factor of 20 in order to obtain an error estimate is clearly unproductive! The technique may however, be useful as

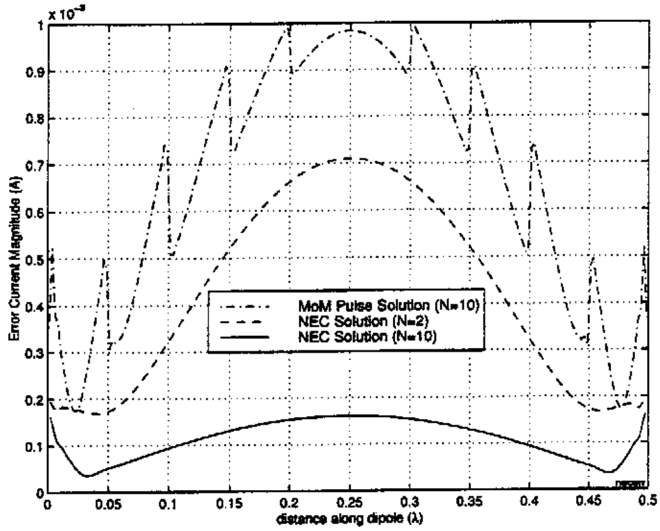


Fig. 4. The error current associated with NEC2 solutions for a 2, 10 segment dipole , as compared to the 10 segment MoM solution using pulses with $\ell = 0.5\lambda$ and $a = 0.005\lambda$.

a research tool; different basis and/or weighting functions, for instance, may be compared in terms of absolute error currents. Such an exercise was performed to compare the NEC2 (Burke & Poggio 1981) basis functions with pulse basis functions. The same dipole antenna was simulated using NEC2 with 2 and 10 segments and the error currents obtained using the same procedure, by applying the tangential **E**-field errors as an excitation to an oversegmented wire of the same dimensions. The error currents associated with these two cases are shown in Figure 4, together with the 10 segment MoM problem, presented earlier. Comparing these currents indicates that NEC2 basis functions achieve roughly the same accuracy for 2 segments as pulse basis functions do for 10 segments. This may be expected since NEC2 uses a superpositioned sine, cosine and a constant term as its basis function, which is quite suitable for approximating the currents on a dipole.

## IV. Using FEP *without* oversegmentation

The previous section showed that the FEP may be used to obtain accurate error currents with major computational overheads associated with overdiscretizing the problem. We now attempt to use the same FEP without the requirement to increase the problem discretization. The main problem with the FEP is that although it is possible to calculate the residual accurately, only $N$ excitations may be used to excite the error currents. It is hence necessary to reduce the residual over each segment to a single value. The first approximation to the accurate FEP was to obtain the residual **E**-fields at 100 points on the dipole and use the average value of ten samples applied as $E_e(s)$ on the original 10 segment problem[1]. Results from using this method were not satisfactory, since the large oscillations in residual **E**-field evident in Figure 2 require a finer discretization in order to obtain a reasonable estimate—which defeats the objective. It should be noted that the estimate was not inaccurate as a result of insufficient field point samples when averaging, since increasing the samples to 200 and 400 did not improve matters.

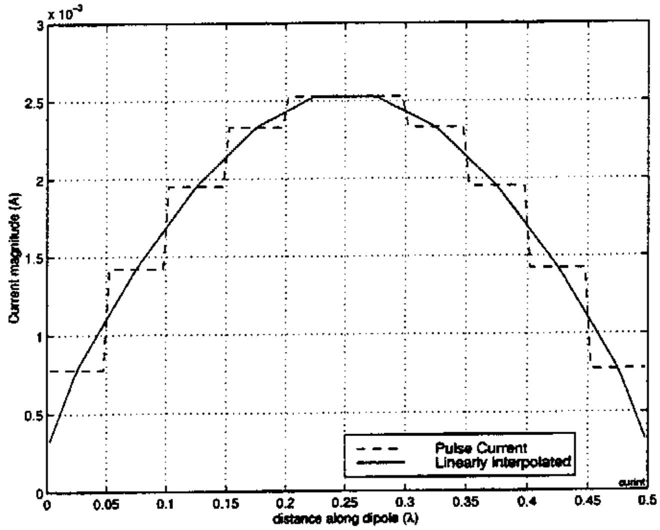


Fig. 5. The current obtained for a 10 segment dipole with $\ell = 0.5\lambda$ and $a = 0.005\lambda$ using pulse basis functions and point matching and a linear interpolation to the original pulse current.

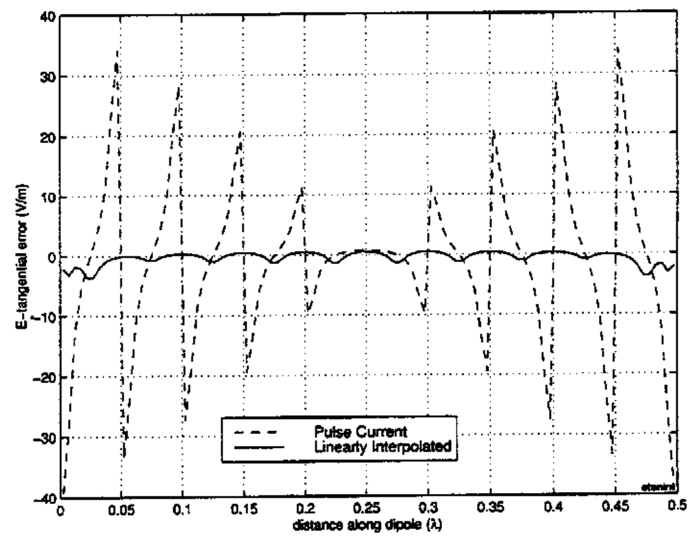


Fig. 6. The tangential **E**-field magnitude from a 10 segment dipole with $\ell = 0.5\lambda$ and $a = 0.005\lambda$ using pulse basis functions and point matching as well as the tangential **E**-field from a linear approximation to the original pulse current

We recognized that the large oscillations in the residual E-fields were mainly due to the pulse basis functions which are discontinuous at segment boundaries. Rather than solving the problem with different basis functions to

[1]Note that using an "average value" is meaningful for a point matching MoM (using Dirac delta weighting functions), however, if pulse weighting functions are used, the error residual computed would be zero. In the case of the pulse weighting function, one sample per segment may result in a more appropriate segment error. It stands to reason that the method used to obtain the segment residual must employ a function other than that of the actual testing functions used in the MoM solution.

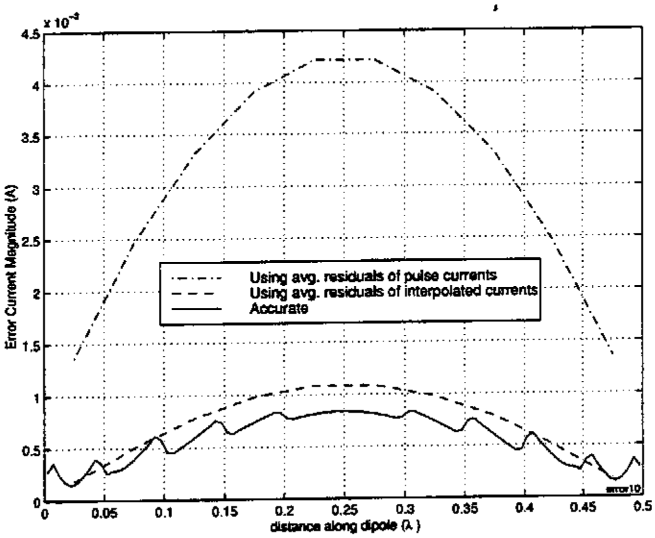


Fig. 7. Error current from applying the averaged **E**-tangential fields obtained from the pulse currents and the linearly interpolated currents as an excitation to the 10 segment problem. The accurate error current obtained earlier is also shown for comparison.

ensure smooth residual **E**-field behaviour, we performed curve fitting to the *actual pulse current solution* before calculating the residual **E**-fields via FEP. Figure 5 shows the original pulse currents obtained from the MoM together with a linear approximation to these currents, which clearly offer a smoother current solution. Figure 6 compares the residual **E**-field due to the original pulse currents as well as to the linearly interpolated currents (using only 10 segments). The residuals from the interpolated currents are clearly much less oscillatory than those resulting from the actual pulse solution.

The average (over a segment) of the residual **E**-fields associated with the linearly interpolated current shown in Figure 6 was applied to the original 10 segment problem to obtain error currents without increasing problem discretization. Figure 7 shows the effectiveness of this approach. Using the average residual **E**-fields obtained from the linearly interpolated currents provides a good estimate of the accuracy of the solution.

It should be noted that, in general, the linear interpolation of the pulse basis function solution will not necessarily yield a more accurate current representation. (It was also not the purpose.) The interpolated current was merely used to produce better behaved (less oscillatory) residuals. These residuals can be averaged over one segment to yield a residual **E**-field vector of the same order as the problem discretization (10 values in our example). The FEP can, hence, be used with the same size matrix, and if matrix factorization is used, without much additional computational effort.

## V. Conclusion

The FEP is definitely suitable for research purposes when applying it "accurately" by increasing the problem discretization. An error prediction scheme for estimating the errors associated with run-of-the-mill simulations using MoM, however, would not be useful if problem order is increased. This is mainly due to memory limitations (since matrix storage space is proportional to $N^2$) and to computational limitations (since execution time is proportional to $N^3$). The latter part of this limited study shows that a reasonable estimate may be obtained without using finer problem discretization provided that the behaviour of the **E**-field residuals are smooth over a segment. This is true even if a discontinuous current due to simple basis functions is artificially smoothed by interpolation to achieve better behaviour of residual **E**-fields.

It seems to us that the relationship between residual **E**-field smoothness and required problem discretization must obey normal sampling criteria for the FEP to provide reasonable error estimates. Problem discretization should allow the residual **E**-field curve shape to be retained by the error excitation vector for good estimates. Residual **E**-field variations along a wire with a period of the order of the segment length ($0.05\lambda$ for 10 segments) are the result of non-physical behaviour of currents at segment boundaries; it should hence, in principle, always be possible to "smooth" current behaviour to obtain slower variations in residuals (changes in the order of a wavelength should be more realistic).

The interpolation approach also illustrates a very interesting aspect of the behaviour of the residual **E**-fields: the large, fast, oscillations are purely associated with local, non-physical current behaviour. Smoothing the current results in much lower variations in the residual **E**-fields, which render them more suitable for error prediction. The interpolated currents are not, however, inherently more accurate—at least in terms of errors in overall magnitude—but they do seem to be a more natural representation of current shape, which one would expect. In the cases investigated here, current interpolation was done in order to obtain error estimates with lower computational overheads. The opposite is clearly also possible: currents may be obtained using some crude basis functions and can then be smoothed *a posteriori* (or altered) while using the FEP method to measure whether a more accurate answer was obtained.

Naturally, errors in the secondary parameters such as input impedance or radiation pattern, etc. can be derived from the error currents using the existing relationships between currents and these parameters—allowing for "error bars" to be placed on these parameters.

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