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Guest Editors
C. J. Reddy and Erik Vedeler

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Performance of Single and Double T-matched Short Dipole Tag Antennas for UHF RFID Systems

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Abstract — The impact of tag antenna and chip impedance tolerances on power transfer between these components is investigated analytically. Means for efficient computation of the minimum and maximum power transmission coefficient under given impedance tolerances are developed. The presented sensitivity analysis is employed to quantify the design uncertainty of single and double T-matched short dipole tag antennas for UHF RFID systems. The simulated and measured performance of the two tag antennas is analyzed and compared.

IndexTerms — Double T-matching, impedance matching, passive UHF RFID, tag antenna, T-matching.

I. INTRODUCTION

In radio-frequency identification (RFID) systems, electromagnetic interaction between a reader and electronic labels, designated as RFID tags, is employed to identify objects. In ultra high frequency (UHF) RFID systems, the mechanism of the interaction is most commonly wave propagation and the RFID tags are antennas loaded with a microchip. Passive RFID tags, which are studied in this article, scavenge energy for their operation from the incident electromagnetic field sent by the reader. In addition to capturing energy with on-chip rectification, the chip stores a unique electronic product code to label the tagged object, demodulates commands from the reader and creates a response to the reader’s queries. Tag’s response is created by switching the chip impedance between two values while the reader illuminates the tag with a single-frequency electromagnetic field. As a result, the tag’s response is modulated in the load-dependent component of the electromagnetic field scattered from the tag antenna [1].

As the passive RFID tags are not equipped with an energy source, maximizing the power delivery from the tag antenna to the chip is often the principal goal in tag design. However, the boundary conditions for the design are stringent. For a globally operable UHF RFID tag, good antenna performance is required over a broad, 10% fractional bandwidth (from 860 MHz to 960 MHz), while compact and low-profile antenna structures are preferred for seamless integration with objects. Thus, small antenna features are a crucial tag antenna design aspect. Furthermore, the unit cost of the tag antenna needs to be minimal when labelling a large asset base with RFID tags. To achieve cost-savings in the antenna manufacturing, most commonly the complex conjugate impedance matching between the tag antenna and the chip is arranged by designing the tag antenna geometry so that appropriate antenna impedance is achieved together with the desired radiation characteristics. In this process computational electromagnetics is extensively employed.

Dipole-type tag antennas are popular in UHF RFID systems. They benefit from being structurally simple radiators with omnidirectional radiation pattern, which allows the detection of
dipole tags from all directions in a plane normal to the tag. Moreover, clever size reduction and impedance matching techniques for them have also been extensively investigated in general context [2-5] as well as for RFID applications [6-8]. This article focuses on RFID tag antenna design verification based on impedance tolerances and on the performance comparison of single and double T-matched short dipole tags.

The rest of the article is organized as follows. Section II introduces the concept of power transmission coefficient and discusses the efficient computation of its limits under given source and load impedance variations. In Section III, the simulation based design of single and a double T-matched short dipole tags is discussed and the prototype antennas as well as simulation and measurement results. Finally, experimental verification of the simulation-based design verification based on impedance tolerances is also been extensively investigated in general context [2-5] as well as for RFID applications [6-8].

The rest of the article is organized as follows. Section II introduces the concept of power transmission coefficient and discusses the efficient computation of its limits under given source and load impedance variations. In Section III, the simulation based design of single and a double T-matched short dipole tags is discussed and the prototype antennas as well as simulation and measurement results. Finally, experimental verification of the simulation-based design verification based on impedance tolerances is also been extensively investigated in general context [2-5] as well as for RFID applications [6-8].

II. IMPACT OF IMPEDANCE TOLERANCES ON POWER TRANSFER

Power transfer from the tag antenna to the chip can be analyzed by considering two complex impedances connected with a transmission line with negligible electrical length. In this case, the ratio of the power available from the tag antenna \( P_{\text{tag}} \) and the power reflected back \( P_{\text{rfl}} \) from the antenna-chip interface due to impedance mismatch is given by [10]

\[
P_{\text{rfl}} = \frac{P_{\text{tag}}}{\left| Z_{ic} - Z_{a} \right|^2} \left( \frac{Z_{ic} + Z_{a}}{Z_{ic} + Z_{a}} \right)^2, \]

where \( Z_{a} = R_{a} + jX_{a} \) and \( Z_{ic} = R_{ic} + jX_{ic} \) are the antenna and chip impedances, respectively and \((\cdot)^*\) denotes complex conjugation. As the delivered power to the chip is the difference \( P_{ic} = P_{\text{tag}} - P_{\text{rfl}} \), using (1), the power transmission coefficient \( \tau \) between the tag antenna and chip is expressed as

\[
\tau = \frac{P_{\text{ic}}}{P_{\text{tag}}} = 1 - \frac{P_{\text{rfl}}}{P_{\text{tag}}} = \frac{4R_{a}R_{ic}}{Z_{a} + Z_{ic}}. \tag{2}
\]

In practice, neither the tag antenna nor the chip impedance is known exactly and due to the nonlinearity of (2) it is difficult to predict the magnitude of the impact of impedance variations on the power transfers without a more rigorous analysis. Thus, it is of practical interest to evaluate the maximum deviation of \( \tau \) from its nominal value while assuming the tag antenna and the chip impedances lie in the neighborhood of their nominal values \( Z_{a0} = R_{a0} + jX_{a0} \) and \( Z_{ic0} = R_{ic0} + jX_{ic0} \), respectively. For the purposes of the presented analysis, these neighbourhoods are defined below as rectangles in the chip and antenna impedance planes.

Let \( 0 < p, r < \infty, 0 < q, s < \infty, \) and \( 0 < \varepsilon < \min(p, r) \) and consider sets defined as

\[
D_{\varepsilon} = \left\{ (x, y) \in R^2 : x \geq \varepsilon, y \in R \right\},
\]

\[
\Lambda_{pq} = \left\{ (x, y) \in R^2 : \left| R_{a0} - x \right| \leq pR_{a0}, \right\}
\]

\[
\Lambda_{rs} = \left\{ (x, y) \in R^2 : \left| R_{ic0} - y \right| \leq sR_{ic0} \right\},
\]

\[
\Lambda_{ps} = \left\{ (x, y) \in R^2 : \left| R_{a0} - x \right| \leq rR_{a0}, \right\}
\]

Under these definitions, \( \Lambda_{pq} \) and \( \Lambda_{rs} \) are rectangles centered at the nominal chip and tag antenna impedances, respectively, and restricted in the half-plane containing the positive resistances. The size of these rectangles is determined by the parameter pairs \((p,q)\) and \((r,s)\) with \( r \) and \( p \) defining the percentage tolerance in \( R_{a} \) and \( R_{ic} \), respectively, and the parameters \( s \) and \( q \) defining the percentage tolerance in \( X_{a} \) and \( X_{ic} \), respectively. In the special case \( 0 < p, r < 1 \), the set \( D_{\varepsilon} \) along with the positive and arbitrarily small number \( \varepsilon \) could be dropped from the definition (3), as in this case the imaginary axis, where (2) is not necessarily well-defined, is always excluded from the sets \( \Lambda_{pq} \) and \( \Lambda_{rs} \). However, as discussed in Section III, large values of \( p \) and \( r \) may be needed in evaluation of platform-tolerance of RFID tag systems. Therefore, \( p \) and \( q \) are not considered upper bounded, but rather the set \( D_{\varepsilon} \) is used to keep (2) well-defined within \( \Lambda_{pq} \), \( \Lambda_{rs} \). Moreover, these sets are nonempty and closed by definition. These properties are also required in the presented analysis.

A. Minimum \( \tau \) under given impedance tolerances

Treating \( \tau \) first as a function of the chip impedance only, while considering the antenna impedance fixed and calculating the directional derivative of \( \tau \) along a vector \( u \) point from the
perfect conjugate match impedance point 
\( (R_{a0}, X_{a0}) \) to an arbitrary point \( (R_c, X_c) \in D \), one finds

\[
D_a \tau = -\frac{4R_a(2X_{a0})}{\tau} \left( \frac{R_a - R_c}{\tau} + (X_{a0} + X_c)^2 \right) < 0, \quad \forall (R_a, X_a) \in \Omega_a \setminus \{(R_{a0}, X_{a0})\}
\]

This shows that \( \tau \) is a strictly decreasing function of the chip impedance in \( D \) towards directions away from the perfect conjugate match impedance point \( (R_{a0}, X_{a0}) \), where it attains its maximum value \( \tau = 1 \). Equation (4) also implies the uniqueness of this maximum within \( D \).

Starting from (2), it is shown that the chip impedances corresponding to a constant \( \tau \) define a circle with center point \( P(\tau) \) and radius \( r(\tau) \) given by

\[
P(\tau) = \left( \frac{2-X_{a0}}{\tau}, -X_{a0} \right)
\]

\[
r(\tau) = 2R_o \sqrt{1 - \frac{\tau}{r}}.
\]

This circle always encloses the perfect conjugate match impedance point \( (R_{a0}, X_{a0}) \), where \( \tau \) is maximized. Since a rectangle can always be enclosed in a circle touching one of its corners, particularly the rectangle \( \Lambda_{pq} \) defined in equation (3) can always be enclosed in a constant \( \tau \) circle touching one of its corners.

To elaborate on the implications of this geometric observation, let \( (R_{pq}, X_{pq}) \) be the corner discussed above. As the perfect conjugate match impedance point \( (R_{a0}, X_{a0}) \) is always contained in the circle (5), equation (4) then guarantees that at any point in \( \Lambda_{pq} \), except for \( (R_{pq}, X_{pq}) \), it holds \( \tau \geq \tau(r_{pq}, X_{pq}) \). This means that the minimum value of \( \tau \) in \( \Lambda_{pq} \) is always attained at a corner of the rectangle \( \Lambda_{pq} \). This is illustrated in Fig. 1.

As seen from equation (2), the expression of \( \tau \) is symmetric with respect to pairs \( (R_a, X_a) \) and \( (R_c, X_c) \). Therefore, all the above conclusions of \( \tau \) as a function of the chip impedance are valid also when \( \tau \) is treated as a function of the tag antenna impedance, while considering the chip impedance fixed. Consequently, for all \( (R_c, X_c), (R_a, X_a) \in \Lambda_{pqrs} = \Lambda_{pq} \times \Lambda_{rs} \), one obtains

\[
\tau(R_c, X_c; R_a, X_a) \geq \tau(R_{pq}, X_{pq}; R, X) \geq \tau(R_{pq}, X_{pq}; R_r, X_r),
\]

where \( (R_{pq}, X_{pq}) \) is a corner of \( \Lambda_{pq} \) and \( (R, X) \) is a corner of \( \Lambda_{rs} \). Obviously, as the lower bound for \( \tau \) obtained in (6) is actually the function itself evaluated at a point in \( \Lambda_{pqrs} \), \( \tau_{min} \) is necessarily a minimum of \( \tau \) within this set.

In practice, this value is calculated as the minimum of \( \tau \) evaluated at the 16 corners of the 4-dimensional rectangle \( \Lambda_{pqrs} \). Equation for \( \tau_{min} \) is given in (7), where the minimum is considered for all the possible sign combinations. Compared with a direct numerical search through a 4-dimensional search grid, much less computations – only 16 evaluations of \( \tau \) – are needed to find \( \tau_{min} \) with this approach. This allows tag antenna designers to perform rapid worst-case tag performance estimation for a large number of frequency points in practical times.

\[
\tau_{min} = \min_{\eta} \frac{4X_c(1+p)X_c(1+p)R_{a0}R_{c0}}{(X_c(1+p)R_{a0} + X_c(1+p)R_{c0})^2 + (X_{a0} + qX_{c0} + qX_{c0})^2}, \quad X_c(x) = \begin{cases} \epsilon_x, & x < 0 \\epsilon_x, & x > 0 \end{cases}
\]

B. Maximum \( \tau \) under given impedance tolerances

Treating \( \tau \) first as a function of the chip impedance only, while considering the antenna impedance fixed, one immediately discovers that the maximum value of \( \tau \) in \( \Lambda_{pq} \) is one, if the
perfect conjugate match impedance point 
\((R_{a0},-X_{a0})\) is contained in \(\Lambda_{pq}\). Otherwise, 
\((R_{ic},X_{ic})\in\Lambda_{pq}\) and 
\((R_{a0},-X_{a0})\in D/\Lambda_{pq}\) can be 
joined with a straight line \(L(R_{ic},X_{ic})\) crossing the 
boundary of \(\Lambda_{pq}\), as illustrated in Fig. 2.

As shown in (4), in \(\Lambda_{pq}\subseteq D\), \(\tau\) is decreasing 
towards every direction from the perfect conjugate 
mismatch impedance point \((R_{a0},-X_{a0})\). From the fact 
that this holds in particular in the direction along 
the line \(L(R_{ic},X_{ic})\), it follows that in the intersection 
\(L(R_{ic},X_{ic})\cap\Lambda_{pq}\), \(\tau\) attains its maximum at the 
boundary of \(\Lambda_{pq}\). Furthermore, the collection of the 
subsets of \(\Lambda_{pq}\) for which this is true is actually the 
whole set \(\Lambda_{pq}\):

\[
\bigcup_{\{R_{ic},X_{ic}\}\in\Lambda_{pq}} \left\{ L(R_{ic},X_{ic}) \cap \Lambda_{pq} \right\} = \Lambda_{pq}. \tag{8}
\]

Thus, if the perfect conjugate match impedance point 
\((R_{a0},-X_{a0})\) is not contained in \(\Lambda_{pq}\), then the 
maximum of \(\tau\) in \(\Lambda_{pq}\) is necessarily attained at its 
boundary.

Since the expression of \(\tau\) is symmetric with 
respect to pairs \((R_{a},X_{a})\) and 
\((R_{ic},X_{ic})\), the same conclusions hold if \(\tau\) is treated as a function of the 
antenna impedance, while considering the chip 
impedance fixed. Based on this observation, a 
chain of inequalities similar to (6) can be 
developed as described below.

Let \(\partial\Lambda_{pq}\) and \(\partial\Lambda_{rs}\) be the boundaries of \(\Lambda_{pq}\) and 
\(\Lambda_{rs}\), respectively and suppose that 
\((R_{a0},-X_{a0})\not\in\Lambda_{pq}\) and 
\((R_{ic0},-X_{ic0})\not\in\Lambda_{rs}\). Under these assumptions 
\(\tau(R_{ic},X_{ic} ; R_a,X_A) \leq \tau(R_{pq},X_{pq} ; R_a,X_a)\) 
\(\leq \tau(R_{rs},X_{rs} ; R_a,X_a)\), \quad \tag{9}
\(= \tau_{\text{max}}\)

for all \((R_{ic},X_{ic} ; R_{a,X})\in\Lambda_{pqrs}\), with \((R_{pq},X_{pq})\in\partial\Lambda_{pq}\) 
and \((R_{rs},X_{rs})\in\partial\Lambda_{rs}\). As the upper bound for \(\tau\) 
obtained in (9) is actually the function itself 
evaluated at a point in \(\Lambda_{pqrs}\), \(\tau_{\text{max}}\) is necessarily a 
maximum of \(\tau\) within this set and it is attained in 
the Cartesian product \(\partial\Lambda_{pq}\times\partial\Lambda_{rs}\). Finally, if 
\((R_{a0},-X_{a0})\in\Lambda_{pq}\) or \((R_{ic0},-X_{ic0})\in\Lambda_{rs}\), then \(\tau_{\text{max}}=1\).

Based on this theoretical insight, the maximum of \(\tau\) under given impedance tolerances 
can be evaluated much more efficiently compared 
with a direct numerical search through a 
4-dimensional grid, since only a very limited 
subset – the Cartesian product \(\partial\Lambda_{pq}\times\partial\Lambda_{rs}\) – of the 
4-dimensional rectangle \(\Lambda_{pqrs}\) needs to be 
considered. With this approach, e.g. a search grid 
of size \(n\times n\times n\times n\) is reduced to significantly smaller 
grid of size \(4n\times 4n\times 4n\), which means a 1-16/\(n^2\) relative 
size reduction.

III. TAG DESIGNS AND SIMULATION

RESULTS

The frontend circuitry of an RFID chip is 
composed of capacitors, diodes and semiconductor 
switches, making the input impedance of the IC 
capacitive, as well as frequency and power 
dependent [11-13]. On the other hand, the input 
impedance of a short dipole tag antenna, operating 
below the fundamental resonance frequency of the 
antenna, is capacitive [2] and needs to be 
transformed to be inductive in order to conjugate 
match the tag antenna with the chip. This can be 
done using T-matching [6-9], which in practice is 
realized by forming a short circuit current path 
parallel to the antenna terminals. With the standard 
(single) T-matching this means adding a conductor 
loop in the structure around the antenna terminals. 
With the embedded T-matching, the short circuit 
current path is formed by a slot, which inscribed in 
the structure around the antenna terminals. The 
input impedance of a T-matched short dipole with 
a fixed length is then controlled by the shape of 
the short circuit current path. For example, with 
the commonly used rectangular path, the antenna 
input impedance can be controlled with only two 
parameters; the length and width of the rectangle.
Further degrees of freedom in the T-matching approach may be added by means of multiple T-matching stages \[6\]. In references \[7-8\] complex dipole antenna configurations with modified double-T matching approaches have been proposed. On the other hand, the present study focuses on a judicious performance comparison of fairly basic dipole antenna configurations with standard (single) T-matching and its simplest possible extension to double T-matching by addition of another identical loop. The comparison is done using quarter wave dipole tag antennas with the same footprint size and very similar radiating geometry. The structure of these tags is shown in Fig. 3. The chip used in both tag designs is the Higgs-3 UHF RFID IC by Alien Technology with the input impedance measured at the wake-up power of the chip \[14\].

The conjugate of this impedance, i.e. the target for the antenna impedance, is shown in Fig. 4. Ansoft high frequency structure simulator (HFSS) was employed in the antenna design.

Substrate material for the antenna designs is Rogers RT/duroid 5880 with the thickness of 3.175 mm, relative permittivity 2.2, and loss tangent 0.0009. This material was chosen due to its well-known microwave properties to reduce the design uncertainties and thereby yield more reliable comparison between the studied antennas. However, for tag antennas aimed for mass markets, thin low cost plastic films are a preferred choice for antenna substrate.

Since it is known that with the standard T-matching approach, good complex conjugate matching at a single frequency can be achieved, the design goal for the T-matched tag (T-Tag) was good performance within the US RFID band centered at 915 MHz. With the added degree of freedom in the impedance tuning with the double T-matching approach, a broader operational bandwidth is expected of DT-Tag. Therefore, a more challenging design goal with good antenna radiation characteristics together with more than 50% power transfer between the antenna and the chip throughout the global UHF RFID frequencies from 860 MHz to 960 MHz was considered.

The initial simulations showed that in practice, the added degree of freedom in the double T-matching approach manifests itself as a non-monotonic frequency response of the antenna reactance. This achievable feature allows a small dip to be tailored in the antenna reactance response to reduce its total variation over a range of frequencies. The reactance response of T-Tag, however, is inherently monotonic. This suggests that the expected broader operable bandwidth of DT-Tag may be realized by utilizing the local reactance dip in the tag antenna impedance in order to create a dual-frequency impedance matching.

In both tag antenna designs, the spiraled dipole arms are used to increase the electrical size of the antenna through the current alignment principle \[4\] and thus the fundamental resonance frequency of the antennas is much affected by the shape of the arms. Therefore, the related parameters \(H, T, q1\) and \(q2\) and \(u1\) and \(u2\) were first chosen in such a way that the fundamental resonance of the tag antennas with lengths \(L1\) and \(L2\) set to 80 mm (close to quarter wavelength) occurred slightly above 1 GHz. In this way, a gradual reactance slope favorable for the design was achieved over the frequencies of interest with both antennas. After this initial step, DT-Tag was optimized for the expected broadband operation, by varying the parameters \(L2, h2, and s2\) with \(L2\).
restricted in the neighborhood of the quarter wave length.

Before optimizing the T-matching loop of T-Tag, the parameter \( L1 \) was set equal to \( L2 \) to achieve exactly the same antenna foot-print size for both tags and thereby enable fair comparison between them. Then the parameters \( h1 \) and \( s1 \), where optimized to satisfy the design goal for T-Tag: good performance within the US RFID band. The built-in genetic optimizer of HFSS version 12 was used in the design.

The optimized antenna impedance of T-Tag and DT-Tag is shown together with the conjugate of the chip impedance (antenna design target) in Fig. 4. The corresponding power transmission coefficients are presented in Fig. 5 with solid “Nominal” curve. These simulation results predict that the T-Tag is well matched near 915 MHz with both resistance and reactance close to their target values. For DT-Tag the same holds in edges of the studied frequency range. In the mid-band, the DT-Tag is also reasonably matched, despite the seemingly large antenna resistance compared with the chip resistance. This is understood by examining the power transmission coefficient given in (2) as a function of the antenna resistance. With small reactance mismatch, the optimal value for resistance is \( R_a \approx R_{ic} \), but beyond this value, the increasing numerator \( 4R_aR_{ic} \) limits the decrease in \( \tau \). For reactance mismatch the rate of decrease in \( \tau \) is determined solely by the square expression in the denominator. This explains the more rapid decrease of \( \tau \) for the T-Tag, despite the good resistance match in the studied frequency range.
The Monte Carlo simulation method used in the chip impedance measurement [14] gives the chip resistance and reactance as means of Gaussian distributions with known standard deviations. The two uncertainty envelopes shown in Fig. 5 are based on 0% and 5% percentage tolerance in the simulated antenna impedance and the one standard deviation uncertainty for the measured chip resistance and reactance. In order to use the analysis from Section II to calculate the minimum and maximum power transmission coefficient under these impedance uncertainties, the standard deviations were first transformed into percentage tolerances. The envelope minimum is obtained from (7) and the maximum with direct numerical search through the set $\partial \Lambda_{pq} \times \partial \Lambda_{rs}$ defined in Section II.

In Fig. 5, the 0% case ($r=s=0$) represents a hypothetically perfectly successful simulation-based design and the 5% case ($r=s=0.05$) represents a more realistic scenario, which could be achieved with good modeling practices. In this study, tolerances beyond 5% were not considered, since the goal of the simulations is to provide judicious performance comparison between the two antennas on a platform with well-known dielectric properties. However, with larger antenna impedance tolerances, the analysis presented in Section II.A can be used for evaluating the platform-tolerance of RFID tags in terms of the minimum $\tau$ under variations in the tag antenna impedance when the tag is attached on different items with different electromagnetic properties.

As seen from Figs. 6-7, both antennas have an omnidirectional radiation pattern in yz-plane (the dipole H-plane) and an 8-shaped pattern in xz-plane (the dipole E-plane). Moreover, both antennas are linearly polarized in yz-plane with predominantly x-directed electric field component. It can also be observed from Figs. 6-7, that the gain of T-Tag decreases slightly with frequency in the yz-plane, while the minimum gain in xz-plane is increasing, whereas the gain pattern of DT-tag is less affected by frequency. However, in the yz-plane the gains of the two antennas are of the same order.

These results show that the matching network does not affect much on the radiation properties of the antenna and thus, the comparison of the antennas’ performance in terms of the matching approach is fair.

All the tag measurements discussed in the next Section were conducted in the forward direction corresponding to the direction of the z-axis in Figs. 6-7. In this particular direction, the forward gain of DT-tag is found to be approximately constant at 1.6 dBi, while for T-Tag, the gain decreases from 1.5 dBi at 860 MHz to $G_{fwd} \approx 0.7$ dBi at 960 MHz.

**IV. MEASUREMENT RESULTS AND DISCUSSION**

In order to verify the tag designs experimentally, the simulated and measured empty space read range is compared. Here it is assumed that the read range is limited by IC’s wake-up power and Friis’ simple transmission equation [15] is used in the calculations. This simple formulation may not be sufficient for estimating the read range in complex real-life environments [16] and in some applications with strong tag-to-tag coupling the receiver sensitivity may be limiting the read range [17]. However, in a controlled measurement environment, the empty space read range allows direct comparison between the measurements and simulations and thereby provides a method for design verification. In the present study, the main goal of the experimental work is to add assurance for the simulation-based conclusions about the performance of the compared tags.

To characterize the forward link performance of the designed tags experimentally, the transmitted threshold power ($P_{th}$) was measured for each tag. This is the minimum transmitted continuous wave power at which a valid response to Electronic Product Code (EPC) Generation 2 protocol’s query command is received from the tag under test. The threshold measurement was conducted in the forward direction in a compact anechoic cabinet with a linearly polarized transmitter antenna. During the measurement, the tag antennas were carefully aligned to match the polarization of the reader to minimize the link loss due to polarization mismatch. In addition, the path loss ($L_{fwd}$) from the generator’s output port to the input port of an equivalent isotropic antenna placed at the tag’s location was measured using the calibration procedure of the measurement device. This allows the compensation of any possible multipath effects in the measurements space, as described below.
According to Friis’ simple transmission equation:

\[ P_{ic,sens} = \tau G_{tag} L_c G_t \left( \frac{\lambda}{4\pi d} \right)^2 P_{th}^*, \quad (10) \]

where \( P_{ic,sens} \) is the wake-up power of the chip, \( P_{th}^* \) is the equivalent transmitted threshold power that would be measured in the perfect empty space conditions, \( L_c \) is the cable loss from the generator’s output (matched to the cable) to the input port of the transmitting antenna (matched to the cable), the gain of the transmitting antenna and the tag antenna are \( G_t \) and \( G_{tag} \), respectively, and the separation between these antennas is \( d \). On the other hand, in the real measurement it holds:

\[ P_{ic,sens} = \tau G_{tag} L_{fwd} P_{th}^*. \quad (11) \]

Thus, multiplying the measured transmitted threshold power with the factor \( \Lambda \) defined as:

\[ P_{th}^* = \Lambda P_{th}, \quad \text{with} \quad \Lambda = \frac{L_{fwd}}{L_c G_t \left( \frac{\lambda}{4\pi d} \right)^2}, \quad (12) \]

the measured \( P_{th}^* \) is mapped to the value that would have been obtained in empty space.

On the other hand, assuming that the measurement was conducted in empty space, the theoretical read range is:

\[ d_{tag} = \frac{\lambda}{4\pi} \sqrt{\frac{EIRP}{L_{fwd} P_{th}^*}}, \quad (14) \]

where the superscript \( m \) indicates that this value is based on measurements.

For comparison between the measurements and simulations, the theoretical empty space read range can be calculated with Friis’ simple transmission equation using the simulated power transmission coefficient \( \tau \), tag antenna gain in the forward direction \( (G_{fwd}) \), and the chip sensitivity \( (P_{ic,sens} = -18 \text{ dBm}) \) provided by the manufacturer. Under these definitions, the simulated value is given by:

\[ d_{tag}^s = \frac{\lambda}{4\pi} \sqrt{\frac{\tau G_{fwd} EIRP}{P_{ic,sens}}}. \quad (15) \]

Comparison of the measured and simulated theoretical empty space read ranges with European power regulation \( (EIRP = 3.28 \text{ W}) \) is shown in Fig. 8.

Simulation results, in Figs. 4-5, predict good reactance matching for DT-Tag at both ends of the studied frequency range, while the tag antenna gain was observed to remain approximately constant. This agrees with the measured frequency response shown in Fig. 8, with peak performance at the edges of the measured frequency range and slightly weaker performance in the middle. The simulated reactance of T-Tag, shown in Fig. 4, increases monotonically through the studied frequencies and consequently, good conjugate impedance matching is achieved only in the neighborhood of 915 MHz. This agrees with the measured frequency response, shown in Fig. 8. Simulations also predict a decreasing slope in the tag antenna gain versus frequency, which agrees with the measured frequency response as well; T-Tag’s performance decays faster towards the higher end of the measured frequency range. In addition, both tag antenna designs are verified within 5% impedance tolerances through the majority of the studied frequency points. This provides further assurance for the performance comparison between them.
V. CONCLUSIONS

The impact of tag antenna and chip impedance tolerances on power transfer between these components was investigated analytically. Means for efficient computation of the minimum and maximum power transmission coefficient under given impedance tolerances were developed and a closed-form expression for the minimum value was derived. This analysis provides tools for tag antenna designers to validate their designs. The presented sensitivity analysis was employed to quantify the design uncertainty of single and double T-matched short dipole tags. Both, the simulation-based and experimental comparisons of these tags showed that the bandwidth of a standard T-matched tag can be significantly improved with the double T-matching approach. Importantly, the modification of the standard T-matching to double-T matching requires only a minimal structural modification, which in our study did not increase the antenna foot-print size.

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FEKO/NEC2 Simulation of Candidate Antennas for the Long Wavelength Array (LWA)

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Abstract — This paper presents FEKO and NEC-2 simulations done on three dipole-like structures: the big blade, the tied-fork and the fork antenna. These antenna elements are considered for the design of the long wavelength array (LWA). The LWA is an interferometer under construction in New Mexico, USA for astronomical observations in the 20 - 80 MHz spectrum. This paper presents the simulation results of a co-polarized antenna gain patterns, impedance values, and mutual couplings for each candidate elements. Coupling results from FEKO and NEC-2 simulations are compared with measurement result of the big blade antenna. The paper also presents S-parameters for 25 elements of the tied-fork antennas.

Index Terms— Blade, dipole-like structures, FEKO, fork, mutual coupling, and tied-fork.

I. INTRODUCTION

The long wavelength array (LWA) is a radio interferometer telescope array under construction in New Mexico, USA for astronomical observation in the 20-80 MHz radio spectrum, within a total range of 10 MHz (ionospheric cut off) to 88 MHz [1, 2]. The array will consist of 53 electrically steered phased array stations. Each station will be constructed using 256 cross-dipole type antennas. The array will cover maximum baselines (distances between stations) up to 400 km of which core stations of 17 are within the center 10 km [3]. The station array has a pseudo-random arrangement that enables large aperture achievement with relatively fewer antenna elements while maintaining low sidelobe levels [4]. The objective of the LWA is to achieve long wavelength imaging with angular resolution and sensitivity comparable to existing instruments operating at shorter wavelengths [5].

Each station will have an elliptical shape with an axial ratio of 1.1:1 (110m in the N-S direction and 100m in E-W direction). This structure enables observation toward declinations that appear in the southern sky of New Mexico. Furthermore, it provides the ability to observe the inner galaxy region. The dimensions of the station array are chosen to balance sampling of a large field efficiently and calibration across the field of view (FOV) [5]. The array spacing d, is 5m which is 0.33\(\lambda\) at 20MHz and 1.33\(\lambda\) at 80MHz. Aliasing at the highest frequency for periodic arrays is avoided by using spacing, \(d < 0.5\lambda\). To avoid aliasing at 80 MHz, the number of antenna elements required for the LWA would have to increase by a factor of three which is economically
prohibitive [5]. Hence, a pseudo-random array arrangement is used to avoid aliasing at the highest frequency.

The number of elements in a station is arbitrarily chosen to be 256, a power of 2. The number of elements can be anywhere between 50 to 2500; however, it is constrained by requirements such as the baseline which affects image quality and the logistics associated with acquiring land, transporting the data, and maintenance of the equipment [5].

The choice of individual element design depends on whether the design meets technical requirements as well as cost limitations. The technical requirements for the LWA interferometer include [5]:

- Sensitivity on the order of arcseconds resolution of 8" and 2" at 20MHz and 80 MHz, respectively
- Field of view of 8° and 2° at 20 MHz and 80 MHz, respectively
- Broad and slowly-varying patterns over the tuning range
- Dimensions on the order of ½ λ at the highest frequency for alias-free beamforming
- Large tuning range for large impedance bandwidth

The technical requirements for candidate elements include [6]:

- Frequency range of 20-80 MHz (3-88 MHz desired)
- Stable, sky noise dominance of 6dB over the frequency range
- Zenith angle coverage, z ≤ 74° (z ≤ 80° is desired), to detect bright transients near the galactic center
- Good axial ratio for circular polarization (This requirement refers to the cross-polarization isolation)
- Durability for 15 year lifespan

The candidate antennas for the LWA system are dipole-like structures. Even though dipoles inherently have narrow impedances, the limitation does not apply to frequencies below 300MHz. Potential and dominant noise contribution comes from natural Galactic noise, not the instrument used [5].

This paper presents FEKO and NEC-2 simulations of the candidate antennas; the big blade, the tied-fork, and the fork antennas. FEKO is an electromagnetic (EM) analysis software suite based on the method of moments (MoM). NEC-2 (the numerical electromagnetic code version 2) is a public domain code also based on the method of moments. Simulation results from this paper as well as measurement data from other studies show the candidate antennas to be comparable in performance. All three candidates have also shown to meet technical requirements for the element design [2].

Section 2 describes the topology of the three candidate dipole type antennas. Sections 3 and 4 present design specification and design process of the LWA, respectively. Section 5 presents the parameters used for simulating the individual antennas as well as the results of the simulation, including the results from the S-parameter simulation of the 25-element array of tied-fork antennas. Section 6 provides the analysis of the results, and Section 7 summarizes the paper.

II. DESCRIPTION OF CANDIDATE ANTENNAS

Each stand of the candidate antennas has two dipoles with collocated feed points oriented at right angles to each other.

The big blade is a complex structure made of two linearly polarized cross dipoles. The element is made of aluminum sheets. Even though the overall performance of this antenna is comparable with the other candidates, since it takes a total of 13,000 elements to construct the entire array system, it makes this candidate unfavorable with respect to cost. The big blade antenna dimensions are shown in Figure 1 and its images from FEKO and NEC-2 simulations are given in Figure 2.

The tied-fork antenna is made of strands of wire that represent the skeletal outline of the big blade. It also has two bars that run across the strands. Since the fork antenna does not involve the use of the aluminum sheet, it is less costly than the complex big blade structure. The tied-fork antenna dimensions are presented in Fig. 1 and its images from FEKO and NEC-2 simulations are shown in Fig. 2.

Like the tied-fork, the fork antenna is made of 3 strands of wire that represent the skeletal outline of the big blade. This antenna is a cost effective candidate. It is also less susceptible to wind effects [7]. The fork antenna dimensions are presented in Fig. 1 and its images from FEKO and NEC-2
III. DESIGN SPECIFICATIONS

The LWA is designed for long-wavelength astrophysics and ionospheric research [5]. The LWA addresses a wide range of research interests including cosmic evolution, solar science, and space weather. Detailed and specific objectives for the LWA are described in [8]. The underlining expectation of the LWA is that it should be able to perform comparably to existing instruments operating in shorter wavelengths with respect to resolution and sensitivity. That is resolution in the order of arcseconds and sensitivity in the orders of mili-janskys, where 1 Jansky = (10^{-26} W)/(m^2 Hz^{-1}), are desired [5]. This means an improvement of several orders of magnitude over existing instruments operating below 100 MHz [5].

In order to achieve the long wavelength imaging required for the exploration of various scientific frontiers, parameters such as dimensions of stations, resolution, collecting area, sensitivity, and field of view (FOV) are considered in the design process. An overview of these parameters is provided here. However, for detailed description and design processes please refer to [5] and [8]. The key design parameters of the LWA stations are collecting area and dimensions of the station beam [5]. Collecting area contributes to image sensitivity while dimensions of the beam...
Resolution (R) is the system’s ability to distinguish between two very close, adjacent and independent objects in the sky. The largest structure that can be imaged by the system is a function of the observational wavelength and the minimum baseline. It is the finest detail an instrument is capable of showing. It is calculated as, \( R = \left( \frac{\lambda}{D} \right) \times (648000/\pi) \), where D is the maximum baseline (400km).

The process of determining the collecting area involves using sufficient number of sources that are detected above a certain flux within the FOV to calibrate the image against the effects of the ionosphere. The effective collecting area of a LWA station is given by \( A = \gamma N_a \xi A_{eo} (\lambda, \theta, \phi) \) where \( \gamma \) accounts for aggregate mutual coupling, \( N_a \) is the number of elements and \( A_{eo} \) is the collecting area of a single antenna in isolation [8].

FOV is the area of the sky being observed. Dimensions of the station array determine the width of the station beam which in turn determines the field of view. The image quality over larger FOV is limited by atmospheric variance in it. The usable FOV is determined by spacings between antennas and it is affected significantly by the ionosphere. The FOV of the LWA can be defined as the area bounded by the half-power beamwidth of a station beam and is calculated as \( \text{FOV} = 4.12 \psi_o^2 \left( \frac{\lambda}{4m} \right)^2 \left( \frac{D}{100m} \right)^{-2} \sec \theta \ [\text{deg}^2] \) where, \( \psi_o = 1.02 \) for a uniformly excited circular array and D is the station mean diameter. For detail derivation of the parameters, please refer to [8].

The sensitivity of a radiotelescope is a measure of the weakest source of radio emission that can be detected; hence, it is directly related to the errors of measurement [9]. Many factors affect sensitivity including the nature of the source signal, antenna characteristics, receiver performance (LWA has Galactic-noise limited receiver), resolution, the medium between the source and the antenna system (atmospheric conditions that are frequency dependent), image forming characteristics, and the size of the region of the sky observed. Sensitivity is parameterized using system temperature where high system temperature value indicates low sensitivity. Sensitivities are calculated for a given integration time. Sensitivity is proportional to the size of the beam, integration time, and total observation bandwidth. It can be improved after observations by averaging channels together. The sensitivity of observation varies across the FOV where it declines away from the center position of the main beam.

IV. DESIGN PROCESS

In general, for short wavelength design if technical issues are overcome, costs will be a major obstacle [8]. Simulations and prototype testing were performed to choose the design of the antenna elements. Each antenna element needed to achieve large tuning range to be considered for the design of the LWA. Previously, low frequency telescopes used antennas that have inherently large impedance bandwidth such as conical spirals [8]. Since the design of the LWA calls for a large number of antennas, such complex and expensive structures are not suitable for the LWA. Hence, simple wire dipoles (folded dipoles) that have inherently narrow impedance bandwidth are chosen. This does not pose significant problems for systems operating below 300 MHz. This is because the natural Galactic noise dominates over the noise contribution of the electrons attached to the antennas [5]. Prototypes of the antennas are used to measure the radio frequency interference environment in the desired frequency band and the result show stable sky noise dominance of 6dB over the frequency range.

The choice of 256 stands distributed over roughly 100m diameter (110mx100m ellipse) balances the desire to efficiently sample large FOV required to image several sources across the sky against the difficulty of ionospheric calibration across the wide FOV [5]. The LWA will be able to image wide FOVs with sufficient diversity of baselines [8]. This choice will also balance cost against quality of image calibration over a broad range of frequencies and zenith angles [8].

Even though it is desirable to have a small number of stations to simplify the process of obtaining land, transporting data, and maintaining instruments since image quality requires diversity of baselines the argument calls for a larger number of stations. The station numbers (53) were chosen based on prior experience and guidance from previous large array systems [5].

The maximum baseline of 400km was chosen in order to obtain the resolution values required to observe detailed structures of extragalactic radio galaxies and avoid confusion that arises due to
unresolved sources or due to plausible long hours of integration times (interval over which data collected are averaged to reduce background noise) [5]. This baseline yields the desired resolution; 8'' at 20 MHz and 2'' at 80 MHz.

Optimization of antenna positions for a pseudorandom station configuration was performed and details of it are found in [10]. Pseudorandom antenna distributions are susceptible to mutual coupling effects. Current simulation does not consider effects of coupling for the LWA; future effort will be focused to include effects of coupling. Furthermore, the effects will be studied when the first station is built [8].

As mentioned earlier, the primary receiving element of the LWA is a fixed stand that incorporates two broadband, crossed, linearly-polarized dipoles. The signal from every antenna is processed by a direct-sampling receiver consisting of an analog receiver and an analog-to-digital converter. Beams are formed using a time-domain delay-and-sum architecture, which allows the entire 10–88 MHz passband associated with each antenna to be processed as a single wideband data stream. A finite impulse response filter which is used to introduce coarse delay is also used to introduce corrections for polarization and other frequency-dependent effects. The raw linear polarizations are transformed into calibrated standard orthogonal circular polarizations, and the signals are then added to the signals from other antennas processed similarly [11].

V. SIMULATION PARAMETERS AND RESULTS

This section presents the parameters used and the results obtained from FEKO and NEC-2 simulations. MATLAB scripts are used to generate the antenna models used in the two simulation tools.

Figure 3 shows co-polarized gain patterns in E- and H-planes along with axial ratios of the candidate antennas at 38 MHz, 74 MHz, and 80 MHz. The axial ratio resulting from a pair of dipoles can be approximated from the difference between the E- and H-plane gain patterns at each elevation angle for a single dipole [12]. Axial ratios are calculated from the co-polarized gain patterns using $\text{AR} (\theta, \phi) = |\text{GE,co} (\theta, \phi) - \text{GH,co} (\theta, \phi)|$, where $(\theta, \phi)$ are observation angles and GE,co and GH,co are co-polarized gain patterns in the E- and H-planes, respectively, of a single dipole expressed in dB. Since the maximum cross-polarized gains for all antenna structures from both simulation tools are very low, their plots are not included in the paper.

Figure 4 presents impedance values obtained for each isolated dipole-like structure. A reference input impedance of 50 $\Omega$ is assumed when calculating the S-parameters in all cases in both FEKO and NEC-2 simulations. Figure 5 shows the S-parameters obtained for two identical elements of each of the antenna types. Dynamic matching at different frequencies would achieve better performance for the significant portion of the required frequency bands instead of the narrow band observed for the S11 values. In Fig. 6, the big blade antenna measurement data from [13] is compared with big blade simulations of NEC-2 and FEKO. A spacing of 6m between two big blade antennas is used when calculating S21.

Figures 7 and 8 present mutual coupling between elements in a periodic 5X5, or 25-element array of tied-fork antennas for the center element and an edge element, respectively, for a range of frequencies. In this coupling calculation all the elements in the array but one are terminated with 50 $\Omega$ impedance and one of the elements (the edge or the center element) is excited.

A conducting ground screen (3mx3m) that can prevent loss through absorption and isolate the antenna from variable ground conditions to stabilize the system temperature will be used in the field. However, it is not included for this simulation. The measurement setup also did not include the ground screen (Fig. 6). Real ground conditions, permittivity, $\varepsilon_r = 13$, and conductivity, $\sigma = 0.005$ S/m, are used in all cases of the simulation in this study. Based on [14], a wire radius ($r_w$) = 0.0099 m is used in all cases. A segment length of ~ 8* $r_w$ is used for NEC-2 simulations and a segment length of ~5* $r_w$ is used for FEKO simulations. The choice of the length of the segments is based on [15] and the final adjustments are made by trial and error. The feed point is assumed to be 1.5 m above the ground for all candidate antenna types. A 0.1m feed point width is used for all antenna types.

FEKO and NEC-2 simulation results for the co-
polarized E-plane and H-plane gain patterns of the big blade, tied fork, and fork antenna (Fig. 3) at 38 MHz, 74 MHz, and 80 MHz are fairly comparable. The H-plane patterns maintain their shape with increased beamwidth as the frequency increases. However, the E-plane patterns exhibit sidelobes at the higher frequencies. The tied fork antenna has the maximum axial ratio for all frequency ranges. The big blade has the lowest axial ratio in all frequencies at all elevation angles.

![Co-polarized gain patterns and axial ratios at 38, 74, and 80 MHz.](image)

Impedance values (Fig. 4) obtained from the simulations show that for all the three element types FEKO simulations results are slightly higher, particularly for higher frequencies than NEC-2 simulations. Overall, the fork antenna has higher impedance values than the other two antennas. There is a spike seen around 55 MHz for the fork antenna, and it is assumed to be a simulation artifact. The tied-fork and fork antenna exhibit higher resonant frequencies as compared to the big blade.

For the mutual coupling calculation of two antennas side by side, a spacing of 6 m between the elements is used; this is because the available measurement data is for 6m spacing. However, 5 m spacing is considered for the LWA design. The mutual coupling measurement of the big-blade [13] is comparable with NEC-2 and FEKO simulations results.
Fig. 4. Big blade, tied-fork, and fork antenna impedances for isolated elements.

Fig. 5. Big blade, tied-fork, and fork antenna S-parameters.
Fig. 6. Measurement data and simulation results of the big blade antenna coupling.

Fig. 7. S-parameters for 5x5 tied-fork antennas – center element.

Fig. 8. S-parameters for 5x5 tied-fork antennas – edge element.
VI. ANALYSIS

The LWA candidate antennas maintain good performance over the required 20 – 80 MHz frequency range if a dynamic input impedance matching is applied for frequency and scan angle changes. This study did not consider antenna performance optimization. Even though, the tied-fork antenna and the fork antenna have simpler topology, they exhibit comparable RF properties to the complex big blade structure.

The co-polarized gain patterns of the antennas in both planes exhibit a single, wide beamwidth lobe with the maximum towards zenith. However, the H-plane patterns better maintain their shape and provide increased beamwidth up to higher frequencies than the E-plane patterns, which develop sidelobes at higher frequencies. The fork antenna seems to be better in keeping its shape at all frequencies. The co-polarized gain patterns are plotted only for E-plane since it is assumed the performance of this plane to be worse than other planes [16].

The beam patterns of the three structures have axial symmetry of 1.2 dB or less for elevation angles down to ± 74 ° from zenith for all frequencies. For higher frequencies, the FEKO simulation results show 1dB or less for all frequencies at these elevations.

The simulated impedance values of the simpler topology antennas are generally higher and shifted down in frequency when compared to the simulated response of the big blade antenna. The big blade exhibits larger impedance bandwidth as compared to the other two antennas.

The S-parameters for the 25 tied-fork antennas show that coupling effects are not prominent enough to cause concern. The worst condition, a coupling of -23 dB, is observed for both center and edge cases at 28 MHz at a distance of 5m when the antennas are in parallel. For the same distance if the antennas are placed diagonally from each other, the coupling is only about -27 dB. This is because the induced current is largest when the two antennas are parallel. The coupling results are obtained by exciting one element (center or the edge) and terminating all the other elements with 50 Ω. Note that parallel implies the maximum radiation is aligned along the line of separation, hence a higher coupling. For all the calculation, the unexcited antenna is terminated with a load of 50Ω.

VII. CONCLUSION

FEKO and NEC-2 simulation results of the big blade, tied-fork, and fork antennas are presented in this paper. The candidate antennas exhibit similar characteristics with slight differences in impedance and gain values. The simulation results obtained in this study are in agreement with the measurement provided in [2]. The S-parameters for the 25 tied-fork antennas indicate that coupling is not a concern. However, further analysis is needed to rule it out completely as an important factor affecting pattern.

A dynamic input impedance matching is recommended for the required range of frequencies to obtain optimum performance of the station.

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Electromagnetic Launch Vehicle Fairing and Acoustic Blanket Model of Received Power using FEKO

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Abstract— Evaluating the impact of radio frequency transmission in vehicle fairings is important to electromagnetically sensitive spacecraft. This study employs the multilevel fast multipole method (MLFMM) from a commercial electromagnetic tool, FEKO, to model the fairing electromagnetic environment in the presence of an internal transmitter with improved accuracy over industry applied techniques. This fairing model includes material properties representative of acoustic blanketing commonly used in vehicles. Equivalent surface material models within FEKO were successfully applied to simulate the test case. Finally, a simplified model is presented using the Nicholson Ross Weir derived blanket material properties. These properties are implemented with the coated metal option to reduce the model to one layer within the accuracy of the original three layer simulation.

Index Terms — FEKO, MLFMM, Nicholson Ross Weir, resonant cavity.

I. INTRODUCTION

With multiple contributions from the range and surrounding radio frequency (rf) emitters, defining the electromagnetic environment for spacecraft can be a daunting task [1]. Determining the environment inside the vehicle fairing presents further challenges as field distribution within the cavity is influenced by resonances which require a full wave solution to achieve a desired accuracy. An added concern is that most spacecraft transmitters are in the GHz frequency range making the structures electrically large and memory requirements a constraint for many of the 3D electromagnetic simulation tools available. Recent research with hybrid physical optics and near-field to far-field transformations, as well as the use of parallelized fast multilevel codes with non-uniform rational B-spline surfaces, have had demonstrated success in modeling complex, electrically large structures [2-3]. This study is focused on solutions to electrically large internal cavity problems related to structures with layers of acoustic blanketing.

In this paper, two structural cases are evaluated: a three layer model, and a one layer model. The three layer model of a vehicle fairing with layered acoustic blanketing materials characterized by thin surface approximations is first presented [4]. For comparison and validation purposes the test case from [5] is summarized here and used as the evaluation data. Next, an equivalent one-layer model is developed using material properties predicted with S-parameters measurement and implemented into the FEKO standard coating option.

II. FAIRING FIXTURE

A fairing test fixture is shown in Figure 1. It is a scaled version with a height of 2 meters and a diameter of 0.6 meters and with industry grade...
aluminum foil lining on the Lexan outer shell [6]. This fixture is representative of typical launch vehicles. The fairing has three sections bolted together and a metal frame outer support structure. Double ridge guide horns were used for transmit and receive and were placed at the bottom and top of the fairing fixture, respectively [7].

Lining materials were added to the inside of the test fixture to simulate typical acoustic blankets inside vehicle fairings. Kapton is commonly used in space applications for its favorable thermal insulating properties. DuPont’s Kapton 160XC, designed to maintain a surface resistance of 377 ohms with inherent RF absorption properties, is utilized as the outer blanket layers while standard ½ inch foam is used as the internal layer.

The test results from this fairing fixture with acoustic blanketing are used for comparison with the three layer and one layer computational models presented here. The goal is to obtain an equivalent one layer model that has similar test data correlation as the three layer model.

**III. THREE LAYER MODEL**

A commercial computational electromagnetic software tool, EM Software Systems, FEKO is utilized in this study. The multilevel fast multipole method (MLFMM) feature is implemented to extend the method of moments (MoM) technique to higher frequencies. MoM is directly implemented for near elements and iterations are used to achieve the desired overall convergence criteria. Figure 2 demonstrates the adequacy of this approach for an aluminum cavity represented by an impedance sheet using both the MoM and MLFMM techniques. The field distribution and power received at 1 GHz using a surface impedance of 0.015 ohms reveals excellent agreement.

![Fig. 2. Field distribution of an aluminum fairing using MLFMM (a) and MoM (b) techniques.](image)

Comparable results were found with FEKO’s lossy metal feature which has a similar implementation as the impedance sheet. FEKO evaluates the input material properties, such as permittivity and conductivity, to obtain a representative impedance term, $Z_s$, which is then added to the standard electric field integral equations used for perfect electric conductor (PEC) structures as in (1) [8, 9].

$$E_{s,tan} - Z_s J_s = -E_{l,tan}, \quad (1)$$

where:

- $E_i$ is the field due to an impressed source in the absence of the scatterer,
- $E_s$ is the scattered field, and
- $J_s$ is the equivalent current density.

The double ridge guide horns were implemented in the simulation using antenna pattern models presented in [4] of the EMCO 3115 horn developed within FEKO. Replacing the horn model with the horn pattern affords a significant savings in computational resources. In addition, parallelization of the FEKO code via preconditioners, such as the sparse approximate inverse, supports solutions for detailed electrically large structures as those considered here. [10].
A combined blanketing and composite fairing structure model was presented in [11]. In this paper, it is desired to first represent the layers separately for direct test comparison. Figure 3 depicts the separate test fixture layers and the composite model used within FEKO.

Fig. 3. FEKO model with acoustic blankets.

The aluminum foil outer layer and acoustic blanketing layers were represented within FEKO as described below:

- The fairing outer walls were represented as a single layer lossy metal with a thickness representing the industry aluminum foil that lined the prototype fairing (0.127 mm thick).
- The Kapton acoustic blanket sheets are modeled with a surface impedance based on industry data at the model frequency.
- The gaps between the impedance sheets represent the foam layer.
- Free space is required on both sides of the impedance sheet thus a thin layer of free space is implemented between the Kapton layer and the aluminum foil outer layer.

Figure 4 shows a comparison of received power between the computational and the test results. The data compares well, with the average variation of 2.43 dB from test data. This is a reasonable result for a test article to model comparisons given uncaptured variations present in the test set-up. The selection of this frequency range is related to the waveguide measurements used in the equivalent one layer approach described in the following section.

Fig. 4. Comparison of received power using computational and test results for the acoustic blanket model.

IV. EQUIVALENT ONE LAYER MODEL

It is desirable to further reduce the required computational resource and run-time requirements of the three layer structure with an electrically large cavity model and simulation by using an equivalent one layer model. Another reason to form a one layer equivalent model is the limited availability of vehicle CAD models with blanket configuration information. It should be noted that the following equivalent layer technique is not needed for simulating waveguide structures in general as there are finite element codes available that precisely model these layers and complex materials in such structures with no simplification [12-13]. This effort uses the waveguide equivalent model to later implement the layered material effects in the computationally intensive electrically large cavity structures where dimensions can be greater than 100 times the transmit wavelength and exact representation of blankets not feasible with existing software packages on available platforms.

A. Methodology selection

Truncation of the scalar Green’s function implemented with the addition theory series in MLFMM introduces an error that can be controlled in open structures, but difficult to
achieve sufficiently accurate results in electrically large reflective cavities [14]. This residual error can, in effect, numerically excite the cavity. Thus, convergence is improved by using layer representations that characterize the material absorption. The absorbing impedance sheets used in the three layer model require a layer of free space on either side; consequently, the one layer model requires a different material representation that can readily be combined with the metallic outer layer. The difficulty in representing the entire vehicle in one layer is the contrast between properties of the aluminum layer and that of the acoustic blankets. Accordingly, an option was used to apply the blanket properties as a coating to the metal outer layer. Material properties of the lossy metals and dielectrics are available in the FEKO material tree. Dielectrics can then be selected as a thin dielectric sheet (TDS) with specified thickness. The coatings were selected from the TDS single layer option. The TDS is implemented within FEKO in a similar way as the impedance sheet in (1) with the $Z_s$ term described in given by [6].

\[
Z_s = \frac{1}{j\omega(\varepsilon_2 - \varepsilon_1)}d. \tag{2}
\]

A TDS is required to be geometrically or electrically thin (approximately 1/10 the smallest element or wavelength, respectively). Due to the this requirement, an inherent limitation is often encountered in the computation when the automatic mesh routine generates fine elements to accurately characterize the respective geometries. However, if the coating is geometrically small with respect to the majority of the elements, the geometrically thin constraint driven by these fine elements is effectively ignored in the model solution. A FEKO utility will perform a validate check, and will return a solution with warnings only. It is also important to note that the electrically thin constraint is relative to a wavelength in the interfacing medium, but the layer does not have to be electrically small relative to a wavelength of the layer itself [9]. Nevertheless, it is often the situation that the actual thickness of the blankets cannot be represented in a coating, and an equivalent method must always be demonstrated and evaluated.

### B. Sample S-parameter measurement

The one layer coating model constraint drives the need to alternately represent the three layer blanket model in a waveguide with a one layer TDS. The Nicholson Ross Weir (NRW) technique is used to derive an equivalent permittivity of the entire layered blanket using S-parameter measurements. A blanket sample was placed in an S-Band waveguide. The S-parameters were then measured with a vector network analyzer as in Fig. 5. These parameters are used in an equation to determine the transmission coefficient and then evaluated in expression (3) to obtain an approximate value of the equivalent permittivity of a homogenous sample with the same length. As most launch vehicle blanketing materials are non-magnetic, setting the permeability, $\mu_r$, to one simplifies the permittivity determination. Moreover, the TDS implementation requires the permeability to be continuous with the surrounding media.

\[
\varepsilon_r = \frac{\lambda_0^2}{\mu_r} \left( \frac{1}{\lambda_c^2} - \frac{1}{2\pi L \ln \left( \frac{1}{T} \right)} \right)^2, \tag{3}
\]

where: $\lambda_0$ is the freespace wavelength for the desired frequency, $\lambda_c$ is the waveguide cut-off wavelength, $L$ is the sample length, and $T$ is the transmission coefficient determined by the measured S-parameters [15].

![Fig. 5. Material sample test fixture.](image)

Determining the permittivity of a homogeneous sample using waveguide measurements and computational models has been verified as being effective in the literature [16]. In this paper, the NRW technique is used to determine a first level approximation of an equivalent permittivity that would apply to a dielectric block with the same measured S-parameters, although the sample itself is layered. Full wave analysis is then used to modify the permittivity at each frequency until a sufficiently
close approximation of the S-parameters is found. This equivalent permittivity data is then used to construct the coating in the one layer model of the fairing.

C. Waveguide sample models

A three layer MoM model was first constructed in FEKO as shown in Fig. 6 to emulate the actual S parameter measurement set-up.

![Fig 6. FEKO MoM model of a three layer fairing blanket sample.](image)

The permittivity and conductivity of each Kapton layer was characterized as a dielectric with the thickness accounted for in the TDS implementation. The foam was represented by air as in the three layer fairing model.

It is straightforward to convert the separate layer model into a multilayer TDS which only uses one face in the geometry representation. However, the multilayer TDS cannot be represented as a coating to a metal. Hence, representation of the material in a single TDS is pursued.

The finite element method (FEM) was employed to verify that the NRW derived equivalent properties derived with (3) represent the S parameters when the waveguide is filled with a homogeneous dielectric block. The FEM model in Fig. 7 effectively reproduced the results as shown in Fig. 9 with some parameter optimization in the model. In this instance, the regions defining the boundary of the block are represented as the dielectric material and implemented with permittivity parameters with respective loss tangents.

The parameters were then implemented with a TDS single layer as shown in Fig. 8 for final implementation into the fairing fixture.

When meshing constraints require a reduced thickness in the TDS layer, a thinner layer can be established by changing the sample length in (3) to achieve a corresponding permittivity. Figure 9 shows a comparison of test, MoM separate layer model, FEM dielectric block model, and the final single layer TDS with original and reduced sample thicknesses. The material parameters can then be adjusted to provide a closer match to the original S$_{21}$ measurements.

![Fig 7. Equivalent homogeneous dielectric block.](image)

![Fig 8. TDS layer in waveguide.](image)

![Fig 9. Comparison of the waveguide S-parameter test data to the FEKO models.](image)

D. Equivalent one-layer vehicle model

Results in Fig. 10 show that incorporation of the permittivity and loss tangent derived from the NRW waveguide technique into a TDS coating of a single metal layer in the vehicle model provides a reasonable correlation to the test data, as does
the three layer model. First, the original sample thickness results are applied directly to the coating properties. Due to layer wavelength related constraints, however, the thickness of the coating is set at three skin depths of the Kapton layer. A closer approximation is achieved by using (3) to provide a different permittivity and loss tangent to correspond to a sample thickness adjusted to a smaller value. Results shown are for a TDS length of 1/6 of the original sample which varied from the test results an average of only 2.5 dB.

The upper and lower bounds represented in Fig. 10 are based on cavity Q equations for aluminum and blanketed walls [17].

![Fig. 10. Comparison of received power using the single layer and the three layer fairing models with the test data.](image)

It is evident that the FEKO models provide significantly better results than approximation results that are generally relied upon. It should be noted that the primary intent of the Q related approximations are to evaluate chambers with very conductive walls with small absorbers present, but the application of these equations are often extended to cavities with more complex material configurations.

The efficiency benefits of using MLFMM in a three and one layer model as compared to MoM are shown in Table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th># Un-knowns</th>
<th>CPU Time/ process (hrs)</th>
<th>CPU Time total</th>
<th>Peak Memory (GB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mom 1 layer</td>
<td>124,377</td>
<td>21.2</td>
<td>339</td>
<td>115</td>
</tr>
<tr>
<td>MLFMM 3 layer</td>
<td>372,622</td>
<td>3.9</td>
<td>60.9</td>
<td>10</td>
</tr>
<tr>
<td>MLFMM 1 layer</td>
<td>124,377</td>
<td>0.066</td>
<td>1.1</td>
<td>2.2</td>
</tr>
</tbody>
</table>

V. CONCLUSION

This paper shows that fairing structures with complex blanketing materials can be modeled effectively with equivalent impedance techniques in a multilayer MLFMM model within the FEKO solution environment by establishing the eigenmodes within the cavity verses an average power approximation. This is important because quantifying fields due to transmission within a vehicle fairing has largely relied on general reverberation chamber average power approximation. The MLFMM more accurately depicts the actual RF energy within the cavity structure. The techniques explored here were the three layer and one layer models. From this data set, both methods appeared to have an improvement over the power approximation techniques for a launch vehicle with simulated acoustic blankets. The equivalent one-layer approach utilized a novel application of NRW formulations to derive an equivalent permittivity of the three layer configuration. Future work includes extending the frequency range beyond S-Band and the application of this technique to other layered materials such as composite vehicle structures.

REFERENCES


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In 1989, Dr. Wahid was named the Tau Beta Pi Professor of the Year. She received the College of Engineering Excellence in Teaching Award in 1994 and 1999. In 1991, she received the University of Central Florida Excellence in Advising Award and in 1997, the University of Central Florida Excellence in Professional Service Award. In 2000, she was awarded the IEEE Region 3 Outstanding Engineer Educator Award and the IEEE Florida Council Outstanding Educator Award. She is a recipient of the IEEE Millennium Award. She was the Technical Program Chair for the 1999 IEEE International AP/URSI symposium and the General Chair for the 1998 IEEE Region 3 Southeastcon conference. She has served many times on the technical program committee for the IEEE AP/URSI conferences. Dr. Wahid is a Senior Member of the IEEE and a member of the Eta Kappa Nu and the Tau Beta Pi Societies.
Evaluation of Lightning Induced Effects in a Graphite Composite Fairing Structure

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Abstract — Defining the electromagnetic environment inside a graphite composite fairing due to near-by lightning strikes is of interest to spacecraft developers. This effort develops a transmission-line-matrix (TLM) model with CST Microstripes to examine induced voltages on interior wire loops in a composite fairing due to a simulated near-by lightning strike. A physical vehicle-like composite fairing test fixture is constructed to anchor a TLM model in the time domain and a FEKO method of moments model in the frequency domain. Results show that a typical graphite composite fairing provides attenuation resulting in a significant reduction in induced voltages on high impedance circuits despite minimal attenuation of peak magnetic fields propagating through space in near-by lightning strike conditions.

Index Terms — Composite, Lightning, Magnetic, Method of Moments, Shielding, Transmission Line Method.

I. INTRODUCTION

A. Background

Direct strike lightning effects have been thoroughly evaluated for composite aircraft structures [1]. In the space industry, launch commit criteria and ground protection systems such as catenary wires shift the focus for launch vehicle protection to indirect effects from a near-by strike. Note that the use of the term indirect effects based on a nearby strike is different than that of the aircraft industry where the effects on internal circuitry from a strike to the airframe is indicated [2]. Aircraft avionics are typically hardened to this environment, but such hardening is not characteristic of typical spacecraft systems that are sensitive by design. Much work in the launch vehicle industry has concentrated on lightning coupling analysis of the large umbilical cable connecting ground support equipment to vehicle/spacecraft power and data circuits as illustrated in Fig. 1. Accordingly, any protection of spacecraft afforded by the composite structure is not well characterized [3].

Minimal shield transfer impedance is required to reduce the common mode coupling to a differential circuit [1]. When design criteria constraints prohibit adequate shielding, voltages induced into sensitive circuitry are primarily driven by the loop area, magnetic field amplitude, and the transient rise time. Thermal constraints can also limit the application of wire twisting, which makes the cancellation of the magnetic field via loop area reduction impractical.

In the event of a near-by lightning strike the spacecraft system must evaluate the retest criteria. This retest criteria is important because only minimal on-pad testing is possible due to limited interface controls. Triggering of this criteria can lead to payload destack and return to processing facilities where mission specific testing can ensue.
False indications of this trigger based on the assumption of zero shielding in composite fairings is costly from a budget and schedule standpoint. Albeit, the consequences of unnecessary retest are severe, the repercussions of an undetected failure are irreversible. As there is no possibility to retrieve a payload on orbit, a conservative, yet easily implementable prediction of attenuation of indirect lightning effects is desired.

Fig. 1. Launch vehicle and umbilical tower.

### B. Lightning Induced Effects

The time varying magnetic and electric fields lead to induced voltages and currents in vehicle and spacecraft circuitry. The governing equation used to approximate the magnetic field from a nearby lighting strike ignoring ohmic losses is given by

\[
\oint H \cdot dl = I_i + I_d
\]

\[
= \iint_A J_i \cdot da + \iiint_A \frac{\partial}{\partial t}(\varepsilon_0 E) da
\]

Where:

- \( E, H = \text{Electric and magnetic fields} \)
- \( A, l = \text{loop area and length} \)
- \( I_i, J = \text{current and current density} \)
- \( \varepsilon_0 = \text{permittivity of free space} \)
- \( i, d = \text{lightning source and displacement} \)

MIL-STD-464 gives the change in the electric field contributed by a near lightning strike 10 m away as \( 6.8 \times 10^{11} \) volts/meter/second (V/m/s) [4]. Assuming a reasonable worst case circuit area, \( A \), of \( 4 \text{ m} \times 0.05 \text{ m} = 0.2 \text{ m}^2 \), the contributing portion of the magnetic field due to the displacement current \( (I_d) \) is \( 1.2 \text{ A/m} \) [1]. This displacement current is relatively insignificant compared to the contribution of the lightning channel, allowing the magnetostatics assumptions to be applied [1], [5,6]. Hence, an approximation of the magnetic field simplifies to \( I/(2\pi r) \), where \( r \) is the distance from the strike and \( 2\pi r \) represents the circumference of the circle with radius, \( r \). For instance, a 50 kA strike at 10 meters would contribute a magnetic field of 795 (amperes/meter) A/m. To determine the induced voltage that arises due to a lightning related magnetic field, the rise time is key as depicted in (2). This rise time varies from 1.4 µs to 50 ns depending on which component of lightning is active (initial severe stroke, return stroke, multiple stroke, or multiple burst). For most launch sites, the range data includes strike magnitude and location (within a 250 to 500 m accuracy), but does not include rise time information. MIL-STD-464 [4] reports the change of magnetic field with respect to time for a near lightning strike 10 m away as \( 2.2 \times 10^9 \) A/m/s and using this, we get

\[
\text{Max } V_{\text{ioc}} = \frac{d(\mu_0 HA)}{dt} = \mu(2.2 \times 10^9)(0.2) = 552.9 V
\]

Where:

\( \mu_0 = \text{free space permeability} = 4\pi \times 10^{-7} \text{ H / m} \).

The differential circuit voltage will be less than predicted by (2) due to actual circuit impedances and common mode rejection; however, the remaining voltage is undesirable for most spacecraft instrumentation circuits. spacecraft retest criteria of 10 – 50 volts is common; however, lower sensitivities have been reported by design constrained spacecraft payloads.

### C. Motivation

Test data and two-dimensional numerical models presented in the literature for a single composite panel in otherwise conductive enclosures, show greater than 40 dB reduction in
dB/dt levels with a composite panel as compared to a fiberglass panel when a nearby transient lightning pulse is simulated [7-9]. The diffusion of direct strikes through composite walls is addressed in evaluation of composite aircraft in [1]. Spacecraft developers and launch vehicle providers have questioned the applicability of panel only studies to the launch vehicle fairing structure. In this study the attenuation of a composite graphite fairing-like structure to the induced effects of nearby lighting strikes is addressed. A physical fairing fixture model is built and test validation is performed as a baseline for the model. Both frequency and time domain testing are performed to anchor the model.

II. FAIRING MODEL
A. Test Fixture
The scaled fairing fixture model shown in Fig. 2 and used for all simulations in this work is ½ to 1/7 the size of typical launch vehicles. The 1.8 m by 0.6 m fairing fixture is made of two composite fairing halves with tabs at the edges for clamping the fairing enclosure. Two 1 mm 4 ply layers of carbon composite material sandwich a 6.35 mm Rohacell®WF foam core. Rohacell®WF is a closed-cell rigid foam based on polymethacrylimide chemistry, which does not contain any carbon fiber composites (CFC’s) and is often utilized in manufacturing advanced composites for aerospace applications [10]. The surface resistivity was measured as 161 mohms. The composite fairing structure was grounded via a metallic flat plate which interfaced with the bottom edges of the fixture.

B. Composite Structure Model
Modeling the layers of the composite fairing individually requires the mesh to be small with respect to the thickness of each layer and is computationally prohibitive with respect to the entire model size. However, although CFC structures are inhomogeneous and tensor formation of permittivity and permeability are needed for accurate representation of electromagnetic shielding, the frequency range of lightning is generally below the interlayer resonance of composite structures, allowing an effective one layer representation of the composite fairing [11,12]. Literature supports modeling composite materials as a single layer if the period of the structure is small with respect to wavelength [11]. This criterion is clearly met with a thin structure and lightning frequency content below 30 MHz [1]. Several composite builds can effectively be modeled as one layer into the GHz frequency range [11]. Each composite 4 ply build was represented as an electromagnetically penetrable thin film with conductivity parameters developed from surface resistivity measurements [13].

In addition, composite material is not uniform in all directions; hence, the volume conductivity cannot entirely be determined from the surface conductivity and thickness. However, if there are several layers of composite materials, then multiple orientations of the fibers will exist allowing the standard volume resistivity calculated from surface resistance to approximate the actual conductivity of the structure [14]. The conductivity for the graphite composite layer was modeled with the uniform material assumption and calculated using (3) shown below

\[
\sigma = \frac{1}{\rho}, \quad \rho = R_s t, \\
\sigma = \frac{1}{(161 \text{mohm})(1 \text{mm})} = 6211 \text{ s/m} 
\]

Where:
\( \sigma = \text{conductivity in s/m} \),
\( \rho = \text{volume resistivity} \),
\( R_s = \text{surface resistivity, and} \)
\( t = \text{thickness} \).
III. MODEL CHARACTERIZATION

Before examining the induced voltages with precise industry lightning models, a characterization of the composite structure was performed with a lab implementable test set-up. The thin layer approach to model the composite fairing was anchored with test data in both the frequency and time domain.

A. Frequency domain

Initially, an industry standard magnetic shielding test was performed [15]. The test set-up was then simulated in the frequency domain using the method of moments (MoM) solver in the electromagnetic simulation software, EM Software & System's FEKO [16], and an imported Pro-E fairing model. Although time domain computational methods dominate lightning related literature, use of the MoM with post-processing has been shown effective [17]. The equivalent layer model was implemented with an infinitely thin impedance sheet based on the direct surface impedance measurement. The impedance sheet represents the relationship between the tangential electric field on the surface and the electric surface current [18].

For both the modeling and test, a sensor is placed 1 meter high in the center of the fairing (see Fig. 2). The baseline case is obtained from measurements with no fairing in place. A small loop was used to provide external excitation and internal sensing at specific frequencies.

Both test and simulation results, shown in Table 1, indicate an increase in magnetic field shielding effectiveness with increasing frequency upto 10 MHz.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Shielding Effectiveness (Test Data) dB</th>
<th>Shielding Effectiveness (Model Data) dB</th>
<th>Difference dB</th>
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</thead>
<tbody>
<tr>
<td>150 kHz</td>
<td>2</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>300 kHz</td>
<td>5</td>
<td>0.8</td>
<td>4.2</td>
</tr>
<tr>
<td>2 MHz</td>
<td>11</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>5 MHz</td>
<td>17</td>
<td>19.5</td>
<td>2.5</td>
</tr>
<tr>
<td>10 MHz</td>
<td>21</td>
<td>21.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

B. Time Domain

Given the limited frequency content in lightning transient pulses, the TLM tool in CST Microstripes is optimally applied for this electrically small structure. Adaptations can be made to the TLM process to account for edge effects, especially for higher frequency applications [19]. TLM divides the physical space into circuits that can be solved for voltages and currents that are related to fields through analogies to Maxwell’s equations [20]. The current source is proximally placed with respect to the composite fairing structure to represent a low impedance magnetic field associated with near field conditions and thus worst case (minimal) shielding of the composite fairing structure. The distal leg of current loop is selected as far as possible away from the fairing in order to limit field cancellation effects as shown in Fig. 3 [13].

The transient source was implemented with a 2 m square PVC structure supporting a 16 gauge wire. An Electrometrics, EM 3410, spike generator was placed at the base of the structure to drive a 10 µsec pulse into the loop. The closest side of the loop was placed 0.5 meters from the fairing, as depicted in the model shown in Fig. 3. This transient current loop was selected rather than a high voltage source for feasibility of implementation in the laboratory setting.

A B-dot sensor (ELGAL MDM-0) was employed in conjunction with a digital oscilloscope, to measure the change in magnetic field with respect to time in the test case. For simulation, this change was determined by examining the time response of the magnetic field data. The baseline comparison case is obtained from measurements with no fairing in place.

The current source in the model was designed to closely characterize the transient generator pulse that could be implemented with a spike generator into an inductive loop. The laboratory loop was modeled with a 10 ohm load impedance to partially account for the inductance created by
the loop. A 100 volt transient pulse source was applied to a loop with a wire conductivity of $5.87 \times 10^7$ s/m and a radius of 0.15 cm.

The difference in the change in magnetic field with respect to time with and without the fairing was 8.06 dB in simulation and 7.4 dB in test, revealing model and test case agreement.

**IV. INDUCED EFFECTS MODEL**

First, to represent a nearby lighting strike, a 1MV/1Mohm source at the top of a 30 foot long simulated lightning channel was substituted for the loop in the model characterization phase as shown in Fig. 4 [21]. To reduce electric field contributions, the source was shielded with a graphite epoxy box as in [21].

The source was driven by the double exponential source characteristics given in (4) which are based on MIL-STD-464 [4].

$$i(t) = I_0(e^{-\alpha t} - e^{-\beta t})$$

(4)

Where: $I_0 = 218,810$ A, $\alpha = 11,354$ s$^{-1}$,

and $\beta = 647,265$ s$^{-1}$.

The TLM model frequency span is set to 20 MHz for broad band evaluations, and the structure mesh size is driven by this frequency. The run time duration is extended beyond the default settings to account for the total waveform time.

In addition, a loop was added in the simulated vehicle to examine currents and voltages on low and high impedance circuitry with respect to magnetic field peak reduction. The emphasis of this paper is the composite fairing attenuation of induced lightning effects. More detailed studies of the lightning induced effects related to loop ground and termination impedances, loop height above ground, and structure surge impedance modeling can be found in recent studies [22-23].

**V. RESULTS**

Figure 5 depicts the low resistance circuit response excited by a simulated nearby (1 m away) lightning strike with and without a composite fairing surrounding the loop.

![Figure 4: Composite vehicle with a simulated lightning strike.](image)

![Figure 5: Composite fairing to air comparison with low impedance loop coupling.](image)
dominant, as in high impedance circuits, the variation in induced effects is influenced by the diffusion process which slows the rise of the magnetic field \([24]\). The effect is much less dominant in the low impedance circuit.

![Composite fairing to air comparison with high impedance loop coupling.](image)

**Fig. 6.** Composite fairing to air comparison with high impedance loop coupling.

Table 2 provides the composite fairing attenuation effects on magnetic field and coupled loop voltage. It also includes the effects of source distance on the internal magnetic field and the coupled voltages.

**Table 2:** Comparison of fairing attenuation of induced effects for varying internal loop impedance and distance from source

<table>
<thead>
<tr>
<th>Loop Impedance Ohms</th>
<th>0.1 Ohms</th>
<th>1M Ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>1.5</td>
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<tr>
<td>1</td>
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<td>Plane Wave</td>
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</tbody>
</table>

The plane wave case provides the greatest attenuation due to the higher source impedance of the field with respect to the composite structures. Nevertheless, significant attenuation of induced voltage in the high impedance loop is achieved at close distances where the source impedance is lower than a plane wave.

**VI. CONCLUSION**

The results presented show that the TLM thin film modeling of the composite structure is effective for the evaluation of attenuation from frequency based and transient based magnetic fields.

The model was modified to align with the industry approach for lightning induced electromagnetic effects. Results shown indicate a typical graphite composite fairing provides significant reduction in induced voltages on high impedance circuits despite minimal attenuation of peak magnetic fields. The energy in the pulse is spread by the diffusion process through the composite material. This spreading slows the incident pulse rise time which in turn reduces the coupling to the circuit.

This study provides a good insight into the differences between literature that specifies attenuation of lightning induced effects and account for any lightning related attenuation for composite structures. The data from this effort is useful for evaluating spacecraft/launch vehicle destack criteria.

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Analysis of Transient Electromagnetic Scattering from an Overfilled Cavity Embedded in an Impedance Ground Plane

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Abstract — In this paper, we consider the time-domain scattering problem of a two-dimensional overfilled cavity embedded in an impedance ground plane. An artificial boundary condition is introduced on a semicircle enclosing the cavity that couples the fields from the infinite exterior domain to those fields inside. The problem is first discretized in time using the Newmark scheme, and at each time step, we derive the variational formulation for the TM polarization, and establish well-posedness. Numerical implementation of the method for both the planar and overfilled cavity models is also presented.

Index Terms — Impedance boundary conditions, overfilled cavity, time domain.

I. INTRODUCTION

Electromagnetic scattering of cavity-backed apertures has been examined by numerous researchers in the engineering community (for example [1-6]) and the mathematical community (for example [7-10]). For overfilled cavities, we mention the works [11-14]. We note that most of the published work deals with either cavities with PEC ground planes or time-harmonic problems. Here, we consider transient overfilled cavities with impedance boundary conditions. Our approach is unique in that we develop a hybrid finite element-boundary integral mathematical model that incorporates an overfilled cavity with impedance boundary conditions. It will be more mathematically challenging yet more physically realistic and, as a result, has numerous applications.

We organize the paper as follows. In Section II, we introduce the problem setting and geometry. In Section III, we discretize the PDE via the Newmark scheme, first decomposing the entire solution domain into two sub-domains via an artificial semicircle, $B_R$, which entirely encloses the overfilled cavity. This requires solving a nonhomogeneous modified Helmholtz equation with nonhomogeneous impedance boundary conditions at each time step via a generalized Green's function approach to obtain an integral representation. In Section IV, we present the integral representation, the Green’s function for an impedance ground plane, and the properties of the Steklov-Poincaré operator. We conclude in Section V, by producing a variational formulation of the problem that is well-posed. In Section VI, we provide numerical results for a simplified planar cavity and an overfilled cavity that demonstrates the analysis can be implemented.

II. PROBLEM SETTING

Let $\Omega \subset \mathbb{R}^2$ be the cross-section (cavity interior) of a $z$-invariant cavity in the infinite ground plane, and the infinite homogenous, isotropic region above the cavity as $\mathcal{U} = \mathbb{R}^2_+ \setminus \Omega$. Furthermore, let $B_R$ be a semicircle of radius $R$, centered at the origin and surrounded by free space, large enough to completely enclose the overfilled portion of the cavity. We denote the region bounded by $B_R$ and the cavity wall $S$ as $\Omega_R$, so that $\Omega_R$ consists of the cavity itself and the homogeneous part between $B_R$ and $\Gamma$. Let $\mathcal{U}_R$ be the homogeneous region outside of $\Omega_R$; that is, $\mathcal{U}_R = \{(r, \theta) : r > R, 0 < \theta < \pi\}$. Refer to Fig. 1 for the complete problem geometry.
The following formulation is modeled after Van and Wood [13]. In this case, the magnetic field \( H \) is transverse to the \( z \)-axis so that \( E \) and \( H \) are of the form \( (0, 0, E_z) \) and \( (E_{x0}, E_{y0}, 0) \). In this case, the nonzero component of the total electric field \( E_z \) satisfies the following boundary value problem:

\[
-\Delta E_z^s + \varepsilon_r \frac{\partial^2 E_z^s}{\partial t^2} = 0 \quad \text{in } \Omega \cup \mathcal{U} \times (0, \infty),
\]

\[
\frac{\partial E_z^s}{\partial t} \bigg|_{t=0} = E_{t,0},
\]

\[
\frac{\partial E_z^s}{\partial t} \bigg|_{t=0} = E_{t,0}^i
\]

where \( \varepsilon_r = \varepsilon / \varepsilon_0 \) is the relative electric permittivity, \( E_0 \) and \( E_{t,0} \) are the given initial conditions and \( \eta = \sqrt{\mu_r / \varepsilon_r} \) is the normalized intrinsic impedance of the infinite ground plane.

We are assuming that we have a non-dispersive material in the cavity, or that the permittivity is not a function of frequency, but could vary with respect to position. That is, we are assuming that the impedance is constant in the time domain. We observe the scattered field \( E_z^s \) solves:

\[
-\Delta E_z^s + \frac{\partial^2 E_z^s}{\partial t^2} = 0 \quad \text{in } \mathcal{U} \times (0, \infty),
\]

\[
\frac{\partial E_z^s}{\partial t} + \frac{\eta}{\mu} \frac{\partial E_z^s}{\partial n} = -\left( \frac{\partial E_z^i}{\partial t} + \frac{\eta}{\mu} \frac{\partial E_z^i}{\partial n} \right),
\]

on \( \Gamma_{\text{ext}} \cup \Gamma \times (0, \infty) \), and also satisfies the appropriate radiation condition at infinity.

The homogeneous region \( \mathcal{U} \) above the protruding cavity is assumed to be air and hence, its permittivity is \( \varepsilon_r = 1 \). In \( \mathcal{U} \), the total field can be decomposed as \( E_z = E_z^i + E_z^s \) where \( E_z^i \) is the incident field, and \( E_z^s \) the scattered field.

### III. SEMIDISCRETE PROBLEM

We will first decompose the entire solution domain to two sub-domains via an artificial semicircle, \( \mathcal{B}_R \), which entirely encloses the overfilled cavity (refer to Fig. 2). These two sub-domains consist of the infinite upper half plane over the impedance plane exterior to the semicircle, denoted \( \mathcal{U}_R \), and the cavity plus the interior region of the semicircle, denoted \( \Omega_R \).

For this problem, as in [13], we will choose to use the Newmark scheme, an implicit time-stepping method that offers the advantage of stability. It is defined by the following: let \( N \) be a positive integer, \( T \) be the time interval, \( \delta t = T / N \) be the temporal step size, and \( t_{n+1} = (n+1)\delta t \) for \( n = 0, 1, 2, ..., N - 1 \). The following are approximations at \( t = t_{n+1} \):

\[
u^{n+1} \approx u, \quad \dot{u}^{n+1} \approx \frac{\partial u}{\partial t}, \quad \ddot{u}^{n+1} \approx \frac{\partial^2 u}{\partial t^2}.
\]
We further define $\gamma$ and $\beta$ as parameters to be determined to guarantee stability of the scheme, 
\[ \alpha^2 = \frac{1}{(\delta t)^2 \beta}, \] 
and $\bar{u}$ denotes predicted values.

Therefore, it can be shown that the scattered field $u^{s, \alpha+1}$ satisfies the following exterior problem:
\[ -\Delta u^{s, \alpha+1} + \alpha^2 u^{s, \alpha+1} = \alpha^2 \bar{u}^{s, \alpha+1} \text{ in } \mathcal{U}_R, \]
\[ u^{s, \alpha+1}(r, \theta) = g(r, \theta) \text{ on } \mathcal{B}_R, \]
\[ \delta t \gamma \alpha^2 u^{s, \alpha+1} + \frac{\eta}{\mu} \frac{\partial u^{s, \alpha+1}}{\partial n} = 0 \text{ on } \Gamma_{ext}, \] 
where $g = u^{n+1} - u^{\alpha+1}$ and the radiation condition is satisfied.

Therefore, we seek the solution for the nonhomogeneous modified Helmholtz equation
\[ -\Delta u(r) + \alpha^2 u(r) = f((r)), \] 
where $r$ denotes location and $f(r) = \alpha^2 \bar{u}^{s, \alpha+1}(r)$. This equation is subject to nonhomogeneous boundary conditions of the form $A u(r_i) + B \frac{\partial u(r_i)}{\partial n} = h(r_i)$, where $r_i$ is on the surface and $n$ is the outward unit normal, $A$ and $B$ are constants defined as $A = \Delta t \gamma \alpha^2$ and $B = \frac{\eta}{\mu}$, and $h(r_i) = \Delta t \gamma \alpha^2 \bar{u}^{\alpha+1} - \bar{u}^{\alpha+1}$.

IV. INTEGRAL REPRESENTATION OF SOLUTION AND GREEN'S FUNCTION

The integral representation of the solution in the exterior domain (the annular sector depicted in Fig. 3) can be shown to be, for $r \in \mathcal{U}_R$ and source location at $r'$:
\[ u(r) = \int_{r'} G(r \mid r') f(r') dS' - \]
\[ \frac{1}{A} \int_{\Gamma_{ext}} h(r') \frac{\partial G(r \mid r')}{\partial n'} dr' + \]
\[ \int_{\mathcal{B}_R} \left( G(r \mid r') \frac{\partial u(r')}{\partial n'} - u(r') \frac{\partial G(r \mid r')}{\partial n'} \right) d\theta'. \] 

The Green's function has been developed from several sources for an impedance half-plane, see for example, [15-17]. We follow Durán, et al., in [11] to obtain the following, noting the integral expression may be simplified through residue analysis:
\[ G(r \mid r') = \frac{1}{2\pi} K_0(\alpha R) - \frac{1}{2\pi} K_0(\alpha R') \]
\[ -\frac{2B}{4\pi} \int_{-\infty}^{\infty} e^{-\sqrt{x^2 + \alpha^2 (y+y')}} \frac{d\xi}{(A - B\sqrt{x^2 + \alpha^2})}. \]

We note $R = \sqrt{(x-x')^2 + (y-y')^2}$ and $R' = \sqrt{(x-x')^2 + (y+y')^2}$. At this point, we implement an integral equation method along the artificial boundary, $\mathcal{B}_R$, to couple the solution along the artificial boundary through the Dirichlet-to-Neumann mapping, or Steklov-Poincaré operator. Following Hsiao, et al., in [18], where we define $\phi(r') = \frac{\partial u(r')}{\partial n'}$, all of the boundary integral operators for $r \in \mathcal{B}_R$ are expressed as follows:
\[ (S\phi)(r) = \int_{\mathcal{B}_R} G(r \mid r') \phi(r') d\theta', \]
\[ (Du)(r) = \int_{\mathcal{B}_R} u(r') \frac{\partial G(r \mid r')}{\partial n'} d\theta', \]
\[ (A\phi)(r) = \int_{\mathcal{B}_R} \frac{\partial G(r \mid r')}{\partial n} \phi(r') d\theta', \]
\[ (Hu)(r) = -\int_{\mathcal{B}_R} u(r') \frac{\partial}{\partial n'} \frac{\partial G(r \mid r')}{\partial n'} d\theta'. \]
We further define the Newton Potential to consist of the following terms, for $r \in B_R$:

$$(Nf)(r) = \iint_{S'} f(r') G(|r| r') dS',$$

$$( Pf)(r) = -\frac{1}{4} \int_{S'} h(r') \frac{\partial G(|r| r')} {\partial n'} dx'.$$

The mapping properties are well-established for the preceding boundary integral operators. As a result, as in [18] or [19], we define $1/2 1/2 : R R \rightarrow \mathbb{C}$ as a bounded Steklov-Poincaré operator as follows:

$$(T_R u)(r) = S^{-1} \left(\frac{1}{2} I + D\right) u(r) = \left[ \left(\frac{1}{2} I + A\right) S^{-1} \left(\frac{1}{2} I + D\right) + H \right] u(r).$$

The second expression is the symmetric expression of the operator. In addition, the following theorem, similar to Cakoni and Colton's Theorem 5.20 in [20], also applies:

**Theorem 1** The Steklov-Poincaré operator, $T_R$, is a bounded, linear operator from $H^{1/2}(B_R) \rightarrow H^{-1/2}(B_R)$. Also, the principal part of $T_R$, referred to as $T_{R,P}$, satisfies the coercivity estimate:

$$-\langle T_{R,P} u, u \rangle \geq C \left\| u \right\|_{H^{1/2}(B_R)}^2$$

for some $C > 0$, such that the difference $T_R - T_{R,P}$ is a compact operator from $H^{1/2}(B_R) \rightarrow H^{-1/2}(B_R)$.

**V. VARIATIONAL FORMULATION**

Instead of enforcing the boundary conditions on the test function space $V$ as in [1], we choose to define the subspace $V$ simply as $H^1(\Omega_R)$. The variational formulation will then be to find $u \in V$ such that:

$$b_{TM}(u, v) = F(v) \quad \forall v \in V. \tag{6}$$

We define the sesquilinear term:

$$b_{TM}(u, v) = \int_{\Omega_R} \nabla u \cdot \nabla v dx dy - \int_{B_R} T_R u \tilde{v} d\ell$$

$$+ \frac{\mu}{\eta} \Delta t \gamma \alpha^2 \int_S \tilde{u} \tilde{v} d\ell$$

$$+ \alpha^2 \int_{\Omega_R} \epsilon \tilde{u} \tilde{v} dx dy, \tag{7}$$

as well as the bounded conjugate linear functional term:

$$F(v) = \int_{B_R} J \tilde{u} \tilde{v} d\ell - \int_{B_R} \Psi \tilde{u} \tilde{v} d\ell$$

$$+ \frac{\mu}{\eta} \Delta t \gamma \alpha^2 \int_S \tilde{u} \tilde{v} d\ell$$

$$- \frac{\mu}{\eta} \int_S \tilde{u} \tilde{v} d\ell + \alpha^2 \int_{\Omega_R} \epsilon \tilde{u} \tilde{v} dx dy, \tag{8}$$

and we further define the bounded terms:

$$\left(\Psi_{\tilde{u}}\tilde{u}^{r,\sigma+1}\right)(r) = S^{-1}((N\tilde{u}^{r,\sigma+1})(r) + (P\tilde{u}^{r,\sigma+1})(r))$$

and

$$J = \frac{\partial u'} {\partial t} \bigg|_{r=\ell} - T_R u'.$$

**Theorem 2** The variational problem (6) is well-posed: a solution $u \in V$ exists, is unique, and for $C > 0$:

$$\left\| u \right\| \leq C \left[ \left\| u' \right\| + \left\| \tilde{u}^{r} \right\| + \left\| \tilde{u} \right\| + \left\| \epsilon \tilde{u} \right\| \right].$$

Using the results of the previous section, and writing $b_{TM}(u, v)$ as the sum of a coercive operator and compact operator, (6) can be shown to be well-posed through variational methods.

**VI. NUMERICAL RESULTS**

**A. Planar cavity results**

We ran a numerical study on a planar cavity to provide a context for the theory. We ran the data for two separate cases: a perfect electrical conducting (PEC) surface on the plane $\Gamma_{ext}$ and PEC surface on the cavity walls, $S$, and a PEC plane and impedance boundary conditions (IBC) on the cavity walls. The idea is to see the progression as we introduce IBC on a strict PEC surface, in which we would expect more attenuation as the surface changes.
The numerical model was set up as depicted in Fig. 4, using an incident Gaussian Pulse with $\delta t = 0.0625$, $\varepsilon_r = 2$, $\mu_0 = 1$, and Newmark parameters $\gamma = 0.95$ and $\beta = 0.5256$ to ensure stability. The visual depictions of the electric field are plotted against time as measured in light meters (LM), which is the amount of time light travels in one meter of free space.

![Fig. 4. Shallow cavity (1m by 0.25m).](image)

The first run is depicted in Fig. 5. With the PEC plane and PEC cavity walls as a benchmark, we observe that the case with IBC enforced at the cavity walls ($\eta = 0.8$) exhibits more attenuated characteristics. We also observe the stability of the Newmark scheme over time.

![Fig. 5. Shallow cavity with $\eta = 0.8$.](image)

We also want to observe the effects as $\eta \to 0$, as we would expect the field to exhibit the characteristics of a strict PEC on the plane and cavity walls. This is clearly evident in Fig. 6 with the PEC plane and IBC cavity walls simulation showing more oscillatory behavior as in the strict PEC case. Again, we observe the stability of the Newmark scheme over time.

![Fig. 6. Shallow cavity with $\eta = 0.2$.](image)

We also wanted to observe the effects of changing boundary conditions on the radar cross section of the cavity model. We ran two simulations, at the selected frequencies of 289.5 MHz and 480.45 MHz, as depicted in Fig. 7 and Fig. 8.

![Fig. 7. Shallow cavity RCS at 289.5 MHz.](image)

In both cases, we observe the expected lobing, and note the fact that as the frequency is increased to 480.45 MHz, the separation in RCS values between the two cases is more apparent. More specifically, for 480.45 MHz, the RCS values are lower as the boundary conditions approach a complete IBC on the cavity walls.
B. Overfilled cavity results

As with the planar cavity, we used the same set of parameters for the overfilled cavity depicted in Fig. 9.

However, in this model, we add a third case of an IBC on both the cavity walls and plane. We also chose an observation point at the origin. We note in both Fig. 10 and Fig. 11 that the depicted scattered field becomes more attenuated as the boundary conditions for the plane and cavity walls approach an IBC surface. The observation is truncated at 50 LM for scaling, but the simulations exhibit the same stability as with the planar case beyond this point. It is also evident that the fields are more oscillatory than the shallow cavity due to the presence of more material both above and below the observation point. We also note in Fig. 11 that as $\eta \to 0$ the field begins to exhibit the characteristics of a strict PEC surface, as seen in the more oscillatory behavior of the IBC plane and IBC cavity walls condition.

We also present the RCS data in Fig. 12 for 289.5 MHz and Fig. 13 for 480.45 MHz. Again, we notice similar behavior at the lower frequency for all three cases; however, at a higher frequency, we observe both the expected lobing and the more attenuated results as the IBC is enforced on both the cavity walls and the plane.
We present a mathematical model for analyzing transient electromagnetic scattering induced by an overfilled cavity embedded in an impedance ground plane. We have established the well-posedness of the problem through a variational formulation, and this sets the foundation for numerical implementation through a hybrid finite element - boundary integral technique. We present electric field and RCS data for both a simplified planar cavity as well as the overfilled cavity model, which both exhibit expected results.

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Coupled Electromagnetic Field Computation with External Circuit for the Evaluation the Performance of Electric Motor Designs

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Abstract -- In this paper, a set of PM machine's designs, having the similar level of nominal input and outputs i.e. voltage, torque, and speed were compared to evaluate the effectiveness of a computational design procedure. The designs include the machines with distributed winding arrangements, different number of slots, different pole widths, and different slot opening shapes. The physical characteristics of machines such as the cogging torque, back emf, flux linkages, and inductances were calculated from a 2D nonlinear transient finite element analysis with motion. The torque and speed profiles of all of the machines were calculated from the phase variable modeling approach. The phase variable model is a database representation of the machine's numerical model and it allows computationally efficient dynamic simulation of the coupled problem with realistic physics-based design. The phase variable models of the machines were linked to the driving circuit to determine the mutual effect of machine design parameter and the drive topology on the performance measures of machines.

Index terms-- Cogging torque, electromagnetic field computation, finite element analysis, motor design, phase variable model, PM machines.

I. INTRODUCTION

Electric machines play an essential role in many industries. PM synchronous motors are widely utilized due to their high power density, low maintenance costs, and high efficiency. From a structural point of view, depending on the setting of the magnets on the rotor, the synchronous motors can be constructed by either burying the magnets within the rotor iron or by mounting them on the rotor surface. Most PM synchronous motors can be categorized into three general categories; surface mounted PM synchronous motors (SPM) which have their permanent magnets mounted on the surface of the rotor, inset PM synchronous motors in which the permanent magnets are inset or partially inset into the rotor, and interior PM synchronous motors which have the permanent magnets completely buried inside the rotor [1]. From machine winding point of view, concentrated windings versus distributed winding for PM motors are widely used because of the low manufacturing cost. Comparisons and quantitative analysis of these two types of windings for two motors with same stator were studied [2, 3]. Nevertheless, it is recognized that in the case of design of the PM motor, infinite number of combinations would result in acceptable outputs. Depending on such outputs and the planned application, different designs could be achieved.

One of the intrinsic characteristics of PM motors is the pulsating torque. This ripple torque is parasitic, and can produce acoustic noise, mechanical vibration, and other problems in electric machine drive systems such as increased iron losses and total harmonic distortion [4]. Therefore, the machine designer must consider these issues in the design process [4, 5]. The study of pulsating torque is important for the application of constant speed or high-precision position control, especially at low speed applications. The torque pulsations are due to the cogging torque and the electromagnetic torque ripple. Many techniques for mitigating the cogging torque were proposed in the literature [6-8]. Some of the methods manipulate the stator or rotor separately or both of them together. This includes employing a fractional number of slots per pole, skewing of the magnets, slots, and/or the opening of slots, shifting and shaping of the main magnets, shifting the slots opening, optimizing the magnet pole-arc to
pole-pitch ratio, and introducing supplementary slots or teeth [6, 7, 8]. The fractional number of slots/pole can change both the amplitude and the frequency of the cogging torque to a desirable value. It also increases the fundamental order of the magnetic flux density in the air gap due to the different relative circumferential positions of the stator slots with respect to the edges of the magnets when this topology is used. On the other hand, the actual back emf waveform of PM motors depends on the conductor distributions and flux density. This in turn is a function of the magnetization characteristic of the magnet stator teeth, and slot structures.

PM machines with trapezoidal back emf have been widely used due to the simplicity in their control [9]. The skew of magnets is effective in reducing the harmonic content in the flux linkage and back EMF waveform, as well as in reducing the cogging torque [8].

As discussed above, different practical and theoretical methods have been proposed for mitigating the cogging torque, and also manipulating the back emf waveform for a simpler and cheaper driving strategy. In fact most of these strategies increase the cost because they alter the conventional manufacturing processes. In this paper, sets of designs representing studied cases were obtained from classic design procedure [1, 10]. Each machine was prepared for an FE analysis as shown in Figure 2(b). Here, the accuracy of manufacturing tolerance is assumed as 0.05 millimeter. Therefore all of the design parameters are rounded to the nearest real value.

Another test set was prepared where the influence of the slot opening geometry on the physical behavior of the machine and its performance measure was investigated. In order to examine the influence of the slot opening geometry, the five 36-slot/6-pole machine designs shown in Figure 1(c) were considered. The details of stator geometry are shown in Table 3. The inter pole angles in all of the designs were two degrees while the rotor geometry remain the same for these five test shown in Table 4. In this set of tests, Design 1 has tapered tooth tips and parallel slot opening. Design 2 has straight tooth tips and parallel slot opening, Design 3 has straight tooth tips and no slot opening, Design 4 has tapered tooth tips and non-parallel slot opening, and Design 5 has straight tooth tips and non-parallel slot opening. All the other stator design parameter remains the same for all of the designs. It is mentioned that, the chosen current density for this range of machine ensure us the thermal limitations; however for a secure and optimal design, a thermoelectric design procedure will be a superior solution.

Here, depending to the voltage value of the DC bus before the conventional 6-switch, 3-phase

II. CASE OF STUDIES
A. Preparation of case studies for field computation

The goal of this section is the preparation of a set of machines with different number of slots, pole widths, different slot opening shape, but with the same range of speed, input power, and voltage. Four different sets of machines were designed where the designs vary in the number of slots and pole widths. The totals of twenty one different machine designs were considered. A schematic view of the designed machine is shown in figures 1(a) and 1(b). All of the machine designs created from a classic design procedure [1, 10] for a WYn winding, 2-hp, 1200-rpm, 6-pole, phase voltage of 111.5-Volt, and current density of 3 A/mm².

The difference in the designs of the machines is shown in Table 1. In the first set, we have eighteen slots with four different pole widths which make four different machines, and all of the coils in the stator are in series together in each phase. In the second set, the number of slots were changed to thirty six, and four different poles width make four different machines, where there is two parallel paths for currents in each phase. The other sets were designed for fifty four and seventy two slots, respectively as shown in Table 1. In this table \(N_p\), \(S_p\), and \(C_s\) stand for number of parallel paths, number of slot per pole, and coil span, respectively. The design details for the various sets were illustrated in Tables 2 and 3. The used magnetic material for all of the machines is Sm2Co17. The magnets are radically magnetized. After a classic design procedure, each machine was prepared for an FE analysis as shown in Figure 2 (b). Here, the accuracy of manufacturing tolerance is assumed as 0.05 millimeter. Therefore all of the design parameters are rounded to the nearest real value.

Here, depending to the voltage value of the DC bus before the conventional 6-switch, 3-phase
DC/AC inverter and also control strategy of the inverter, the maximum amplitude of the back emf should be chosen otherwise the desired speed and torque characteristics will not be achieved. The four well-known, modulation strategies for the inverter connected to the motors are: six-step inverter control, sinusoidal PWM, space vector PWM, harmonic elimination, hysteresis, Delta and Third harmonic injections. The respective phase-neutral DC bus utilization of each of these control strategy are $2V_{dc}/\pi$, $V_{dc}/2$, $V_{dc}/\sqrt{3}$, $V_{dc}/2$, $V_{dc}/2$, $V_{dc}/\pi$, and $V_{dc}/2$. The rule is that the maximum amplitude of the back emf voltage of each of the phases should be always lower than the phase-neutral voltage in order to ensure a proper speed control. If this rule is ignored then unwanted fluctuations will appear in the torque and speed profiles and more probably the speed control will lost. Therefore, in this paper the number of coil’s turns per phase is chosen based upon this criterion. In this work, the voltage of the DC bus for the inverter is chosen as 300-V, and the control strategy is chosen as Hysteresis current regulated control, therefore the maximum back emf voltage is limited to a value lower than 150-V. However, for a secure current hysteresis control for a wide speed range, the maximum back emf should be chosen with a good security margin. Here, it is assumed that the drive is not equipped with the flux weakening control therefore in the designs the maximum amplitude of the fundamental component of the back emf were limited to 100-V to offer more security margin for over speed conditions, likely 150% of the nominal speed. The hysteresis band in the hysteresis current control is fixed as 0.3 Ampere.
Table 4: Rotor geometry of studied case (mm)

<table>
<thead>
<tr>
<th>Set number</th>
<th>Rro</th>
<th>Sr</th>
<th>Wry</th>
<th>Lm</th>
<th>g</th>
</tr>
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<tr>
<td>All sets</td>
<td>66.6</td>
<td>44.1</td>
<td>16.4</td>
<td>5.8</td>
<td>3</td>
</tr>
</tbody>
</table>

### III. THE PHASE VARIABLE MODEL

The phase variable model of PM machines is an accurate and fast model for the purpose of integrated drive system simulations. This model uses transient FE solutions to establish a detailed block description of the implemented machines in a Simulink environment as shown in Figure 2(a). This model accounts for flux weakening as well as other performances [11, 12, and 14]. The model is essentially a database representation of the nonlinear transient operation of the machine to allow the use of a detailed computational model for dynamic simulation.

The creation of the phase variable model consists of two discrete steps. In the first step, a linear transient FE analysis is performed to calculate the cogging torque, back emf, flux linkage, and the inductance matrix of the machine. The FE-based phase variable model is rotor–position dependent, therefore, the FE analysis must take the transient analysis and the motion of rotor into account. In the FE domain, the corresponding magnetic vector potential formulation is calculated as:

\[
-\nabla \left( \sigma \frac{\partial A}{\partial t} - \sigma \vartheta \times (\nabla \times A) - J^e \right) = 0
\]

\[
\sigma \frac{\partial A}{\partial t} + \nabla \times (\mu^{-1} \nabla \times A - M - \sigma \vartheta \times (\nabla \times A)) = J^e
\]  

(1)  

(2)

where \( \sigma \) is the conductivity, \( A \) is the vector potential, \( \vartheta \) is the velocity of the modeled object, \( J^e \) is the external current density, \( \mu \) is the permeability, and \( M \) is the magnetisation. The constitutive relation considering ferromagnetic saturation is:

\[
B = \mu_0(H + M)
\]

(3)

where \( B \) is the flux density, \( H \) is the field strength, and \( \mu_0 \) is the permeability of air.

Following the FE analysis, the FE output parameters are collected into lookup tables in the circuit environment. The second step is the implementation of the machine equations, equations (4) - (8), in the circuit environment. The values are retrieved via look-up tables to create the database and implement it. In the circuit environment, the back emf, and the flux linkage, cogging torque as well as the inductances are updated for each rotation position varying with the speed of the machine. Rotor-position-dependent inductance matrix is calculated by the incremental method [13]. Following the implementation of the phase variable model in Simulink, a hysteresis current regulated drive with speed controller were linked with the phase variable model of the machine to control the speed as given in Figure 2(b).

\[
V_{abc} = R_{abc}i_{abc} + \frac{d\varphi_{abc}(i_{abc}, \theta)}{dt}
\]

\[
\varphi_{abc}(i_{abc}, \theta) = \varphi_{sabc}(\theta) + \varphi_{rabc}(\theta)
\]

\[
T_M = [p(0.5i_{abc}^2L_{abc}(\theta)/d\theta).i_{abc} + i_{abc}^2d\varphi_{rabc}(\theta)/d\theta)] + T_{cog}(\theta)
\]

\[
\frac{J\omega}{dt} = T_m - F\omega - T_L
\]

\[
\omega = \frac{d\theta}{dt}
\]

(4)  

(5)  

(6)  

(7)  

(8)

In the above equations, \( V_{abc}, R_{abc}, \) and \( i_{abc} \) are the terminal voltage, resistance, and current of the stator winding, respectively. The flux linkage \( \varphi_{abc} \) is composed of two parts, see Eq. (5). The first part is related to the inductance \( L_{abc} \) of the stator winding, while the other part is contributed by the permanent magnets on the rotor, represented by \( \varphi_{rabc} \). The cogging torque \( T_{cog} \) is added to obtain the total output torque \( T_m \), as shown in Eq. (7). The rotation angle of the rotor position is represented by \( \theta \). Here, \( L_{abc}, \varphi_{abc} \) and \( T_{cog} \) are considered as rotor-position-dependent parameters. Also, in these equations \( p, J, \omega \) and \( F \) are the number of pole pairs, inertia, angular speed, and friction factor, respectively. The load torque is \( T_L \) in Eq. (7).

![Fig. 2. (a) Phase-variable model of PM machine [14].](image)
IV. SIMULATION RESULTS

A. The cogging torque

The cogging torque is the consequence of the interaction between the permanent magnet fields and the stator in the neighbor of the air gap that is because of the reluctance variations with the rotor position and that is independent of the stator current [11]. Figure 3 (A) to (D) show the simulated cogging torque for different angle α. It can be observed that the maximum value of cogging torque and its frequency is dependent to the number of slots and pole widths. In this study, as α is changed, the inner radius of the poles are kept unchanged. From Figure 3, it is concluded that for the eight slots and by decreasing the pole width, the maximum of the cogging torque is reduced, but its frequency increases. Therefore, it is concluded that a change in the angle α would change the cogging torque to a better value. As can be seen in Figures 3 (a) to (d), it is concluded that the cogging torque can be a function of (α, Ns) which has a minimum that, in this case, occur at α=2 degree. Moreover, by comparison of Figures 3 (a) to (d), it is concluded that the number of slots in the design procedure would highly have an influence on the maximum amplitude of the cogging torque.

The cogging torque calculation in the 5th set, Figure 3 (e), shows that the 2nd design has the highest cogging torque and the 3rd design has the lowest cogging torque. The cogging torques of the 1st, 4th, and 5th designs are almost similar with the maximum amplitude equal to 0.6 (n. m).
Fig. 3. (d) Cogging torque of the 4th set.

Fig. 3. (c) Cogging torque of the 5th set.

B. The back emf and flux linkage as a function of design parameters:

The back emf is the consequence of the induced voltage in the coils due to the rotation of the magnetic field produced by the magnets. The waveform of the back emf has a direct influence on the machine current and therefore the torque ripple [11-12]. As can be seen from Figure 4, as the pole width is decreased, i.e. when \( \alpha \) is increased, the flux opens a new path through the iron of the inter-pole to the rotor yoke. Therefore, the concentration of flux inside the coils is decreased and therefore the root mean square value of the induced back emf in the coils is reduced.

Fig. 4. The flux picture as a function for the 1th set.

Figures 5 (a) to 5 (d) show the back emfs for different pole width and number of slots. As can be seen from these figures, when the number of slots is increased the influence of the pole width becomes more visible. For example, for 72 stator slots, as the angle \( \alpha \) increase, the back emf waveforms become more similar to a sinusoidal waveform than for the 36 slots machine. Also one can observe that the maximum value of the back emf is independent of the angle \( \alpha \). Therefore, it is concluded that the back emf of a motor with a higher number of slots is more sensitive to the pole width. In fact the shape of the back emf has an essential role for choosing the driving strategy. By comparison of Figures 8(a) to (d) it can be concluded that, if a trapezoidal waveform of the back emf is required, the lower inter-pole angle is preferable and if a sine waveform of the back emf is required, a higher inter-pole angle would be preferable although this conclusion is almost validated for radial and parallel magnetized permanent magnets with equal inter-pole angles [1]. Moreover, by comparison of the Figures 5(a) to (d) concerning different number of slots it is seen that the machines with higher number of slots can provide the drive circuit with better trapezoidal or sine back emf waveform; for example, in Figure 5 (a) it is seen that achievement of a pure sine or a pure trapezoidal back emf waveform is indeed unachievable. In fact, creation of a proper current for ripple free performance with the used drive topology and winding arrangement maybe not be feasible.

On the other hand, a closer look at the effect of the number of slots reveals that with smaller number of slots, a smaller number of coils are required. However, the number of turns per coil and the size of the slot would be larger. A small number of slots may lead to a small savings in cost. However, the effect of the stator slots on the air gap flux and therefore back emf in small machines is considerable.

Figure 6 (a) show the flux linkage of the 1st to 4th set. As can be seen from this Figure, the number of slots has minor influence on the maximum amplitude of flux linkage, but as the pole width is decreased the maximum amplitude of the flux is decreased. Moreover, the flux linkage of the 5th set, Figure 6 (b), shows that, the 3rd set has the lowest flux linkage that the reason is that the flux closes its path in the added iron to the slot openings area. The flux linkages of other test set are relatively
remained unchanged.

Figures 7 (a) and 7 (b) show the self and mutual inductances of the 5th set. As it is seen from the inductance profiles, the 3rd design has the highest self and the lowest mutual inductance. The self and the mutual inductance of the other designs are at the same level. It is evident that the slot opening geometry has a noticeable effect on the inductances. This shows that, the machines with the highest inductances have higher start-up time but have lowest speed fluctuations. Moreover, the inductance study of the 1th to 4th set show that, the higher number of slot/pole ratio, the lower will be the winding inductance ratio. This is due to the number of the series turns per phase reduces in order to achieve a given back emf.

Figures 8(a) and 8 (b) show the total output torque against different number of slots and slot openings shapes respectively. The output torque is obtained from simulation of the phase variable model. All of the simulations had the same drive system. A hysteresis band speed controller was used to control the speed. The torque is calculated for the pole width in which the cogging torque has minimum values in all of the cases, i.e. the value of $\alpha$ is equal to 2. As illustrated in Figure 8 (a), as the number of slots is increased, the torque ripples and the setting time of the torque is decreased. In figure 8 (b) less torque ripple compared to other designs is seen.

Figures 9(a) and 9(b) show the speed profile with different number of slots and slot openings shapes, respectively. From Figure 9 (a), it is concluded that the higher number of slots has lower speed fluctuations which can be an important factor for some special application. In Figure 9 (b), design 3, lower speed fluctuation but higher start-up time compared to other machines is seen. The hysteresis current loop control used for the speed control of the machine has higher dynamic comparing to other modulation strategies such as SVPWM, SPWM, harmonic elimination, etc. This method is also sensitive to the inductance of the electrical load. As the inductance of the machine is increased the current ripple is decreased. As a result, the electrical torque ripple and the speed fluctuations is decreased. However, the higher inductance the start-up time of the machine and the setting time of the speed during start-up or any change in speed reference is increased.
V. CONCLUSION

We investigated the effect of the number of slots, pole width, and slot openings shape as design parameters on the cogging torque, back emf, flux linkage, total torque, and speed for various PM motors controlled by a sinusoidal current regulated drive. The study was performed on twenty one different machines with similar nominal range of inputs/output. The simulation results shown that as the number of slots was increased, the cogging torque was generally decreased. In addition, it was shown that the pole width has a good potential to effectively change the cogging torque. It was found that a minimum cogging torque can be found for a specific pole width. Moreover, it was shown that a machine with higher number of slots has lower setting time compared to a machine with lower number of slots where the speed fluctuations and torque ripples were also relatively reduced. It can be concluded that a machine with higher number of slots can show better performance measures than a machine with lower number of slots, although it may increase the cost and require more accuracy in manufacturing tolerances.
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REFERENCES


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Adaptive Design Specifications and Coarsely-Discretized EM Models for Rapid Optimization of Microwave Structures

Slawomir Koziel

Abstract — A simple and efficient procedure for EM-simulation-driven design optimization of microwave devices is discussed. Our approach exploits recently introduced adaptively adjusted design specifications technique that shifts the optimization burden into a relaxed-accuracy and computationally cheap (low-fidelity) model of the structure under consideration, evaluated using the same EM solver as the original (high-fidelity) model but with coarse discretization. The unavoidable misalignment between the low- and high-fidelity models is accounted for by suitable adjustments of the design specifications. The presented method is simple to implement and allows rapid design improvement as demonstrated through examples.

Index Terms — Adaptive design specifications, computer-aided design (CAD), electromagnetic simulation, simulation-driven design.

I. INTRODUCTION

Simulation-driven design and design optimization is ubiquitous in contemporary microwave engineering. For many classes of microwave structures no systematic design procedures are available so that EM-based design becomes the only option. Examples include ultrawideband (UWB) antennas [1], dielectric resonator antennas [2] and substrate integrated circuits [3]. On the other hand, increasing complexity of microwave devices and the demand for high accuracy make the direct optimization involving numerous electromagnetic (EM) simulations impractical because of the computational cost of such a process. Co-
simulation [4-6] is only a partial solution because the circuit models with embedded EM components are still directly optimized.

A cost-efficient design of microwave structures exploiting EM solvers can be realized using surrogate-based optimization (SBO) [7, 8]. In SBO, the direct optimization of the CPU-intensive EM-evaluated structure of interest (high-fidelity model) is replaced by iterative updating and re-optimization of its computationally cheap representation, the surrogate. The successful SBO approaches in microwave area are space mapping (SM) [9-16] and various forms of tuning [17, 18] as well as combinations of both [19, 20]. Other SBO methods used in microwave engineering include manifold mapping [28] as well as techniques exploiting variable-fidelity EM simulations [29, 30]. Space mapping builds the surrogate using a physically-based low-fidelity EM model — typically an equivalent circuit. Tuning approaches are based on embedding circuit-theory-based tuning elements into the structure of interest using properly located internal ports [18]. Both approaches can be very efficient and yield satisfactory designs after a few full-wave EM simulations of the structures under consideration [9, 18].

Unfortunately, implementation of both SM and tuning may not be straightforward. In particular, modification of the structure being optimized and engineering experience may be required (tuning), additional mapping and more or less complicated interaction between various auxiliary models is necessary (SM). In order to take advantage of space mapping, the low-fidelity model should be computationally much cheaper
than the high-fidelity model, therefore, equivalent-circuit models are preferred [9]. Reliable equivalent-circuit models, however, may be difficult to develop for certain types of microwave devices (e.g., antennas). Also, an extra simulator must be involved in the process and linked to the optimization algorithm. Moreover, space mapping performance heavily depends on the selection of the SM transformations used to construct the surrogate. On the other hand, tuning cannot be directly applied to radiating structures.

In this paper, an efficient technique for simulation-driven design of EM-simulated structures is discussed that is based on the SBO principle, coarsely-discretized EM low-fidelity models, and adaptive adjustment of the design specifications [21]. Original design specifications are modified to take into account the difference between the high-fidelity and low-fidelity model responses at the current design. The low-fidelity model is then optimized with respect to the modified specifications to produce a new design that—assuming sufficient quality of the low-fidelity model—gives a good prediction of the optimal high-fidelity model design with respect to the original specifications. The above assumption is typically satisfied for coarsely-discretized EM models. The presented method is simple to implement, and, as demonstrated through examples, it is able to yield a satisfactory design after a few high-fidelity EM simulations of the structure under considerations.

II. SIMULATION-DRIVEN DESIGN METHODOLOGY

In this section, we formulate the optimization problem (Section II. A), describe the concept of adaptively adjusted design specifications technique (Section II. B), as well as comment upon the use of coarse-discretization EM simulations as the low-fidelity model guiding the optimization process (Section II. C).

A. Design optimization problem

Let \( R_f(x) \) and \( R_c(x) \) denote the response vectors of a high- and low-fidelity models of the microwave structure of interest at the design vector \( x \). For example, \( R_f(x) \) may consist of the values of \( |S_{21}| \) evaluated at set of frequencies. The high-fidelity model is evaluated using CPU-intensive electromagnetic simulation. The low-fidelity model is a relaxed-accuracy and computationally cheap representation of \( R_f \). In particular, \( R_c \) may be evaluated using the same solver as \( R_f \), but with coarser mesh.

We want to optimize the high-fidelity model with respect to a given set of design specifications. Figure 1(a) shows the high- and low-fidelity model responses at the optimal design of \( R_r \), corresponding to the microstrip bandstop filter [21] used here as an illustration example; design specifications are indicated using horizontal lines.

B. Optimization through adaptively adjusted design specifications

The optimization procedure based on adaptively adjusted design specifications, originally introduced in [21], consists of the following two simple steps that can be iterated if necessary:

1. Modify the original design specifications in order to take into account the difference between the responses of \( R_f \) and \( R_c \) at their characteristic points.
2. Obtain a new design by optimizing the low-fidelity model with respect to the modified specifications.

Characteristic points of the responses should correspond to the design specification levels. They should also include local maxima/minima of the respective responses at which the specifications may not be satisfied. Figure 1(b) shows characteristic points of \( R_f \) and \( R_c \) for our bandstop filter example. The points correspond to –3 dB and –30 dB levels as well to the local maxima of the responses. As one can observe in Fig. 1(b), the selection of points is rather straightforward.

In the first step of the optimization procedure, the design specifications are modified so that the level of satisfying/violating the modified specifications by the low-fidelity model response corresponds to the satisfaction/violation levels of the original specifications by the high-fidelity model response. More specifically, for each edge of the specification line, the edge frequency is shifted by the difference of the frequencies of the corresponding characteristic points, e.g., the left edge of the specification line of –30 dB is moved to the right by about 0.7 GHz, which is equal to the length of the line connecting the corresponding characteristic points in Fig. 1(b). Similarly, the specification levels are shifted by the difference between the local maxima/minima values for the
respective points, e.g., the –30 dB level is shifted down by about 8.5 dB because of the difference of the local maxima of the corresponding characteristic points of $R_f$ and $R_c$. Modified design specifications are shown in Fig. 1(c).

![Graphs](image)

Fig. 1. Bandstop filter example (responses of $R_f$ and $R_c$ are denoted using solid and dashed line, respectively) [21]: (a) responses at the initial design (low-fidelity model optimum) as well as the original design specifications, (b) characteristic points of the responses corresponding to the specification levels (here, -3 dB and -30 dB) and to the local maxima, (c) responses at the initial design as well as the modified design specifications.

The low-fidelity model is subsequently optimized with respect to the modified specifications and the new design obtained this way is treated as an approximated solution to the original design problem (i.e., optimization of the high-fidelity model with respect to the original specifications). Steps 1 and 2 can be repeated if necessary. As demonstrated in Section III, substantial design improvement is typically observed after the first iteration, however, additional iterations may bring further enhancement. In practice, the algorithm is terminated once the current iteration does not bring further improvement of the high-fidelity model design.

Figure 2 shows the flow diagram of the optimization procedure. It should be emphasized, that unlike in case of other simulation-driven techniques popular in microwave engineering (particularly space mapping [9]), the low-fidelity model is not modified or corrected in any way. The discrepancy between the models is “absorbed” by means of modifying the design specifications.

![Flow diagram](image)

Fig. 2. A flow diagram of the optimization procedure exploiting adaptively adjusted design specifications and coarse-discretization EM models.

The operation of the adaptively adjusted design specifications technique can probably be best explained using the example. Figure 3 illustrates an iteration of the procedure used for design of a CBCPW-to-SIW transition [31]. One can observe...
that the absolute matching between the low- and high-fidelity models is not as important as the shape similarity.

C. Coarsely-discretized EM-simulation models

While in general, the low-fidelity model can be any physically-based model that is available (e.g., equivalent circuit [9]), the coarse-discretization EM models are used here. This has several advantages: (i) coarse-discretization models using the same EM solvers as corresponding fine models are typically more accurate than any other models of a given structure (e.g., equivalent circuits), (ii) optimization procedure is easy to implement because it exploits one software package (in the case of circuit-based low-fidelity models, an extra simulator is necessary which complicates the algorithm implementation because interaction the simulators has to be realized), (iii) coarse-discretization EM model typically provides better initial design than any other conceivable low-fidelity model type, (iv) coarse-discretization model is available for any microwave structure; in particular, the optimization can performed for devices where finding equivalent circuit model may be problematic (e.g., antennas).

One of the possible problems is that coarse-discretization EM models are relatively expensive so that minimizing the number of low-fidelity model evaluations is crucial in reducing the computational cost of the optimization process. It should be emphasized, however, that the procedure described here is quite efficient with this respect. In particular, it does not have a parameter extraction step—typical for space mapping approaches [12]—that normally requires consumes a substantial number of low-fidelity model evaluations.

D. Practical issues

The adaptively adjusted design specifications technique is very simple to implement and quite efficient as demonstrated in Section III. It should be emphasized, however, that the quality of the low-fidelity model is essential for the performance of this design procedure. More specifically, it is necessary that the high- and low-fidelity models are similar in shape (as functions of frequency) so that modification of the design specifications can be a relevant tool reflecting their misalignment. This requires that the discretization density for the low-fidelity model is sufficient; otherwise, the method may fail to find a satisfactory design. In practice, a parametric study of the mesh density and visual comparison of the high- and low-fidelity model responses are necessary to select the meshing parameters for the latter.
III. EXAMPLES

A. Double annular ring antenna [22]

Consider the stacked probe-fed printed annular ring antenna [22] shown in Fig. 4. The antenna is printed on a printed circuit board (PCB) with electrical permittivity $\varepsilon_{r1} = 2.2$, and height $d_1 = 6.096$ mm for the lower substrate, and $\varepsilon_{r2} = 1.07$, $d_2 = 8.0$ mm for the upper substrate. The radius of the feed pin is $r_0 = 0.325$ mm. The design parameters are $x = [a_1, a_2, b_1, b_2, \rho_1]^T$.

The fine model is evaluated using FEKO [23]. Its response is the modulus of the reflection coefficient, $|S_{11}|$, evaluated over the frequency band 1.75 GHz to 2.15 GHz. The number of meshes for $R_f$ is 1480 and its evaluation time is 2 hours and 5 minutes. The design specifications are $|S_{11}| \leq -10$ dB for $1.75$ GHz $\leq \omega \leq 2.15$ GHz. The coarse model $R_c$ is also simulated in FEKO. The number of meshes for $R_c$ is 300. The coarse model evaluation time is 6 minutes and 30 seconds. Initial design $x(0) = [10 8 30 30 20]^T$ mm.

Optimization of the antenna was performed using the adaptively adjusted design specifications technique described in Section II. Figure 5 shows the fine and coarse model responses at the initial design (minimax specification error +6.0 dB), as well as the fine model response at the final design $x(2) = [10.81 5.75 28.5 32.25 19.5]^T$ mm (specification error is –0.2 dB) obtained after two iterations of our procedure. The total number of evaluations of the coarse model in the optimization process is 87. Table 1 shows the computational cost of the optimization: the total optimization time corresponds to only 6.5 evaluations of the fine model. For comparison purposes, the direct optimization of the fine model using Matlab’s $\text{fminimax}$ routine was performed using $x(0)$ as a starting point. A slightly better design was obtained (with the specification error of –0.5 dB) at much higher cost of 55 fine model evaluations (almost 115 hours of CPU time).

![Fig. 4. Geometry of a stacked probe-fed printed double annular ring antenna [22].](image)

![Fig. 5. Double annular ring antenna: High- (- - -) and low-fidelity (⋅⋅⋅) model responses at the initial design $x(0)$, and the high-fidelity model response at the final design found by the adaptive design specifications technique (—).](image)

Table 1: Computational cost of optimizing the double annular ring antenna

<table>
<thead>
<tr>
<th>Algorithm Component</th>
<th>Number of Model Evaluations</th>
<th>Optimization Time</th>
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</thead>
<tbody>
<tr>
<td>Evaluation of $R_f$</td>
<td>87</td>
<td>565 min</td>
</tr>
<tr>
<td>Evaluation of $R_c$</td>
<td>2*</td>
<td>250 min</td>
</tr>
<tr>
<td>Total optimization time</td>
<td>N/A</td>
<td>815 min</td>
</tr>
</tbody>
</table>

* Excluded evaluation of the fine model at the initial design

* Number of high-fidelity model evaluations
B. Miniature dual-mode bandpass microstrip filter [24]

Consider the miniature dual-mode bandpass filter [24] shown in Fig. 6. The design parameters are \(\mathbf{x} = [L \ s \ p \ g]^T; \ W = 1 \text{ mm}, \ W_c = 0.5 \text{ mm}\). Both the fine and coarse models are evaluated in FEKO [23]. The total mesh number for the fine model is 646 (evaluation time 20 min), the total mesh number for the coarse model is 68 (evaluation time 26 seconds). The design specifications are \(|S_{21}| \geq -1 \text{ dB for } 2.35 \text{ GHz} \leq \omega \leq 2.45 \text{ GHz, } |S_{21}| \leq -20 \text{ dB for } 1.6 \text{ GHz} \leq \omega \leq 2.2 \text{ GHz and for } 2.6 \text{ GHz} \leq \omega \leq 3.2 \text{ GHz.}\) The initial design is \(\mathbf{x}^0 = [12.0 2.0 2.0 0.2]^T \text{ mm.}\)

The adaptively adjusted design specifications technique of Section 2 was used to optimize the filter. Figure 7 shows the fine model and coarse model responses at the initial design (minimax specification error +19.6 dB), as well as the fine model response at the final design \(\mathbf{x}^3 = [12.65 1.99 1.38 0.145]^T \text{ mm}\) (specification error is –0.2 dB) obtained after three iterations of the optimization procedure. The total number of evaluations of the coarse model in the optimization process is 137. The total cost of the design process corresponds to about 5.9 evaluations of the fine model (Table 2).

C. UWB monopole antenna

The monopole is on a 0.508 mm thick Rogers RO3203 substrate. Design variables are \(\mathbf{x} = [h_0 \ w_0 \ a_0 \ s_0 \ h_1 \ l_{gm} \ w_1]^T\) (Fig. 8). Other parameters: \(l_s = 25, \ w_m = 1.25, \ h_p = 0.75\) (all in mm). The microstrip input of the monopole is fed through an edge mount SMA connector [26] having a hex nut. The ground of the monopole has a profiled edge. Both high- and low-fidelity models are evaluated using the time-domain solver of CST Microwave Studio [27]. Simulation time of \(\mathbf{R}_c\) (152,640 mesh cells) is 2 min, and that of \(\mathbf{R}_f\) (1,151,334 mesh cells) is 45 min (both at the initial design). The design specifications for reflection are \(|S_{11}| \leq -10 \text{ dB for } 3.1 \text{ GHz to } 10.6 \text{ GHz.}\) Additionally, the radiation pattern of the monopole is to be omnidirectional in the \(XOY\) plane.

Initial design \(\mathbf{x}^0 = [18 \ 12 \ 2 \ 0 \ 5 \ 1 \ 15 \ 40]^T \text{ mm.}\) Optimization performed using the adaptively adjusted design specifications technique yields the final design \(\mathbf{x}^3 = [18.27 \ 19.41 \ 2.02 \ 1.34 \ 1.95 \ 5.83 \ 15.74 \ 35.75]^T \text{ mm}\) (\(S_{11} < -14.5 \text{ dB in the frequency band of interest}) obtained after three iterations of our procedure. Figure 9 shows reflection responses of the high- and low-fidelity models at the initial design as well as the \(\mathbf{R}_f\) response at the final design. The far-field response of the final design is shown in Fig. 10. The total number of evaluations of \(\mathbf{R}_f\) in the optimization process is 252. Table 3 shows the computational cost of the optimization: the total optimization time corresponds to about 14 evaluations of the high-fidelity model.
Table 2: Computational cost of optimizing the miniature dual-mode bandpass filter

<table>
<thead>
<tr>
<th>Algorithm Component</th>
<th>Number of Model Evaluations</th>
<th>Optimization Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluation of $R_c$</td>
<td>137</td>
<td>59 min</td>
</tr>
<tr>
<td>Evaluation of $R_f$</td>
<td>5</td>
<td>60 min</td>
</tr>
<tr>
<td>Total optimization time</td>
<td>N/A</td>
<td>119 min</td>
</tr>
</tbody>
</table>

* Excluded evaluation of the fine model at the initial design
# Number of high-fidelity model evaluations

Fig. 8. UWB monopole: top view, substrate shown transparent. $H$-symmetry wall is shown with the dash-dot line.

Table 3: Computational cost of optimizing the UWB monopole antenna

<table>
<thead>
<tr>
<th>Algorithm Component</th>
<th>Number of Model Evaluations</th>
<th>Optimization Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluation of $R_c$</td>
<td>252</td>
<td>8.4 hour</td>
</tr>
<tr>
<td>Evaluation of $R_f$</td>
<td>3*</td>
<td>2.3 hour</td>
</tr>
<tr>
<td>Total optimization time</td>
<td>N/A</td>
<td>10.7 hour</td>
</tr>
</tbody>
</table>

* Excluded evaluation of the fine model at the initial design
# Number of high-fidelity model evaluations

IV. CONCLUSION

An efficient procedure for design optimization of EM-simulated microwave devices is discussed. The presented approach exploits a computationally cheap model of the structure under consideration, evaluated using the same electromagnetic solver but with coarse discretization. The misalignment between the low- and high-fidelity EM models is absorbed by suitable adjustments of the design specifications. The performance of the presented technique is demonstrated through the design of the double annular ring antenna, the microstrip bandpass filter, and the UWB monopole antenna. Satisfactory designs are obtained at the computational cost corresponding to a few high-fidelity EM simulations of the respective structures.

ACKNOWLEDGMENT

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Abstract — A fast solver based on multilevel Lagrange interpolation of homogenous space electric and magnetic field Green’s functions is discussed. Broadband applications are possible due to a wavelength adaptive multilevel scheme. By an FFT-technique, the pertinent translation operators are diagonalized. An impedance boundary condition (IBC) is employed considering electric and magnetic currents for the approximate treatment of non-metallic objects. The common mixed-potential integral equation and a direct field formulation are both discussed. In general, the direct field formulation leads to more accurate results in conjunction with interpolated Green’s functions, especially for low frequency problems. The efficiency of the algorithm is shown in several numerical examples.

Index Terms — Electromagnetic radiation, electromagnetic scattering, fast integral solvers, integral equations.

I. INTRODUCTION

Surface integral equation methods belong to the most efficient techniques for solving electromagnetic scattering or radiation problems. When employing method of moments (MoM) discretization, the integral equation (IE) operators are converted into matrix vector products [1, 2]. Unfortunately, these IE operators are in general fully populated, causing bad numerical complexities. This makes the computation of problems with many unknowns very challenging. Hierarchical fast solvers have been introduced to overcome this problem. One of the most popular methods is the multilevel fast multipole method (MLFMM) [2]. In this method, discretization elements with small separations are grouped hierarchically and far-range interactions are computed among groups of basis functions on appropriate levels in the accordant hierarchy. Interactions between the single basis/testing functions are just contained in the translation operators between elements, which are very close to each other. Due to the strongly increasing magnitude of the Hankel functions, the MLFMM suffers a low frequency breakdown. This makes the numerical evaluation of the MLFMM diagonal plane wave based translation operators very difficult [2]. Hence, other approaches have to be found. One possibility to overcome this drawback is to include evanescent waves within the respective translation operators to better capture reactive fields [3]. Furthermore, it is also possible to work with the standard multipole-based translation operators at low frequencies [4], which are full operators. With respect to the diagonal MLFMM, both approaches have increased computational complexity.

For approximating smoother fields at lower frequencies, polynomial field representations appear to be very appropriate as low interpolation orders may be sufficient. One popular method working with a non-hierarchical approximation on a grid of equally spaced points covering all objects is the adaptive integral method (AIM) [7]. Although the approximation itself is non-hierarchical, a hierarchical acceleration of the translation step is performed by an FFT. Due to the AIM grid structure, empty portions of the solution domain are also covered causing unnecessary computations.

A hierarchical method employing multilevel Taylor series expansion of the respective Green’s functions is presented in [5]. In [6], a method based on Lagrange interpolation of the Green’s functions is proposed. The advantage of Lagrange interpolation is its rather constant approximation...
error throughout the interpolation domain due to several interpolation points, whereas the approximation error of the Taylor expansion increases with distance from the single expansion point.

In this contribution, a method working with multilevel Lagrange interpolation based polynomial factorization of Green’s functions, which is fully compatible with the well-known MLFMM oct-tree is presented. Firstly introduced in [8] for perfectly electric conducting (PEC) objects and mixed-potential electric field integral equation (MPIE) formulation, the method has been expanded for the treatment of IBC objects. The method is also applied to magnetic field integral equation (MFIE). Mixed-potential and dyadic integral equation formulations for electric and magnetic currents are considered and compared. The translations are only performed between non-empty box levels, translations are accelerated by FFT with the inner product

\[
\mathbf{L}\{\mathbf{v}(r')\} = \iint \overline{\mathbf{G}}(r,r') \cdot \mathbf{v}(r') \, da
\]

and the operators

\[
\mathbf{K}\{\mathbf{v}(r')\} = \iint \nabla \mathbf{G}(r,r') \times \mathbf{v}(r').
\]

The excitation vector elements are

\[
e_m(r) = -\iint \{\mathbf{b}_m(r) \cdot \alpha \{\mathbf{E}^{inc}(r)
\]

\[
+ Z(1 - \alpha)\left(\hat{n} \times \mathbf{H}^{inc}(r)\right)\} da.
\]

The parameters \( \mu, \varepsilon, k \) and \( Z = \sqrt{\mu/\varepsilon} \) are the permeability, permittivity, wavenumber, and wave impedance of free-space. \( \mathbf{G}(r,r') = e^{-jk|r-r'|}/(4\pi|r-r'|) \) is the homogeneous space scalar Green’s function, \( \overline{\mathbf{G}}(r,r') = (\mathbf{I} + 1/k^2 \nabla \nabla \mathbf{G}(r,r')) \) the electric field dyadic Green’s function, and \( \alpha \) the so-called combined field integral equation (CFIE) combination parameter with \( 0 \leq \alpha \leq 1 \). \( \mathbf{E}^{inc}(r) \) and \( \mathbf{H}^{inc}(r) \) are the electric and magnetic field strength due to the respective excitation.

No magnetic surface currents \( \mathbf{M}_A \) with the expansion coefficients \{\mathbf{M}\} occur at the respective Huygens’ surface if a PEC object is analyzed and equations (1) and (2) determine a unique solution for \{\mathbf{J}\}. However, in general this set of equations is under-determined and additional equations are required. For this purpose, the common impedance boundary condition [10] is utilized for dielectrically coated PEC objects. The IBC is defined on the boundary between the exterior of the coating and the surrounding medium. It is formulated as

\[
\mathbf{M}_A = Z_A (\mathbf{J}_A \times \hat{n}).
\]

\( Z_A \) is the characteristic surface impedance which can be approximated according to [10] as

\[
Z_A \approx j\sqrt{\mu_r/\varepsilon_r} \tan(kd/\sqrt{\mu_r \varepsilon_r})
\]

with the relative permeability and permittivity \( \mu_r \) and \( \varepsilon_r \) and the thickness \( d \) of the coating. The IBC (7) is then
The discretized according to [11] and the resulting equations are directly considered within an iterative solver.

The hyper-singular integrals in (2) and (3) can be avoided by the so-called mixed-potential formulation which has only weak $1/R$-singularities with $R = |\mathbf{r} - \mathbf{r}'|$. After applying some vector-analytic manipulations to (2) and (3), especially the surface divergence theorem [1], the inner products involving $\nabla \nabla G(\mathbf{r}, \mathbf{r'})$ are rewritten in the following manner [11]:

\begin{align}
\left\langle a(\mathbf{r}), L_{\mathbf{r}'}(b(\mathbf{r'})) \right\rangle &= \oint_{C_m} a(\mathbf{r}) \cdot \hat{u}_m L_{\mathbf{r}} \left( \nabla \cdot b(\mathbf{r'}) \right) dC_m \\
\left\langle b(\mathbf{r}), L_{\mathbf{r}'}(b(\mathbf{r'})) \right\rangle &= \left\langle \nabla \cdot b(\mathbf{r}), L_{\mathbf{r}} \left( \nabla \cdot b(\mathbf{r'}) \right) \right\rangle
\end{align}

(8)

with the operators

\begin{align}
L_{\mathbf{r}'}(v(\mathbf{r'})) &= \int \int \nabla \nabla G(\mathbf{r}, \mathbf{r'}) \cdot b(\mathbf{r'}) da' \\
L_{\mathbf{r}}(s(\mathbf{r'})) &= \int \int G(\mathbf{r}, \mathbf{r'}) s(\mathbf{r'}) da'.
\end{align}

(9)

$C_m$ is the boundary curve of the test domain and $\hat{u}_m$ is the unit vector in the tangent plane and perpendicular to $C_m$. The mixed-potential coupling integrals with (8) and (9) applied to (2) and (3), respectively, are named $Z_{\text{mixed}}^{\text{mn}}$ and $Z_{\text{mixed}}^{\text{mn,M}}$ in the following.

For reducing redundancy of the discretized IE operators, which are in general fully-populated with respect to the contained far-interactions, an appropriate basis change is performed for these far-interactions. The basis change is achieved by multilevel Lagrange interpolation of the pertinent Green's functions, where the current basis functions are mapped on the interpolation samples. A further speed-up is reached by FFT acceleration for the computation of the multilevel interactions among the interpolation samples.

III. LAGRANGE INTERPOLATION OF GREEN'S FUNCTIONS

The electric field dyadic Green’s function $\tilde{G}(\mathbf{r}, \mathbf{r'})$, the scalar Green’s function $G(\mathbf{r}, \mathbf{r'})$ and its gradient $\nabla G(\mathbf{r}, \mathbf{r'})$ can be factorized by Lagrange interpolation with respect to $\mathbf{r}$ and $\mathbf{r'}$ employing Lagrange interpolation factors according to

\begin{align}
\tilde{G}(\mathbf{r}, \mathbf{r'}) &= \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} A_i(\mathbf{r}) A_j(\mathbf{r'}) \tilde{G}_i(\mathbf{r}, \mathbf{r'}) \\
G(\mathbf{r}, \mathbf{r'}) &= \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} A_i(\mathbf{r}) A_j(\mathbf{r'}) G_i(\mathbf{r}, \mathbf{r'}) \\
\nabla G(\mathbf{r}, \mathbf{r'}) &= \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} A_i(\mathbf{r}) A_j(\mathbf{r'}) \nabla G_i(\mathbf{r}, \mathbf{r'}).
\end{align}

(10)

with $A_i(\mathbf{r})$ being the respective Lagrange polynomials and $N_p$ the number of interpolation points. When inserted in equations (2) and (3), the resulting integrals can be pre-computed and do not have to be evaluated in every matrix-vector product within an iterative solver.

By this Lagrange interpolation point representation, an accelerated evaluation of the discretized IE operators can be achieved in two ways. First, the necessary number of interpolation points can be considerably smaller than the number of basis functions. This is in particular the case for low-frequency applications where very many discretization steps per wavelength are needed in order to represent fine geometrical details. Second, the coupling computation effort can be considerably reduced when employing an FFT-based coupling computation.

IV. DIAGONALIZATION OF THE TRANSLATION OPERATOR

The computational effort of the presented Lagrange interpolation based integral equation representation has a disadvantageous large computational effort for the far-interactions of $O(N_p^3 N_p^3 \lambda^3) = O(N_p^6)$ for one pair of interpolation domains (3D cubes) with $N_p$ being the number of interpolation points. This is because the interpolations must in general be performed in three dimensions, even if a surface integral equation is considered. Furthermore, the corresponding translation operators are still full operators.

To overcome this drawback, an FFT-based method for diagonalizing the corresponding translation operators is employed. In particular the formulation for $Z_{\text{direct}}^{\text{mn}}$ according to (1) and (2) is considered but everything applies in similar form also for $Z_{\text{mixed}}^{\text{mn}}$ and all other matrices. The approximated matrix $\tilde{Z}_{\text{mn},J}$ with the interpolated dyadic Green’s function according to (10) can be formulated as

\begin{align}
\tilde{G}(\mathbf{r}, \mathbf{r'}) &= \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} A_i(\mathbf{r}) A_j(\mathbf{r'}) \tilde{G}_i(\mathbf{r}, \mathbf{r'}) \\
G(\mathbf{r}, \mathbf{r'}) &= \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} A_i(\mathbf{r}) A_j(\mathbf{r'}) G_i(\mathbf{r}, \mathbf{r'}) \\
\nabla G(\mathbf{r}, \mathbf{r'}) &= \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} A_i(\mathbf{r}) A_j(\mathbf{r'}) \nabla G_i(\mathbf{r}, \mathbf{r'}).\n\end{align}

(10)
\[
\tilde{Z}_{mn,j} = \sum_{i=1}^{N^2} \int_{A} \Lambda_i(r) b_m(r) da \cdot \sum_{i=1}^{N^2} \tilde{G}(r, r_{j}) \cdot \int_{A} \Lambda_i(r) b_n(r') da'.
\] (11)

As it can be seen, the Green’s function is factorized by the Lagrange interpolation with respect to \( r \) and \( r' \) so that the integrals in (11) can be pre-computed.

Employing tensor notation with third order tensors, each order corresponding to one Cartesian dimension, the \( j \) summation in (11) can be rewritten as

\[
[R]_{i_k,j_k,l_k}^3 = \sum_{i_\alpha \in [i]} \sum_{j_\alpha \in [j]} \sum_{l_\alpha \in [l]} [G]_{\alpha,\alpha,\alpha} \cdot [S]_{i_\alpha,j_\alpha,l_\alpha}^3,
\] (12)

with the third order tensors \([S]^3\) and \([R]^3\) containing the source or receive integrals, respectively, and \( \Delta i_k = i_k - i'_k \). This discrete convolution in space domain with equidistant sampling can efficiently be computed in spectral domain according to

\[
[R]_{i_k,j_k,l_k}^3 = \sum_{i_\alpha = 1}^{2N_x} \sum_{j_\alpha = 1}^{2N_y} \sum_{l_\alpha = 1}^{2N_z} \mathcal{F}^{-1} \left[ \tilde{G} \right] \otimes \mathcal{F} \left[ [S]^3 \right].
\] (13)

Zero-padding has to be performed in order to avoid aliasing errors. The symbol \( \otimes \) denotes the Hadamard (tensor element-wise) dyadic-vector product in the discrete Fourier domain. It is essential, that the necessary forward transformations \( \mathcal{F} \) and backward transformations \( \mathcal{F}^{-1} \) are performed by FFT to obtain enhanced computational efficiency when employing (13) instead of (11). In the end, the receive contributions in \([R]^3\) must be multiplied with the test integrals. Since the translations are computed as Hadamard products in the discrete Fourier domain, diagonalization of the translation operators has been achieved. However, the computational complexity is then dominated by the transformations instead of the coupling computations itself. Together with the necessary zero-padding, the overall procedure appears to be efficient only for relatively large numbers of interpolation points, as e.g. obtained if the whole radiation or scattering object is covered with one regular grid.

By numerical experiments it was found that employing (13) is advantageous even for the small number of 3 interpolation points per Cartesian dimension. For cubic interpolation with \( N_p = 4 \), which is mostly a good choice for accurate computations, there is a reduction in translation time by a factor of about 7. Due to this observation, FFT-based translations are employed within this fast solver. The complexity of the translation is \( O(N_p^6 \log N_p^3) = O(N_p^3 \log N_p) \) with respect to the number of interpolation points \( N_p \) instead of \( O(N_p^6) \) for direct translation in space domain.

VI. MULTILEVEL ALGORITHM

One major drawback of grid based fast solvers is the general necessity to work with 3 dimension-al grids although the problem itself is a 2 dimensional surface problem. Especially dominant is this drawback in AIM or pre-corrected FFT techniques. There, all coupling computations are performed by a global 3D FFT on a single grid covering the whole computation domain. Hence, most computations are performed for grid points located in the empty space inside the computation domain. To relieve this problem, a multilevel algorithm following a hierarchical oct-tree grouping strategy is developed, where regular grids within the individual groups are considered and where the translations among non-empty groups are computed by employing FFT-acceleration.

A cubic oct-tree structure as known from the MLFMM algorithm [2] is assumed. The distances \( d \) between different box centers on a given level \( lev \) are found to be

\[
d_{lev}^{near} = n_1 a_{lev},
\]

\[
d_{lev}^{far} = n_2 a_{lev}.
\]

\( d_{lev}^{near} \) is the near-coupling range for each level \( lev \), where the necessary interactions are computed on finer levels or by direct MoM integration. \( d_{lev}^{far} \) is the far-coupling range up to which far translations are performed on the corresponding level and \( a_{lev} \) is the edge length of each cube in the pertinent level. On all levels, the relation \( a_{lev+1} = 2 a_{lev} \) is valid.

The described Lagrange interpolation algorithm with FFT acceleration is employed for boxes with separations \( d_{lev}^{near} \leq d \leq d_{lev}^{far} \). Interactions among boxes with \( d > d_{lev}^{far} \) are computed on the next coarser level \( lev + 1 \). For levels with a small box-size as compared to wavelength, the number
of interpolation points per dimension $N_{p}^{lev}$ on a certain level $lev$ can be kept constant on different levels as suggested in [12]. As the resulting impedance matrix is then an $H^2$-matrix [12], computational and storage complexity per matrix-vector product are $O(N)$. This strategy is only valid for fine levels with box sizes significantly smaller than the wavelength. Hence, we use this strategy if $a_{lev+1}/\lambda$ is below a constant threshold $r_{const}$. Following this strategy, a coarser level source tensor $[S^{lev+1}]^{3}_{x',y',z'}$ is Lagrange anterpolated for aggregation case

$$[S^{lev+1}]^{3}_{x',y',z'} = \sum_{i=1}^{N_{s}} \sum_{j=1}^{N_{s}} \Lambda_{i,j}^{lev+1} \sum_{i=1}^{N_{s}} \sum_{j=1}^{N_{s}} \Lambda_{i,j}^{lev} [S^{lev}]^{3}_{x,y,z},$$ (15)

where the interpolation points on all levels are equally spaced in each dimension in order to realize a regular grid. The disaggregation procedure is performed in the same manner by interpolating finer level receive contributions from corresponding coarser level receive contributions. All aggregation factors can be pre-computed before evaluating the matrix-vector product. Due to the orthogonality of the Lagrange polynomials, the operators contain a lot of zero entries, which leads to computation time and memory reduction.

If the box size is not small compared to the wavelength, the absolute interpolation point distance is maintained on coarser levels. No Lagrange interpolation is necessary as all coarser level points are at the same position as at least one finer level interpolation point and the accordant Lagrange polynomials are orthogonal. Hence, (15) reduces to

$$[S^{lev+1}]^{3}_{x'+\Delta i,x+y+\Delta j,z+\Delta z} = [S^{lev}]^{3}_{x,y,z},$$ (16)

for the aggregation case, whereas $N_{p}^{lev} = N_{p}$ at the finest level following this strategy and $N_{p}^{lev+1} = 2N_{p}^{lev} - 1$ for all subsequent coarser levels. $\Delta i$, $\Delta j$, and $\Delta z$ are index offsets dependent on the position of the finer level group within the coarser level group.

For disaggregation case, the pertinent receive contributions from the coarser level have just to be copied on the finer level. Following this strategy, the computational complexity increases as the required number of interpolation points roughly doubles in each direction. The computational complexity following this strategy is $O(N^{3/2}\log(N))$ in conjunction with FFT-based translations and is thus significantly worse compared to fast high frequency solvers like MLFMM with $O(N\log(N))$ complexity. The proposed algorithm is thus especially suited for low-frequency problems where the fine-level strategy can be employed for most levels.

### VI. Dyadic versus mixed-potential MLIPFFT

The presented multilevel interpolatory FFT accelerated method (MLIPFFT) is employed to direct field integral equations according to (2) and (3) as well as to the more common mixed potential integral equations (MPIE) according to (8)-(10). In a previous work [9] it has been shown, that the MPIE is preferable for interpolation in well-conditioned problems as the computation time per matrix-vector product is smaller than for the EFIE and MFIE with dyadic Green's function formulation. For ill-conditioned problems, the direct field EFIE and MFIE with dyadic Green’s functions are preferable as the isolated electric scalar potential and the superposition of the potential contributions of the MPIE usually need a higher interpolation accuracy.

### VII. NUMERICAL RESULTS

The efficiency of the presented algorithm is shown for several computation examples. All computations shown in this section have been carried out on one core of a Dell Precision T7500 workstation (2.53 GHz clock speed, 96 GByte RAM).

As a linear equation system solver, a flexible generalized minimal residual solver with Given’s rotations was used. As an iteration stop criterion, a residual error of $10^{-4}$ was configured.

#### A. Sphere with dielectric coating

A PEC sphere with diameter 1 m and a dielectric coating of 2.5 cm thickness ($\varepsilon_r = 4 - j100$) is computed as a first example. It is discretized by 176 472 unknowns (88 236 electric and magnetic current unknowns, respectively). The object is illuminated by a plane wave with a frequency of 500 MHz. In Fig. 1a, the bistatic radar cross section (RCS) of the sphere is depicted for an MLIPFFT computation employing the IBC (direct field formulation) compared to an analytical Mie
series solution. As can be seen, both results show excellent agreement. Figure 1b shows the object and the resulting magnetic current distribution. For the MLIPFFT, the total solution time was 2 171 sec with a memory consumption of 1 522 MByte.

![Bistatic RCS of Coated Sphere](image1)

Fig. 1a. Bistatic RCS of a sphere with dielectric coating.

![x-component of magnetic current distribution on dielectric sphere (real part)](image2)

Fig. 1b. $x$-component of magnetic current distribution on dielectric sphere (real part).

**B. Parabolic reflector**

The second example is a $3\lambda$ PEC parabolic reflector, which is vertically illuminated by a plane wave. The reflector is densely discretized resulting in 222 583 unknowns. The MLIPFFT computation employing a mixed-potential EFIE was performed within 5 577 sec and with a total memory consumption of 1 873 MByte. For comparison, the computation was also performed by an MLFMM within 9 066 sec and with 9 475 MByte memory consumption. Both RCS results are shown in Fig. 2a and the accordant current distribution in Fig. 2b.

![Bistatic $\theta\varphi$-RCS of a parabolic reflector.](image3)

Fig. 2a. Bistatic $\theta\varphi$-RCS of a parabolic reflector.

![Electric current distribution on parabolic reflector (real part).](image4)

Fig. 2b. Electric current distribution on parabolic reflector (real part).

**C. P3-Orion**

The last example is the flight object “P3-Orion” with a length of 35 m. At first, the object’s surface material is PEC. The object is discretized with 536 250 electric current unknowns and illuminated by a plane wave incident from $\theta_i = 90^\circ$, $\varphi_i = 20^\circ$ with a wavelength of 36 m. The accordant MLIPFFT computation time was 12 675 sec and the required memory 7 718 MByte. Second, the problem was computed for a finite conducting coating ($\sigma = 10^{-1} \text{ S/m}$) of the object employing IBC. The computation time was 18 793 sec and the memory demand 13 610 MByte. The accordant bistatic $\varphi\theta$-RCS results are depicted in Fig. 3a. Figure 3b shows the object and the electric current distribution.
VIII. CONCLUSIONS

A fast integral equation fast solver, which is especially suited for low frequencies, has been presented. By 3D FFT, the accordant translation operators are diagonalized. Even for small numbers of interpolation points, this FFT-based technique has shown to be effective. Furthermore, an oct-tree based adaptive multilevel scheme reduces the computation of empty space and makes the algorithm useful for broadband applications and combinable with a high-frequency fast solver. The interpolation-based fast solver has demonstrated excellent efficiency and accuracy for PEC and impedance boundary body problems.

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An Implementation of King’s Green Functions in Thin Wire Scattering Problems

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Abstract — We investigate electromagnetic scattering from metallic thin wire structures located over planar and spherical lossy dielectric half-spaces by applying Green’s function formulation and method of moments in the resonance region and under “high contrast approximation” (HCA). For this purpose, in the calculations of the impedance matrix and the potential column of the moment system, we employ the Green functions of King valid for arbitrary range under HCA and the asymptotic (far field) Green functions for planar and spherical impedance surfaces delivered by Norton and Wait, respectively. For a verification of the developed codes, the current distributions obtained under plane wave illumination on the arms of a cross shaped thin wire structure are compared to the same results obtained by the commercial software SNEC™. Various illustrations for the scattered electrical field from a thin wire plate located over planar and spherical half-spaces are also presented.

Index Terms — Electromagnetic scattering, method of moments, Sommerfeld problem, thin wires.

I. INTRODUCTION

Ever since the pioneering work [1] by Sommerfeld over a century ago the interest in the derivation of computationally efficient solutions for the radiation fields of a Hertzian dipole in inhomogeneous media has constantly grown in parallel to their applications in diverse areas of electrical engineering. While it is impossible to provide a satisfactory list of all such attempts in literature to date, a wide account can be reached in [2]. The class of solutions to the Sommerfeld problem that constitute the topic of the present investigation is the Green functions derived by King in 1982 [3] for Hertzian dipoles radiating over a planar lossy dielectric half-space. The most distinctive aspect of King’s fully analytical solutions, which have been collected in [4] for various different properties of ambient medium, is that they apply for arbitrary range under high contrast approximation (HCA). Following 1999 to date, King’s method has been applied to many new geometries involving stratified spherical grounds in numerous works [5-17] initiated by his co-workers.

The thin wire mesh electromagnetic model of an arbitrarily shaped conducting body was first introduced and tested experimentally in 1966 by Richmond [18]. This pioneering work was followed by numerous theoretical as well as experimental investigations [19-27] to specify the ranges of validity of wire mesh models for certain canonical structures. Following the development of the method of moments (MoM) in 1967 by Harrington [28], there has appeared many papers through the 70’s on the MoM formulation of scattering problems for the wire mesh structures over a dielectric half-space due to their importance in radar applications [29] Since a computationally efficient analytical solution of the Sommerfeld problem was not available until 1982 [3], in such
works the surface wave components of Green functions were generally ignored (as called “reflection coefficient method”) for a practical computation of the impedance matrix without estimating the relative error. This gap was then filled by the famous open software NEC-2 [30], which was developed in 1981 in Lawrence Livermore National Laboratory, CA with extensive numerical/asymptotic libraries.

In the present work, we provide the analytical backbone of a software which incorporates the Green functions of King alternative to a similar role of the extensive numerical/asymptotic libraries of NEC-2 whenever HCA applies. While the developed codes equally have the ability to read NEC-2 formatted input files, their main advantage lies in the capability to evolve by proper substitutions of Green functions to take into account various terrain features in any scenario. Accordingly, in Section 2 we provide the MoM formulation of the scattering problem, while the details of the calculation of the elements of the impedance matrix are presented in Section 3. In Section 4 the elements of the potential column in MoM formulation are provided for three different scenarios of propagation over planar and spherical impedance surfaces, and their numerical implementations are presented in Section 5. For a verification of the developed codes we provide the amplitude and phase distributions of currents on the arms of a crossed wire over a planar lossy ground with reference to the same results obtained by the commercial software SNEC™ [31].

A time convention \( \exp(-i\omega t) \) is assumed and suppressed.

II. FORMULATION

Let regions I \((z > 0)\) and II \((z < 0)\) be free-space and a simple lossy dielectric with constitutive parameters and wave numbers given as \((\varepsilon_0, \mu_0), k_1 = \omega \sqrt{\mu_0 \varepsilon_0}\) and \((\varepsilon_2, \mu_2, \sigma_2), k_2 = \omega \sqrt{\mu_2 (\varepsilon_2 + i \sigma_2/\omega)}\), respectively. The complex refractivity of ground is defined by \(N = k_2/k_1 = \sqrt{\varepsilon_r + i \sigma_r/\omega}\) with \(\varepsilon_r = \varepsilon_2/\varepsilon_0\). The HCA is defined as \(|N^2| > 1\) (or equivalently \(|N| \geq 3\)). Analytically, the lowest value that \(|N|\) can take in any simple medium is limited by \(\sqrt{\varepsilon_r}\). Therefore, in any medium with \(\varepsilon_r \geq 9\) (especially seawater with \(\varepsilon_r \approx 75–80\)), it can always be satisfied regardless of conductivity and the operating frequency.

Under HCA, the King formulation of Green functions for a Hertzian dipole located at \(\vec{r}' = (x', y', z')\) and calculated at \(\vec{r} = (x, y, z)\) constitutes “direct” (d), “perfect image” (i), and “surface wave” (s) components, which can be represented in tensorial form by

\[
G(\vec{r}, \vec{r}') = G^{(d)}(\vec{r}, \vec{r}') + G^{(i)}(\vec{r}, \vec{r}') + G^{(s)}(\vec{r}, \vec{r}')
\]

\[
G^{(d,i,s)}(\vec{r}, \vec{r}') = \hat{x}g_{x}^{(d,i,s)} + \hat{y}g_{y}^{(d,i,s)} + \hat{z}g_{z}^{(d,i,s)}
\]

\[
+ \hat{x}g_{x}^{(d,i,s)} + \hat{y}g_{y}^{(d,i,s)} + \hat{z}g_{z}^{(d,i,s)}
\]

\[
+ \hat{x}g_{x}^{(d,i,s)} + \hat{y}g_{y}^{(d,i,s)} + \hat{z}g_{z}^{(d,i,s)}
\]

Here, \(g^{(d)}\) stands for the total \(b\)-axis electric field component of the Hertzian dipole with unit moment directed along \(a\)-axis. The entire set is given for \(z, z' \geq 0\) as follows:

\[
g_{x}^{(d)} = \frac{e^{ik_{R}R}}{4\pi R_{1}} \left[ \frac{x - x'}{R_{1}} - \frac{(x - x')^{2}}{R_{1}} \right] - \frac{e^{ik_{R}R}}{4\pi R_{2}} \left[ \frac{y - y'}{R_{2}} - \frac{(y - y')^{2}}{R_{2}} \right]
\]

\[
+ \frac{1}{2\pi R_{2}} \left[ \frac{z + z'}{R_{2}} \right] - \frac{1}{N} \left[ \frac{y - y'}{R_{2}} \right] \left[ \frac{z + z'}{R_{2}} \right] - \frac{1}{N} \left[ \frac{y - y'}{R_{2}} \right] \left[ \frac{z + z'}{R_{2}} \right] - \frac{1}{N} \left[ \frac{y - y'}{R_{2}} \right] \left[ \frac{z + z'}{R_{2}} \right]
\]

\[
g_{y}^{(d)} = \frac{(x - x')(y - y')}{R_{2}^{2}} \left[ e^{ik_{R}R} \right] - \frac{(x - x')(y - y')}{4\pi R_{1}} \left[ e^{ik_{R}R} \right]
\]

\[
+ \frac{(x - x')(y - y')}{4\pi R_{2}} \left[ e^{ik_{R}R} \right] - \frac{(x - x')(y - y')}{2\pi R_{2}} \left[ e^{ik_{R}R} \right]
\]

\[
\times \frac{1}{N^{2}} \left[ \frac{1}{N} R_{2}^{3} \left( \frac{ik_{R}R_{2}}{N^{3}} \right) \right]
\]
with
\[ R_{22}(\vec{r}, \vec{r}') = |\vec{r} - \vec{r}'| \]
\[ = \left[ (x-x')^2 + (y-y')^2 + (z+z')^2 \right]^{1/2}, \]
\[ P = \left[ (x-x')^2 + (y-y')^2 \right]^{1/2}, \]
\[ U = k_{11}R_2 \left[ \frac{R_2 + N(z+z')}{P} \right]^2, \]

and the dimensionless parameters
\[ \xi_1 = 1 - \frac{1}{ik_{11}R_1} - \frac{1}{k_{11}^2R_1^2}, \quad \xi_2 = 1 - \frac{3}{ik_{11}R_1} - \frac{3}{k_{11}^2R_1^2}, \]
\[ \eta_1 = 1 - \frac{1}{ik_{12}R_2} - \frac{1}{k_{12}^2R_2^2}, \quad \eta_2 = 1 - \frac{3}{ik_{12}R_2} - \frac{3}{k_{12}^2R_2^2}, \]
\[ \eta_3 = 1 - \frac{1}{ik_{22}R_2}, \quad \Xi = \left( \frac{\pi}{k_{22}R_2} \right)^{1/2} e^{-U} F(U), \]

where
\[ F(U) = 1 + i(\pi U)^{1/2} e^{-U} \text{erfc}(-iU^{1/2}), \]
is known as the Norton attenuation function. The surface wave components vanish in the limit \[ |N| \to \infty. \]

In thin wire approximation, we assume the wire mesh structure comprises cylindrical segments with fixed length \( \ell << \lambda \) and radius \( a << \ell \), where \( \lambda \) is the wavelength in the ambient medium, as depicted in Fig. 1.
In virtue of our choice of pulse basis functions in MoM formulation, we may assume the \( j \)-th thin wire segment supports a constant current \( I_j \), whose density function can be expressed in the local cylindrical coordinates \( \rho \phi \) through the Dirac delta distribution \( \delta \) and the unit step function \( H \) as

\[
\phi_j = J(\rho, \phi, z) = \frac{z^j I_j}{2\pi a} \delta(\rho^j - a) \quad \times \left[ H(z^j + \ell/2) - H(z^j - \ell/2) \right].
\]

The radiation field of \( j \)-th segment is given by the volume integral

\[
E_j (\rho, \phi, z) = i\omega \mu_0 \int G(\rho, \phi, z, \rho', \phi', z') d\rho' d\phi' d z',
\]

and the total radiated field by a total of \( M \) segments in a mesh is expressed by

\[
E(\rho, \phi, z) = \sum_{j=1}^{M} I_j \phi_j (\rho, \phi, z),
\]

based on the principle of superposition. Accordingly, the total electrical field at any point in space reads

\[
E^{\text{tot}} (\rho, \phi, z) = E^{\text{inc}} (\rho, \phi, z) + E(\rho, \phi, z),
\]

where \( E^{\text{inc}} (\rho, \phi, z) \) is the total field calculated at any point in the absence of the scatterer. Applying the collocation method, the boundary condition on the segments yields the linear system of equations

\[
\sum_{j=1}^{M} Z_{mj} I_j = V_m, \quad m = 1, 2, \ldots, M,
\]

where

\[
Z_{mj} = \int_{-\ell/2}^{\ell/2} G(\rho, \phi, z; \rho', \phi', z') d\rho' d\phi' d z',
\]

\[
V_m = -E_{\text{inc}} (\hat{r}_m) \cdot \hat{\ell}_m.
\]

At each junction the corresponding junction condition on currents increases the dimension of the linear system by one. In case of \( L \) junctions in a wire mesh, the currents are calculated by multiplying the extended system by the Hermitian transpose of the extended impedance matrix before inversion as follows:

\[
[I]_{M+L \times 1} \times \left( [Z]_{M+L \times (M+L)} \right)^{-1} = \left( [Z]_{M \times (M+L)} \right)^{-1} \times \left( [V]_{M \times (M+L)} \right).
\]

Alternative models for junctions can be reached at [32-34].
which require to be calculated separately through
\[
Z_{\nu m}^{d,i}(t) = \text{i} \omega \nu_0 \int_{-t/2}^{t/2} \int_{m} G \left( \tilde{r}_m, \tilde{r}_m' \right) \frac{d \phi'}{2 \pi} dz',
\]
(4)

Since \( Z_{\nu m}^d \) and \( Z_{\nu m}^i \) are space wave components, their calculations can be carried out directly in the local reference frame \( O'x'y'z' \). Regarding \( Z_{\nu m}^d \), the expressions of the difference vector directed from the central point of \( j \)-th to \( m \)-th segment in local and outer reference frames are given as
\[
\tilde{r}_m - \tilde{r}_j = (x_m - x_j, y_m - y_j, z_m - z_j),
\]
while they are related by
\[
\tilde{r}_m - \tilde{r}_j = T_j \cdot \tilde{r}_m,
\]
through the Euler transformation matrix \( T_j \)
\[
= \begin{bmatrix}
\cos \alpha_j & \cos \beta_j & -\sin \beta_j \\
-\sin \alpha_j & \cos \alpha_j & 0 \\
\sin \beta_j & \sin \alpha_j & \cos \beta_j
\end{bmatrix},
\]
whose inverse is equal to its transpose: \( T_j^{-1} = T_j^T \).

The explicit expressions of the 3-D transformation angles \( \alpha_j \) ve \( \beta_j \) are as follows:
\[
\sin \alpha_j = (y_j - y'_j) \left[(x_j - x'_j)^2 + (y_j - y'_j)^2\right]^{1/2},
\]
\[
\cos \alpha_j = (x_j - x'_j) \left[(x_j - x'_j)^2 + (y_j - y'_j)^2\right]^{1/2},
\]
\[
\sin \beta_j = (x_j - x'_j) \left[(x_j - x'_j)^2 + (y_j - y'_j)^2\right]^{1/2} / \ell,
\]
\[
\cos \beta_j = (z_j - z'_j) / \ell.
\]

Accordingly, under the thin wire approximation one has
\[
R_j^2 (\tilde{r}_m; \tilde{r}_m') \cong (x_m)^2 + (y_m)^2 + (z_m)^2 + a^2
\]
\[
-2z_m z_m' + (z_m')^2,
\]
and the \( \hat{z}' \)-directed Green functions read
\[
g_{x}^{z,d} (\tilde{r}_m, \tilde{r}_m') = -\frac{(x_m - x_m')(z_m - z_m')}{{R}_j^2} \frac{e^{ik_{R_1}}}{4 \pi R_1} \Xi_2, \quad (5)
\]
\[
g_{y}^{z,d} (\tilde{r}_m, \tilde{r}_m') = -\frac{(y_m - y_m')(z_m - z_m')}{{R}_j^2} \frac{e^{ik_{R_1}}}{4 \pi R_1} \Xi_2, \quad (6)
\]

Substituting the polar transformations \( x' = a \cos \phi' \) ve \( y' = a \sin \phi' \) in (5) and (6), a full period integration in (4) yields the resultant regular integral
\[
Z_{\nu m}^d = \text{i} \omega \nu_0 \int_{-t/2}^{t/2} + \left[ e^{ik_{R_1}} \Xi_2 \right] d\ell',
\]
with
\[
i \nu \nu_0 \left[ e^{ik_{R_1}} \Xi_2 \right] = -\frac{x_m (z_m - z_m')}{{R}_j^2} \frac{e^{ik_{R_1}}}{4 \pi R_1} \Xi_2,
\]
\[
\nu \nu_0 \left[ e^{ik_{R_1}} \Xi_2 \right] = -\frac{y_m (z_m - z_m')}{{R}_j^2} \frac{e^{ik_{R_1}}}{4 \pi R_1} \Xi_2.
\]

In calculating the surface wave components \( Z_{\nu m}^i \), we express the source points in the local reference frame and the observation points in the outer reference frame. For this purpose, we set
\[
\tilde{r} = \tilde{r}_m, \quad \tilde{r}' = \tilde{r}_j + T_j \cdot \tilde{r}_m', \quad \text{in King's Green functions},
\]
where \( \tilde{r}_j = (x'_j, y'_j, z'_j) \) and \( \tilde{r}_m' = (x'_m, y'_m, z'_m) \) (see Fig. 2).

Fig. 2. \( j \)-th and \( m \)-th thin wire segments and position vectors.
Under the thin wire approximation one has
\[ R_{i,j}(\vec{r}_i, \vec{r}_j) \cong (x_m - x_j)^2 + (y_m - y_j)^2 \]
\[ + (z_m - z_j)^2 + a^2 + z''^2 \]
\[ - 2z'' \sin \beta_j \]
\[ + (z_m - z_j) \cos \beta_j \]
\[ P^2 \cong (x_m - x_j)^2 + (y_m - y_j)^2 \]
\[ - 2z'' \sin \beta_j \]
\[ + (z''^2) \sin^2 \beta_j, \]
\[ U \cong \frac{k_i R_j}{2N^2} \left( \frac{R_j + N I_j}{P} \right)^2, \]
\[ I_z = z_m + z' \cong z_m + z_j + z'' \cos \beta_j. \]

By describing the following parameters
\[ I_x = \int_0^{2\pi} (x_m - x_j) \frac{d\phi''}{2\pi} = x_m - x_j - z'' \cos \alpha_j \sin \beta_j, \]
\[ I_{xx} = \int_0^{2\pi} (x_m - x_j)^2 \frac{d\phi''}{2\pi} = (x_m - x_j - z'' \cos \alpha_j \sin \beta_j)^2 \]
\[ + (a^2/2) \cos^2 \alpha_j \cos^2 \beta_j + \sin^2 \beta_j), \]
\[ I_y = \int_0^{2\pi} (y_m - y_j) \frac{d\phi''}{2\pi} = y_m - y_j - z'' \sin \alpha_j \sin \beta_j, \]
\[ I_{yy} = \int_0^{2\pi} (y_m - y_j)^2 \frac{d\phi''}{2\pi} = (y_m - y_j - z'' \sin \alpha_j \sin \beta_j)^2 \]
\[ + (a^2/2) (\sin^2 \alpha_j \cos^2 \beta_j + \cos^2 \alpha_j), \]
which emerge from the full period integration of the surface wave components of the Green tensor, one reaches the resultant regular integral
\[ Z_{m_i} = i \omega \mu_i \]
\[
\times \left[ \ell m_i t_x^x z_{jx} + \ell m_i t_x^y z_{jy} + \ell m_i t_x^z z_{jz} \right] \int_{-\ell/2}^{\ell/2} \left[ +\ell m_i t_y^x z_{jx} + \ell m_i t_y^y z_{jy} + \ell m_i t_y^z z_{jz} \\
+\ell m_i t_z^x z_{jx} + \ell m_i t_z^y z_{jy} + \ell m_i t_z^z z_{jz} \right] dz', \]
with \( z' = z_{jx} \hat{x} + z_{jy} \hat{y} + z_{jz} \hat{z} = T_{j} \cdot \hat{z}' \), which requires to be calculated numerically.

**IV. THREE DIFFERENT SCENARIOS**

The influence of the geometrical and physical properties of the ambient medium in scattering phenomenon appears in the expression (3), which is determined by the incident field. In this section, we consider three different scenarios for the incident field and the ambient medium for a numerical investigation.

**A. Scenario I: Homogeneous plane wave incidence and planar ground**

Let the electrical field of an incoming homogeneous plane wave in an arbitrary direction \( \hat{n}_i \) in region I be given by
\[ \vec{E}^i = \hat{e} e^{j \omega \mathbf{r} \cdot \hat{n}_i}. \]

The normal of the interface is \( \hat{n} \equiv \hat{z} \), while the normal of the incidence plane is calculated as \( \hat{q} = \hat{n} \times \hat{n}_i \). By use of the identity \( \hat{e}_i = \hat{q} \times (\hat{q} \cdot \hat{e}_i) = \hat{q} \times (\hat{q} \times \hat{e}_i) \) one can decompose the incident wave into TE and TM components as
\[ \vec{E}^i = \vec{E}^i_{TE} + \vec{E}^i_{TM}, \]
where
\[ \vec{E}^i_{TE} = -\hat{q} \times (\hat{q} \times \hat{e}_i) e^{j \mathbf{r} \cdot \hat{e}_i} = (\hat{q} \times \hat{e}_i) \times \hat{q} e^{j \mathbf{r} \cdot \hat{e}_i}, \]
\[ \vec{E}^i_{TM} = \hat{q} (\hat{q} \cdot \hat{e}_i) e^{j \mathbf{r} \cdot \hat{e}_i}, \]
while their reflected components read
\[ \vec{E}^r_{TE} = (\hat{q} \times \hat{e}_i) \times \hat{q} \Gamma_{TE} e^{j \mathbf{r} \cdot \hat{e}_i}, \vec{E}^r_{TM} = \hat{q} (\hat{q} \cdot \hat{e}_i) \Gamma_{TM} e^{j \mathbf{r} \cdot \hat{e}_i}, \]
with the Fresnel coefficients
\[ \Gamma_{TE} = \frac{N \cos \psi - (N - \sin^2 \psi) \rho_{1/2}}{N \cos \psi + (N - \sin^2 \psi) \rho_{1/2}}, \]
\[ \Gamma_{TM} = \frac{\cos \psi - (N - \sin^2 \psi) \rho_{1/2}}{\cos \psi + (N - \sin^2 \psi) \rho_{1/2}}. \]

Here, \( \psi \in [0, \pi/2) \) stands for the angle between the unit vectors \( \hat{n} \) and \( \hat{n}_i \). Accordingly, the total incident field can be expressed by
\[ \vec{E}^{inc} = \vec{E}^{inc}_{TE} + \vec{E}^{inc}_{TM} + \vec{E}^{inc}_{TM}. \]

**B. Scenario II: A monopole antenna and planar impedance ground**

In virtue of (1), the incident far field of a monopole antenna located at \( \mathbf{r}_M = \rho \hat{\lambda} \) along \( \hat{\lambda} \) direction with moment \( \mathbf{p}_M \) can be expressed as
\[ \vec{E}^{inc} = i \omega \mu_0 \mathbf{p}_M \mathbf{G}(\mathbf{r}_M; \hat{\lambda}) \cdot \hat{\lambda} \quad (7) \]
The elements of the Green tensor can be specified as the Green functions delivered by Norton [35] under HCA and grazing wave incidence as follows:
\[
\begin{align*}
G_x^x &= \frac{e^{ik_R}}{4\pi R_i} \left[ 1 - \frac{(x - x')^2}{R_i^2} \right] \\
+ \frac{e^{ik_R}}{4\pi R_i^2} \left[ \frac{R_h}{P^2} - \frac{R_v}{(x - x')^2} \left( \frac{1}{P^2} - \frac{1}{R_i^2} \right) \right] \\
G_y^z &= -\frac{(x - x')(y - y')}{P^2} e^{ik_R} \\
&+ \frac{(x - x')(y - y')}{4\pi R^2} \left[ \frac{R_h + R_v}{P^2} \right] \\
G_z^x &= -\frac{(x - x')(z - z')}{P^2} e^{ik_R} \\
&+ \frac{(x - x')(z + z')}{4\pi R_i^2} \left[ \frac{R_h + R_v}{R_i^2} \right].
\end{align*}
\]
In Norton’s formulation the ground is modeled by a scalar impedance boundary condition (cf.\cite{36, Sec.1.15}) with normalized surface impedance
\[ \Delta_N = \left(1/N\right)\left[1-(1/N^2)(P/R)^2\right]^{1/2} \]
With respect to free space characteristic impedance \( Z_0 = 120\pi \), while the reflection coefficients and ground parameters therein are given as
\[ R_v = \frac{z+z'-R_N\Delta_N}{z+z'+R_N\Delta_N}, \quad R_h = \frac{z+z'-R_0\delta_0}{z+z'+R_0\delta_0}, \]
\[ W_N = \frac{ik_0R_2}{2}\left(\frac{z+z'+R_2\Delta_N}{P}\right), \]
\[ \delta_0 = N^2\Delta_N, \quad q = \frac{ik_1R_2}{2}\left(\frac{z+z'+R_2\Delta_0}{P}\right). \]

C. Scenario III: A monopole antenna and spherical impedance ground

In this case, the expression \( (7) \) still applies, while the Green functions can be adopted as the set given by Wait \( 37,38 \) which are derived based on the Pol and Bremmer theory \( 39,40 \). Accordingly, the \( z, x, \) and \( y \) axes of the outer Cartesian coordinates in the vicinity of the scatterer can be coincided respectively with the spherical coordinate curves \( r, \theta, \phi \) of the globe, whose origin is the central point as depicted in Fig. 3.

Fig. 3. Spherical earth and its global coordinate system \( \theta \phi \).

Thereby, the Cartesian tensor components of the Green functions are suitable for our purposes with \( \theta' = 0 \) can be calculated as
where \( \Delta = (1/N) \left[ 1 - (1/N^2) \right]^{1/2} \) represents the surface impedance normalized w.r.t. free space characteristic impedance \( Z_0 = 120\pi \); \( b = (4/3) \times 6378 \text{ [km]} \) is the effective radius of earth taking into account first order tropospheric refractions; \( A = (k_i b/2)^{1/3} \), \( q_i = iA\Delta \), and \( q_i = N^2 q \) are ground constants; \( h_i = r^i - b \) and \( h_2 = r - b \) are the heights of the source and observation points above the ground; \( y_i = k_i h_i/A \); \( y_2 = k_i h_2/A \); and \( w(t) = \sqrt{\pi} \left[ B_i(t) + iA_i(t) \right] \)

with \( A_i(t) \), \( B_i(t) \) denoting standard Airy functions. The Green functions are derived under the natural assumption \( h_i, h_2 \ll b \) and the Rayleigh hypothesis \( |\Delta|^2 << 1 \), which fits well with HCA. The parameters \( t_s \) and \( t_m \) correspond to the discrete complex roots of the Stokes equations \( w'(t) - qw(t) = 0 \) and \( w'(t) - q w(t) = 0 \), respectively. They are the eigenvalues of the ground wave modes which are located in the first quadrant of the complex plane and their magnitudes increase with index number. In their determination, we apply the algorithm available in [41, pp. 340-343].

The critical distance, beyond which the influence of the curvature of earth on wave propagation cannot be disregarded, has been determined by Houdzoumis [42] as \( \rho_c = b (k_i b/2)^{-1/3} \).

In order to enrich any scenario by including any terrain feature, either land to sea transitions or obstacles (“islands”) along the propagation path as devised by Furutsu [43-49] or layered media, it is sufficient to substitute the appropriate set of Green functions into (2) and (3).

The oceanographic parameters such as mean wind speed, fetch length, and wave directionality as described in any sea spectrum [50], [51, pp. 386-403], [52, pp. 109-139], [53-55] can also be taken into account by modifying the normalized surface impedance as \( \Delta = \Delta + \Delta_{\text{add}} \), where the additional term \( \Delta_{\text{add}} \) was first calculated analytically by Barrick [56] as a 2-D spectral integral involving the sea spectrum using perturbation technique and the Rayleigh hypothesis.
V. NUMERICAL IMPLEMENTATIONS

In this section, we provide certain numerical results for the three scenarios in Sec. 4 in the frequency range $3-45 \text{ MHz}$ for propagation over seawater with $\epsilon_r = 80$, $\sigma = 4 \text{ [S/m]}$. First, for a verification of the developed codes we consider the case depicted in Fig. 4 where a crossed wire above planar sea surface is illuminated by a homogeneous plane wave with incidence angle $\psi = 45^\circ$ and unit electrical field amplitude.

![Fig. 4. A crossed wire located above planar sea surface and illuminated by a homogeneous plane wave.](image)

The four arms of the cross are assumed to have the same length $3.33 \text{ [m]}$ while the height of the bottom arm from ground is $h = 8 \text{ [m]}$. The horizontal arms are assumed to lie along the $y$-axis.

In the first set of illustrations, the operating frequency is taken $f = 3 \text{ [MHz]}$ ($\lambda = 100 \text{ [m]}$) for which each arm length is $\lambda/30$ and $h = 2\lambda/25$. In virtue of thin wire approximation, the geometrical parameters of the segments are picked as $a = 0.5 \text{ [m]} = \lambda/200 \text{ [m]}$ and $a = 1/40 \text{ [m]} = \lambda/4000 \text{ [m]} = \ell/20$. They fall into the range in which the values of the computed fields remain insensitive. Under this parameterization, the total number of segments read 27.

In Figs. 5 and 6, we provide the amplitude and phase distributions of currents on the arms of the crossed wire and relative errors calculated by $\%100 \|(\text{SNEC} - \text{CODE})/\text{SNEC}\|$ with reference to the same results obtained by the commercial software SNEC<sup>TM</sup>.

![Fig. 5. The amplitude distributions of currents on vertical and horizontal arms and relative errors at 3 [MHz].](image)

![Fig. 6. The phase distributions of currents on vertical and horizontal arms and relative errors at 3 [MHz].](image)

In the second set of illustrations in Figs. 7 and 8, the operating frequency is taken $f = 15 \text{ [MHz]}$ ($\lambda = 20 \text{ [m]}$) for which the arm length is $\lambda/6$ and $h = 2\lambda/5$, while $\ell = 0.5 \text{ [m]} = \lambda/40 \text{ [m]}$ and $a = 1/40 \text{ [m]} = \lambda/800 \text{ [m]} = \ell/20$.

The relative errors in the two sets of illustrations, which are restricted by 10%, stem from the choice of poorly converging pulse basis functions in the MoM scheme, as opposed to the more realistic sinusoidal basis functions employed in SNEC<sup>TM</sup>. It is seen that the error due to theoretical failure of pulse basis functions in satisfying the zero current tip condition reflects on
the entire geometry through matrix inversion in the current calculation.

Fig. 7. The amplitude distributions of currents on vertical and horizontal arms and relative errors at 15 [MHz].

As an application of the second scenario, a wire mesh plate with side length 7 [m] and diagonal length \( D \approx 10 \) [m] is illuminated by a monopole with unit moment at a distance of 10 [km] as depicted in Fig. 9.

In Figs. 10 and 11, we plot the elevation (\( Oxz \)) and azimuth (\( Oxy \)) patterns of the total (normalized) scattered field \( 10\log_{10}(4\pi r^2|\vec{E}|^2) \) at 15, 30, 45 [MHz] for which the operating wavelength corresponds to \( 2D, D, 2D/3 \), respectively. The symmetries observed in the patterns are due to the symmetric structure of the plate as well as the thin wire approximation that the current flows only along longitudinal direction in every segment.

Fig. 9. A wire mesh plate illuminated by a monopole residing on planar sea surface.

Fig. 10. Elevation patterns for the total scattered field at a)15, b) 30, c) 45 [MHz].

Fig. 11. Azimuth patterns for the total scattered field at a)15, b) 30, c) 45 [MHz].
In Fig. 12, we consider the same scatterer and source as in Fig. 9 on the spherical sea surface illuminated from a distance $10^3 \text{[km]}$.

![Fig. 12. A wire mesh plate illuminated by a monopole on spherical sea surface.](image)

The total scattered field $20\log_{10} |\vec{E}|$ measured over the sea surface from the origin in the direction of the monopole is plotted in Fig. 13 at $15, 30, 45 \text{[MHz]}$, for which the critical distances are calculated respectively as $77.216 \text{[km]}, 61.287 \text{[km]}, 53.539 \text{[km]}$. As expected physically, there is an increased attenuation beyond the critical distances proportional with frequency.

![Fig. 13. The total scattered field by a wire mesh plate calculated over sea surface at 15, 30, 45 [MHz].](image)

**VI. CONCLUDING REMARKS**

In the present work, we provided a MoM formulation for thin wire structures located over a lossy dielectric ground under HCA employing King’s range independent Green functions. Since we are focused on the analytical aspects of the formulation in the first place, our choice of pulse basis functions as the simplest option in a MoM scheme has resulted in a predictable and unavoidable relative error in current calculation as compared to the same results by the commercial software SNEC™, which employs (more realistic) sinusoidal pulse basis functions. Since such a deficiency is not associated with the success of the analytical calculations, the relative error can totally be removed by picking the same set of sinusoidal basis functions as in NEC softwares. While the current codes equally have the ability to read NEC-2 formatted input files, they are developed in the MATLAB™ environment with no commercial concern on total computational time at the time being. However, the numerical implementations put it very clearly that an electromagnetic simulation software that incorporates the Green functions of King may not only provide an alternative to the similar role of the extensive numerical/asymptotic libraries of NEC-2 whenever HCA applies (see also [57,58]), but also provides a capability to evolve by proper substitutions of Green functions to take into account various terrain features in any scenario. This is especially important since the physical (antenna) measurements around critical distances over earth have been reported to diverge seriously from those calculated by NEC-3 and NEC-4 while they follow the analytical results derived by King smoothly (see [59, Sec. 1] and the references cited therein). In light of the expertise gained the research is planned to pursue along the following areas of investigation:

i) Replacing the pulse basis functions in MoM scheme with sinusoidal basis functions following reference works as [60] to eliminate the current relative error with reference to NEC based softwares completely;

ii) Providing a time domain analysis ability to investigate the scattering of actual radar wave forms (as in [61-63]) from mesh structures above sea surface;

iii) Extending the impedance matrix for dielectric coated mesh structures for stealth applications. This is managed by reformulating the MoM matrix by describing a first order impedance boundary condition on each wire segment. For an analytical demonstration of the validity of the impedance boundary condition on arbitrarily shaped surfaces one may refer to [64];

iv) Incorporating Green functions of layered and complex media available in literature ([4-17]) for arbitrary range;

v) Investigation of most efficient NEC2 pre-processors (cf.[65], modeling guidelines (cf.[66,67]), programming platforms and
algorithms to minimize the computational time for integration and linear algebraic operations at the stage of developing a commercial product.

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REFERENCES


Simulation and Design of a Tunable Patch Antenna

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Abstract – A method for designing a tunable microstrip patch antenna is presented, suggesting cooperation between a theoretical transmission line model and a professional electromagnetic simulation tool. Tunable impedance elements are used to perturb the microstrip patch to alter the tuning range of the antenna. Results show excellent correlation between theoretical calculations and simulation data from Sonnet. Finally, guidelines for designing an antenna to be excited by an IMPATT diode are discussed.

Index Terms – Tunable Antenna, IMPATT Diode, Sonnet, Microstrip, Patch Antenna

I. INTRODUCTION
Tunable antennas offer several intriguing properties for wireless communications [1-5], ranging from cost savings by combining several analog components into one to introducing new uses, such as an adaptive element for smart antenna systems. The ideal element would offer dynamic control of a significant tuning range of resonance frequencies and bandwidths on a single antenna. To date, attempts to design a truly tunable antenna have been rudimentary, attaining a piece of the goal, such as a shift in resonance frequency at the cost of bandwidth, or vice versa. Some focus is needed to determine the type of excitation that offers total control over the electrical properties of an antenna.

Previously, a theoretical method was explored for tuning microstrip patch antennas by way of a dynamic “black box” impedance element: an ideal component with an unlimited set of values for both resistance and reactance [6]. Such an idealistic approach helped find the bounds of tunability with respect to the antenna’s field pattern and identify the most useful implementations of tunable antenna elements. Subsequently, the transmission line model previously used has been improved to include the impact of feedline width and mutual effects, which in turn allows for more accurate design simulations.

The transmission line model offers enough accuracy to quickly optimize not only the antenna dimensions, but also to search for tuning features such as complex impedance values and diode locations to meet design specifications. While more rigorous optimization techniques are available, (such as demonstrated for patch antenna sensitivity analysis with a method of moments [7]) these tools focus specifically on evaluating changes to the physical structure of the antenna only. Significant effort would be needed to modify this technique to include perturbations from an active tunable impedance device, as modeled in this presented research.

As a check against this work, a professional software tool, Sonnet, was used to evaluate the antenna baseline and tuning results derived from the theoretical model. Sonnet uses a method of moments [8] to analyze the electromagnetic properties from the physical dimensions of a circuit. By using Sonnet for this comparison, the previous work can be confirmed with a different computer-aided design methodology. It was found that the two design tools offered complementary benefits, so a natural conclusion was to create a cooperative procedure for designing tunable antennas. This new procedure takes advantage of the optimization flexibility of the theoretical model and the analytical rigor from the easy-to-use Sonnet package.

While the basis of the work was an ideal impedance “black box”, the IMPact Avalanche
Transit Time (IMPATT) diode [9-11] has shown significant promise as a real-world means for achieving tunable antennas. Sonnet design and simulation considerations for integrating an IMPATT diode with a microstrip patch antenna are also discussed.

II. THEORY

The cornerstone of the tunable antenna model is the classic Pues and Van de Capelle [12-14] transmission line model for microstrip antennas. This method uses the dimensions as well as four imaginary radiating slots to represent the radiation properties of the antenna, and is well-known for being simple yet accurate. The model allows one to visualize the antenna as a network of elements as seen in Fig. 1.

Procedurally, the Pues model computes effective parameters for line width \( W_e \), length \( L_e \), dielectric constant \( \varepsilon_{\text{eff}} \), and loss tangent \( \delta_e \) that have been adjusted to compensate for the total dimensions of the strip, the strip thickness, and dispersion at the operating frequency. These parameters are then used to find the appropriate characteristic admittance \( Y_c \), propagation constant \( \gamma \), self-admittance \( Y_s \) of the equivalent radiating slot, and mutual admittance effects \( Y_m \). They combine via:

\[
Y_n = \frac{Y_c^2 + Y_s^2 - Y_m^2 + 2Y_cY_m \coth(\gamma L) - 2Y_cY_m \csch(\gamma L)}{Y_c + Y_m \coth(\gamma L)},
\]

(1)

to yield an input admittance for the designed antenna at a microstrip feedline located at one edge of the patch.

The relationship in (1) leads directly to the input impedance of the patch antenna, and can be somewhat simplified by use of the transmission line admittance transfer function [5]:

\[
Y_i = Y_c \frac{Y_s + Y_i \coth(\gamma L)}{Y'_i + Y_c \coth(\gamma L)},
\]

(2)

where \( Y_c \) is still the characteristic admittance, \( Y_i \) is the transferred admittance at that intermediate point. If \( Y_i \) is set as the sum of \( Y_m \) and \( Y_i \), (2) can be used to find a \( Y_i \) at the input edge that has been transferred from the slot and mutual effects of the far edge of the antenna. Then slot and mutual admittances from the input edge can be added to yield the expected input admittance.

In regards to tunability, the transfer function in (2) can be exploited to introduce a tunable element along the length of the patch, connecting the radiating slots of the antenna. For instance, consider some \( Y_d \) that represents the admittance of a tunable diode placed at distance \( L/2 \) from the patch input. This becomes a new intermediate point, and the transfer function needs to be used twice. First, the far edge admittance, \( Y_{I_{\text{far}}} \), is “rolled” a distance \( L/2 \) with (2). This creates a \( Y_{I_{\text{diode}}} \) that can be added with \( Y_d \) (a shunt device) to give a \( Y_{I_{\text{diode}}} \). Then, (2) is used again to “roll” the remaining \( L/2 \) to the near edge, where it is added with the near edge admittance \( Y_{I_{\text{input}}} \) to give a new \( Y_{\text{in}} \) for the tuned antenna.

This effort suggests another factor impacting antenna tunability: the location of the “black box” on the patch at design time. The addition of another parameter to the design considerations evolves the simple \( L \) and \( W \) search of the standard patch antenna design procedure to a new process requiring an optimization routine that takes into account not only the antenna dimensions, but \( Y_d \) and its location as well.

Generally speaking, the tunable impedance element (or admittance based on preference) provides a perturbation that, in turn, modifies the input impedance to the patch. With the relationship between input impedance and patch dimensions (in light of the documented Pues model and (2)), this tuned antenna system behaves like a static antenna but with a new \( W \) and \( L \) at the operating frequency. It is as if the tunable “black box” impedance can electrically stretch or squeeze the patch antenna.

For the tunable microstrip patch antenna designer, the following process is recommended:

i. Design a basic microstrip patch antenna to establish a baseline for the design using (1)

ii. Set design goal(s) for the complete tunable antenna system, such as frequency operating range or a desired \( Z_{\text{in}} \)

iii. Begin iterative loop to step through different locations and tuning values for the tunable impedance (admittance) element
iv. Use (2) to find the transferred shunt admittance of the radiating slot plus mutual effects at the location of the “black box”

v. Combine the tunable and transferred admittances; use (2) to transfer to the input

vi. Combine the new transferred admittance with the near radiating slot and mutual effects to get the new input admittance

vii. Repeat the design process in reverse if the ‘tuned’ width and length are desired

viii. Compare to design goal(s) and continue iterations until desired effects are achieved

This procedure can be used to evaluate the impact of the tuned impedance values from the “black box” at several different locations on the baseline antenna. Checking this work against an accepted software tool is vital to ensuring its reliability. Comparison with results from Sonnet is presented below.

III. RESULTS

As a baseline, a microstrip patch antenna is designed with a resonance frequency at \( f_0 = 2.4 \) GHz. The dimensions of the patch are \( W = 2.42259 \) cm and \( L = 4.15922 \) cm with strip thickness \( t = 35 \) \( \mu \)m on a substrate having \( \varepsilon_r = 2.2 \) and height \( h = 0.15875 \) cm. These substrate parameters mimic Rogers RT/duriod 5880, a well-known, commercially available copper substrate. The feedline is matched to 50 \( \Omega \) at 2.4 GHz with a width of 0.484517 cm. Fig. 2 illustrates the antenna layout with a “black box” at the input.

Fig. 2. Physical layout of microstrip patch antenna with black box element at the input.

Using the implemented model based on Pues [12-14], the baseline antenna is calculated to have \( Z_{\text{in}} = 979.9 \) \( \Omega \), \( Z_c = 14.3269 \) \( \Omega \), \( Z_s = 15.432 - j185.92 \) \( \Omega \), \( Z_m = 1150.1 + j4130.6 \) \( \Omega \), and \( \gamma = 0.0326 + j72.3651 \). Sonnet simulated this antenna to resonate slightly higher at 2.404 GHz with \( Z_{\text{in}} = 979.9 \) \( \Omega \). The nearly identical \( Z_{\text{in}} \) responses for the theoretical model and Sonnet are plotted in Fig. 3.

![Fig. 3. Comparison of input impedance vs. frequency for designed patch antenna from the theoretical model and equivalent in Sonnet.](image)

To evaluate the impact of placing the tuning element at various locations along the midline of the patch antenna, the Sonnet geometry for the patch was modified to include a via port at the desired locations. It is at this port the “black box” will be added in shunt to perturb the antenna. The first step was to choose a cell size for Sonnet since it is desirable to make the via as small as possible to reduce parasitic capacitance, yet shrinking the cell multiplies the memory requirement for Sonnet’s calculations. For a single cell implementation, the final dimensions of the via were 202.889 \( \mu \)m long by 230.723 \( \mu \)m wide by 158.75 \( \mu \)m deep (10% of the dielectric height).

A new geometry was created to include the via port along the midline at one-tenth increments of the patch length from the feedline input to the far edge of the antenna. The net parasitic capacitance is seemingly significant, shifting the resonance frequency of the new geometries lower by about 10-15 MHz, yet the field patterns are comparable at 2.4 GHz, as shown in Figs. 4a and b. By comparing the \( Z_{\text{in}} \) responses for the basis and via-port geometries, it was determined the shunt capacitance introduced averages about 64 \( \text{fF} \) around the 2.4 GHz operating range.

Analyzing the impact of the tuning elements was carried out by stepping through values within a desired impedance range for the “black box” and one-tenth increments of the antenna length. It is
assumed that the tunable “black box” has two extremes: 1) positive resistance with inductance (extreme value of 30 + j190 Ω), 2) negative resistance with capacitance (extreme value of -5 - j100 Ω). The tuning range bounds were chosen to provide a bit of reality to this theoretical search, as it is hoped to replicate these results in the lab. Thus, the values were picked to emulate measurement results from previous IMPATT diode work [9-11], and also result in a good degree of tunability for the proposed antenna system.

Using the procedure given in Section II for an element tuned to -5 - j100 Ω and placed at the input to the patch, the resultant $Z_{in}$ is calculated and plotted in Fig. 5, highlighting a downward resonance frequency shift of 107 MHz to 2.293 GHz. Correspondingly, the Sonnet Netlist function is used to combine the antenna with a .z1p data file and plot the results in Figs. 6 and 7.

Fig. 5. Input impedance versus frequency for patch antenna tuned with $Z_d = -5-j100\Omega$ from theoretical model with resonance frequency at 2.293 GHz.

Fig. 6. Input resistance vs. frequency for tuned antenna ($Z_d=-5-j100\Omega$) in Sonnet: networked components vs. stretched dimensions. Arrow indicates match at frequency of 2.4 GHz.

Additionally, Figs. 6 and 7 display a “stretched” antenna based on the tuned input impedance at 2.4 GHz (labeled ‘stretch’). For this scenario, the equivalent patch dimensions are a 7.4% longer length of 4.46902 cm and a 28.2% shorter width of 1.74059 cm, demonstrating the ability to tune beyond the physical area of the
antenna. The Netlist and “stretched” curves differ, with comparable resonance frequencies at 2.278 and 2.279 GHz, respectively, but have a significant discrepancy in input impedance. It should also be noted that the Netlist curve included the additional capacitance from the via port, while the ‘stretched’ analysis does not.

Fig. 7. Input reactance vs. frequency for tuned antenna (Z_d=−5−j100Ω) in Sonnet: networked components vs. stretched dimensions. Arrow indicates match at frequency of 2.4 GHz.

The difference between the Netlist and “stretched” input impedance results highlights an initial flaw in the proposed analysis procedure: there is an important distinction between the operating range and the designed resonance frequency. Subsequent research will focus on the equivalent “tuned” dimensions over an entire desired operating range, and reporting how well that range is met for the “new” length and width.

To summarize the comparison, results from the two extreme tuning scenarios, -5-j100Ω and 30+j190Ω, at each of the predetermined locations are captured in Table 1 to assess the impact to moving the tuning element along the midline of the patch antenna. The first observation to be made is to note that adding -5-j100Ω anywhere causes the resonance frequency to shift towards DC, while 30+j190Ω moves f_res higher. Next, the shift seen due to the inclusion of the via port in the Sonnet geometries was consistent through all results: not only in the difference of the 2 tuning scenarios, but also as the delta between the model and Sonnet. There is an excellent correlation between the results of the model and those of Sonnet, validating the use of the model as a design guide for placing tunable elements.

### Table 1: Impact of tuning at several locations – no load (nl), -5-j100Ω (nrc), and 30+j190Ω (prl)

<table>
<thead>
<tr>
<th>Case</th>
<th>f_res (GHz)</th>
<th>Z11 (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basis</td>
<td>2.4/2.404</td>
<td>979.6/979.9</td>
</tr>
<tr>
<td>0.0L, nl</td>
<td>2.4/2.394</td>
<td>979.6/1102</td>
</tr>
<tr>
<td>0.0L, nrc</td>
<td>2.293/2.278</td>
<td>1982/7406</td>
</tr>
<tr>
<td>0.0L, prl</td>
<td>2.456/2.445</td>
<td>541/505</td>
</tr>
<tr>
<td>0.4L, nl</td>
<td>2.4/2.392</td>
<td>979.6/969.8</td>
</tr>
<tr>
<td>0.4L, nrc</td>
<td>2.39/2.381</td>
<td>982.4/958.6</td>
</tr>
<tr>
<td>0.4L, prl</td>
<td>2.405/2.397</td>
<td>932/933.2</td>
</tr>
<tr>
<td>0.6L, nl</td>
<td>2.4/2.391</td>
<td>979.6/1023</td>
</tr>
<tr>
<td>0.6L, nrc</td>
<td>2.39/2.381</td>
<td>1068/1148</td>
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<tr>
<td>0.6L, prl</td>
<td>2.405/2.395</td>
<td>896.2/935.1</td>
</tr>
<tr>
<td>1.0L, nl</td>
<td>2.4/2.394</td>
<td>979.6/971.5</td>
</tr>
<tr>
<td>1.0L, nrc</td>
<td>2.294/2.273</td>
<td>2046/2843</td>
</tr>
<tr>
<td>1.0L, prl</td>
<td>2.456/2.448</td>
<td>538.8/553.0</td>
</tr>
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</table>

A third observation can be seen from Table 1 that the closer the location is to 0.5L, the lesser the impact to the f_res shift. There may be other benefits to placing the tuning element at these inner locations such as dynamic input impedance tuning, but this was not evaluated during the analysis. For this work, the input feedline meets the patch at the very edge (as seen in Fig. 2 and not indented), creating an impedance mismatch. The input may be notched further inside to provide a better Z match in subsequent work.

The final observation is that there is symmetry about the 0.5L point, as can be seen by comparing the results between 0.0L and 1.0L as well as 0.4L and 0.6L in Table 1. Each case offers very similar results whether it is on the near or far side of the midpoint, and analysis of the impedance response in the surrounding spectrum confirms comparable results. In a way, this makes the choice of tuning location a bit easier, since the impact on one side mirrors the other side. In turn, this may offer more degrees of freedom to choose locations that bring other benefits such as system layout convenience.

The largest discrepancies in the results can be seen in the magnitude of Z_m at resonance. Of strongest note is the difference in Z_m at the 0.0L location for the -5-j100Ω scenario. Much of the disparity can be linked to the resolution of the calculations, as the input impedance behaves...
asymptotically near resonance.

As a final remark, the work presented intuitively leads to a design procedure that leverages the advantages of the two models. The designer can begin constructing the dimensions of the desired antenna and tuning characteristics with an optimization tool like the theoretical transmission line model presented. Once a framework is determined, Sonnet’s advanced layout and simulation features can be used to verify performance and establish a design feedback loop with physical data such as area and boundaries. Such a complementary effort between the two tools could be integrated via SonnetLab, which offers a link between MATLAB and Sonnet for automating their interaction.

IV. IMPATT DIODES

Several methods for tuning antenna elements have been proposed over the years, most notably the usage of varactors and/or specially-biased FETs [15]. These are often limited to just resistance or capacitance and tend to fall short for improving antenna robustness.

The tuning range chosen for this investigation is based on values that have been achieved with the IMPATT diode [9-11], a promising method for achieving a range of impedance values through a single device. A DC bias, which should be well isolated from the RF signal, controls the avalanche frequency, and, hence, the diode impedance. Fig. 8 shows the model for the IMPATT diode before and after the avalanche frequency.

![Fig. 8. Model for IMPATT impedance vs. frequency, for both sides of avalanche frequency.](image_url)

The impedance capabilities of the IMPATT diode range from $R + j\omega L$ to $-R + 1/j\omega C$, as modeled with the “black box”. The necessary tuning range can be achieved by careful design of the diode dimensions. This means that when combined with a patch antenna, the properties can be tuned solely by the biasing of the IMPATT diode. Using the model to find ideal locations for IMPATT diodes along the patch, designers could quickly layout and analyze a tunable antenna with co-located IMPATT diodes in Sonnet.

V. CONCLUSIONS AND REMARKS

In summary, the work presented demonstrates a good marriage between results from the transmission line model and Sonnet. This relationship naturally leads to a collaborative design procedure for tunable patch antennas. To further advance the study of exciting a microstrip patch with impedance elements, Sonnet could improve the means of locating shunt impedances inside a structure (not just at edges) for evaluation. Finally, it should be pointed out that the final step of implementing the proposed tunable antenna with an IMPATT diode is made significantly simpler with the advanced layout and analysis features of the Sonnet tool.

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