A Resonance Prediction Method for a Shielding Enclosure with Apertures Illuminated by a Plane Wave

Bao-Lin Nie¹, Zhong Cao², and Ping-An Du¹

¹School of Mechanical and Electrical Engineering
University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China
blnie@uestc.edu.cn, dupingan@uestc.edu.cn

²Chongqing Vehicle Test & Research Institute Co., Ltd.
Chongqing, 401122, China
caozhong@cmhk.com

Abstract — Electromagnetic resonances in a shielding enclosure with apertures could result in significant degradation of its shielding performance. In this paper, an analytical method is proposed to predict the resonances of a shielding enclosure with aperture arrays illuminated by a plane wave. Firstly, an obliquely incident and arbitrarily polarized plane wave is decomposed into several normally incident plane waves each polarized in the axis direction. Then, the transmitted fields through each aperture are equivalent to fields radiated by a magnetic current located at the center of the corresponding aperture. Finally, the resonant modes and resonant frequencies of the enclosure are determined according to the direction of the magnetic current and the properties of the radiated fields. This method is verified by comparing its results with the results of full-wave simulations. Compared with numerical methods, the method is easier to implement and more efficient in predicting resonances.

Index Terms — Aperture array, electromagnetic shielding, resonances, shielding enclosure.

I. INTRODUCTION

Resonances play an important role in some devices such as microwave resonators and resonant circuits while producing an adverse effect in others. For example, electromagnetic resonances occur in enclosures of electronic equipment, which pronouncedly enhance the amplitude of electromagnetic fields and deteriorate the shielding performance of these enclosures. The shielding performance of an enclosure is quantified by its shielding effectiveness (SE), defined as the ratio of the field strength at observation points in the presence and absence of the enclosure. It is known that an enclosure may have negative SE at resonant frequencies, which means that the presence of the enclosure increases the field strength. Electromagnetic resonances in shielding enclosures thus draw great attention from researcher. Up to present, numerous methods have been reported in predicting SE and resonances of shielding enclosures, including both analytical formulations and numerical methods.

The majority of analytical formulations are based on the equivalent circuit method proposed by Robinson, et al. [1]. In this method, the aperture is represented as a length of shorted coplanar strip transmission line, and the enclosure is modeled as a length of rectangular waveguide shorted at the end. It is expected to be valid beyond the resonant frequency of the next higher order mode of the enclosure [2]. Some attempts tried to extend the original method to handle arbitrary plane wave [3], to account for the thickness [4], and to deal with multiple apertures [5]. Unfortunately, all these methods fail to predict the entire resonances excited in the concerned higher frequency band. Moreover, numerical methods such as the finite-difference time-domain (FDTD) method [6], the transmission-line modeling (TLM) method [7], the finite element method (FEM) method [8, 9], and the method of moments (MoM) [10] can accurately capture the higher order modes propagation effect and deal with arbitrarily complex geometry. However, numerical methods are computationally expensive for this multi-scale configuration.

There are other works reveal the relationship between the resonances of the cavity-slot coupled system and incident plane waves [11-13]. In Ref. [11], the coupling of an incident plane wave through a slot into a lossy rectangular cavity was analyzed by using a generalized network formulation. It was found that, apart from the cavity’s natural resonances, two types of cavity-slot coupling resonances may occur. In Ref. [12], explicit formulas for the conditions of cavity-slot resonances were derived based on the duality between a slot and a strip. The formulas are simple to use and do not involve
the numerical evaluation of derivatives and integrals. In Ref. [13], analytic approximation of the complex resonances of a slot-fed rectangular cavity was derived using just one aperture expansion function. However, most of these efforts were put on one slot case and mainly on the cavity-slot coupling resonances. It has been found that the coupling resonances would not be excited when the diameter of the aperture is very small compared with wavelength [11].

It is known that the electromagnetic fields in a rectangular cavity can be regarded as the superposition of its transverse electric (TE) and transverse magnetic (TM) resonant modes. However, for a rectangular enclosure with aperture arrays excited by an external plane wave, not all the possible resonant modes in the concerned frequency band will be actually excited. So the relationship between the properties of the plane wave and the subsistent resonant modes is important for predicting the resonances in the concerned frequency band. This paper investigates the resonant condition of a shielding enclosure with aperture arrays and provides designers of shielding enclosures with a method to predict resonant frequencies and resonant modes.

II. THE PROBLEM DESCRIPTION

A. Shielding geometry and excitation

As shown in Fig. 1, a shielding enclosure mimics a computer box. The interior dimensions of the enclosure are \( a \times b \times d \) mm (360 \( \times \) 300 \( \times \) 120 mm) in \( x \), \( y \), and \( z \) directions. The thickness of the enclosure walls is 1.5 mm. The material of the enclosure is aluminum. Two aperture arrays, with aperture radius of 6 mm, center-to-center spacing of 20 mm, and total number of apertures 21 and 27, are residing on the front \( y-z \) wall and the side \( x-z \) wall. A plane wave acting as an excitation source is obliquely incident with propagation vector \( \mathbf{\beta} \), incident angles \( \phi \) and \( \theta \), and polarization angle \( \varphi \) to the \( \theta \)-axis, as shown in Fig. 2. The monitor point of the electric SE is located at the center of the enclosure (-180mm, 150mm, 60mm). The studied frequency band is 0-2GHz.

![Fig. 1. Geometry of a rectangular enclosure with aperture arrays on multiple walls.](Image)

![Fig. 2. The coordinate system and the incident plane wave.](Image)

As shown in Fig. 2, the propagation vector and the electric field can be decomposed into three components in the rectangular coordinate system as:

\[
\mathbf{\beta} = -\beta_0[\hat{x}\cos\phi\sin\theta + \hat{y}\sin\phi\sin\theta + \hat{z}\cos\theta],
\]

\[
E = E_0[\hat{x}(\cos\varphi\cos\theta\cos\varphi - \sin\phi\sin\varphi) \\
+ \hat{y}(\sin\phi\cos\theta\cos\varphi + \cos\phi\sin\varphi) \\
+ \hat{z}(-\sin\theta\cos\varphi)],
\]

where \( \beta_0 \) and \( E_0 \) are magnitudes of the electric field and the propagation vector, \( \hat{x} \), \( \hat{y} \), \( \hat{z} \) are unit vectors in the direction of each coordinate axis. Then, the oblique incident and arbitrarily polarized plane wave can be decomposed into several normally incident plane waves.

B. Resonant frequency analysis

It is well known that the resonant frequency of each resonant mode in a rectangular cavity of size \( a \times b \times d \) can be expressed as:

\[
f_{\text{res}} = \frac{c}{2\pi} \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 + (\frac{p\pi}{d})^2},
\]

where \( c = 1/\sqrt{\mu_0\varepsilon_0} \) is the speed of light in free space, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability in free space, \( m \), \( n \), and \( p \) are mode indexes in \( x \), \( y \), and \( z \) directions. A resonant frequency usually corresponds to a unique resonant mode of electromagnetic fields in the cavity. Theoretically, the number of modes in a rectangular cavity is a function of the frequency. The number of modes has a smoothed approximation as [14]:

\[
N_s(f) = \frac{8\pi abdf^3}{3c^2} - (a + b + d) \frac{f}{c} + \frac{1}{2}.
\]

Equation (4) shows that the number of modes and the mode density increase markedly as the frequency increases.

For the shielding enclosure excited by an external plane wave shown in Fig. 1, its shielding performance is,
to a large extent, determined by its resonant properties. Therefore, it is necessary to analyze the resonant properties of the enclosure under specified excitation. For simplicity, we remove the aperture array residing on the side x-z wall, and keep the aperture array residing on the front y-z wall. The related parameters are set to be $\phi = 0^\circ$, $\theta = 90^\circ$, and $\varphi = 90^\circ$, thus the plane wave propagates in -x direction and has its electric field polarized in y direction.

The Full-wave TLM numerical method in the time domain can accurately deal with arbitrarily complex geometry [15]. Therefore, the TLM method is adopted to compute the electric SE of the enclosure in the following examples. During the TLM simulation, we slowly increase the mesh density until the results converge. The material of the enclosure is assumed to be aluminum with a conductivity of $3.54 \times 10^7$ S/m. Take this particular configuration for example, hexahedron absorbing boundary which has all its face 109 mm away from the enclosure is adopted. The whole solution domain is discretized into a Cartesian mesh of grid cells, and the total number of grid cells is $6.34 \times 10^6$. The time step is $7.51 \times 10^{-13}$ s, and the total number of time step is $3.58 \times 10^5$.

In Fig. 3, the actual resonant frequencies indicated by the troughs of the electric SE curve are compared with all the possible resonant frequencies. It can be observed from Fig. 3 that only a few of the possible resonant modes are really excited. Specifically, the resonant frequencies for this configuration are 1317 MHz, 1501 MHz, 1600 MHz, 1653 MHz, 1767 MHz, and 1804 MHz. Therefore, this paper investigates the physical mechanism of resonances in shielding enclosures and proposes an effective method for resonance prediction.

### III. Resonance Prediction

It is known that the electromagnetic wave transmitted through a small aperture into an enclosure is equivalent to the wave radiated by an infinitesimal electric current and an infinitesimal magnetic current without the aperture present [16],

$$\mathbf{J} = j \omega \varepsilon_0 \mu_0 \mathbf{n} \mathbf{E} \delta(x-x_0) \delta(y-y_0) \delta(z-z_0),$$  \hspace{1cm} (5)

$$\mathbf{M} = -j \omega \varepsilon_0 \mu_0 \mathbf{H} \delta(x-x_0) \delta(y-y_0) \delta(z-z_0),$$  \hspace{1cm} (6)

where $\alpha_e$ and $\alpha_m$ are electric and magnetic polarizability of the aperture, $E_x$ and $H_z$ are the undisturbed normal electric field and transverse magnetic field at the position of the aperture, $\hat{n}$ is the unit vector normal to the surface of the aperture. $(x_0, y_0, z_0)$ are the coordinates of the aperture, and $\delta(x), \delta(y), \delta(z)$ are Dirac delta functions.

The enclosure in Fig. 1 is adopted here to derive the radiated fields and predict the resonances. For simplicity, the apertures residing on the side x-z wall are again removed, the incident angles and polarization angle are set to be $\phi = 0^\circ$, $\theta = 90^\circ$, and $\varphi = 90^\circ$. Since $E_x = 0$ for normal incident case, we need only consider the fields produced by the magnetic current. The magnetic field of the incident plane wave can be expressed as:

$$\mathbf{H} = -\frac{E_0}{\eta_0} e^{j \beta x},$$  \hspace{1cm} (7)

where $\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$ is the intrinsic impedance of free space, $\beta$ is the phase constant. Equation (7) represents a plane wave polarized in y direction and propagation in -x direction. The enclosure with apertures closed will approximately cause a total reflection of the incident fields. Thus the total magnetic field outside the enclosure with apertures closed can be expressed as

$$\mathbf{H} = -\frac{E_0}{\eta_0} (e^{j \beta x} + e^{-j \beta x}).$$  \hspace{1cm} (8)

Then, 21 equivalent magnetic currents each located at the center of the corresponding aperture can be introduced to replace the aperture array. The equivalent magnetic current that represents the aperture located at the center of the y-z wall is analyzed here in detail. The magnetic current density of the equivalent magnetic current is:

$$\mathbf{M} = \frac{2 \mu_0 \varepsilon_0 \alpha_m}{\eta_0} \delta(x) \delta(y - \frac{b}{2}) \delta(z - \frac{d}{2}).$$  \hspace{1cm} (9)

In this case, the presence of the conducting wall is easily accounted for using image theory, which has the effect of doubling the current strengths and removing the wall. Thus the electric field radiated by the equivalent magnetic current can be expressed as [17]

$$\mathbf{E} = \sum_{n,p} A_{n,p} E_{n,p}^e,$$  \hspace{1cm} (10)

Fig. 3. Electric SE of the enclosure; all possible resonances are indicated with vertical dashed lines.
where $n$ and $p$ represent the mode indexes in $y$ direction and $z$ direction, $E_{n,p}^{x}$ is the electric field component of the $(n, p)$ mode propagating in $-x$ direction, $A_{n,p}$ denotes the corresponding coefficient that can be expressed as

$$A_{n,p} = \frac{1}{F_{n,p}} \int_{V} H_{n,p}^{y} \cdot M \, dv,$$  \hspace{1cm} (11)

where $H_{n,p}^{y}$ denotes the magnetic field component of the $(n, p)$ mode propagating in $x$ direction, $F_{n,p}$ is a normalization constant proportional to the power flow of the corresponding mode [17].

It can be seen from equation (11) that only the $z$ component of $H_{n,p}^{y}$ has a contribution to the coefficient $A_{n,p}$, which makes the analysis more straightforward. The $z$ component of $H_{n,p}^{x}$ has a form of

$$(H_{n,p}^{x})_{z} = B \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d} e^{-j\beta x},$$

where $B$ is a constant. In order to ensure the coefficient $A_{n,p}$ of the $(n, p)$ mode does not vanish when $y$ is set to be $0.5b$ and $z$ is set to be $0.5d$ in evaluation of equation (11), $n$ must be an even number and $p$ must be an odd number. When it comes to the multiple apertures case as shown in Fig. 1, each aperture can be replaced by a corresponding equivalent magnetic current. All magnetic currents point in the same direction, thus the conclusion still holds. This is the physical mechanism that only a few of the possible resonant modes are really excited.

The resonant frequencies shown in Fig. 3 exactly validate this conclusion. The resonant frequencies for this configuration are 1317 MHz for TE, 101, 1501 MHz for TE, 201, 1600 MHz for TM, 021, 1653 MHz for TE, 121, 1653 MHz for TM, 121, 1767 MHz for TE, 301, 1804 MHz for TE, 221, and 1804 MHz for TM, 221. The RMS magnitude of the magnetic field on the plane $z = 60$ mm in the entire computational domain is shown in Fig. 4, where the horizontal direction and vertical direction correspond to the $y$-axis direction and $x$-axis direction in Figure 1. In Fig. 4, the magnetic field distributions at 1653 MHz and 1804 MHz are shown out, which intuitively indicate the mode indexes of these two resonant modes.

**IV. VERIFICATION FOR OTHER CONFIGURATIONS**

**A. Normal incidence in -$y$ direction**

In this subsection, we remove the aperture array residing on the front $y$-$z$ wall, and keep the aperture array residing on the side $x$-$z$ wall. The related parameters are set to be $\phi = 90^\circ$, $\theta = 90^\circ$, and $\varphi = 0^\circ$, thus the plane wave propagates in -$y$ direction and has its electric field polarized in -$z$ direction. In this case, the electromagnetic fields transmitted through apertures can be equivalent to fields radiated by magnetic currents in -$x$ direction. The equivalent magnetic current that represents the aperture located at the center of the $x$-$z$ wall is analyzed here in detail. The magnetic current density of the equivalent magnetic current is

$$M = -\frac{\varepsilon}{\eta_{0}} \frac{2}{\pi} j \omega \mu_{0} \sum_{m} E_{m} \delta(x - a) \delta(y) \delta(z - \frac{d}{2}).$$  \hspace{1cm} (13)

It can be seen from equation (11) that only the $x$ component of $H_{n,p}^{y}$ has a contribution to the coefficient $A_{n,p}$. The $x$ component of $H_{n,p}^{x}$ has a form of

$$(H_{n,p}^{x})_{x} = C \sin \frac{m\pi x}{a} \cos \frac{p\pi z}{d} e^{-j\beta x},$$

where $C$ is a constant. In order to ensure the coefficient $A_{n,p}$ of the $(m, p)$ mode does not vanish when $x$ is set to be $0.5a$ and $z$ is set to be $0.5d$ in evaluation of equation (11), $m$ must be an odd number and $p$ must be an even number.

It can also be deduced that the resonant frequencies are larger than 2GHz when $p$ is greater than or equal to 2, so $p$ must be equal to 0, thus $m$ must be nonzero. Consequently, the resonant modes must be TE$_y$ type and $n$ must be nonzero. The predicted resonant modes and resonant frequencies are indicated in Table 1 as gray cells. The simulated shielding effectiveness revealing resonances shown in Fig. 5 again confirms the prediction.

**Table 1: Resonant frequencies in MHz predicted by the proposed method**

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<td>4</td>
<td>1739</td>
<td>1942</td>
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Fig. 5. The resonant frequencies and corresponding modes of the enclosure with aperture array residing on the side x-z wall.

B. Aperture arrays residing on the multiple walls

In this subsection, the aperture arrays residing on the front y-z wall and the side x-z wall are both kept. The related parameters are set to be $\phi = 30^\circ$, $\theta = 90^\circ$, and $\varphi = 0^\circ$, thus the plane wave is obliquely incident and has its electric field polarized in z direction. In this case, the plane wave can be decomposed into two plane waves, one propagates in -x direction and another propagates in -y direction, both have their electric field polarized in -z direction.

For the plane wave propagating in -y direction, its analysis is the same as that in subsection 4.1. Therefore, $m$ must be an odd number and $p$ must be an even number. For the plane wave propagating in -x direction, its magnetic field is in -y direction, thus the electromagnetic fields propagating through apertures can be equivalent to fields radiated by magnetic currents in y direction. The equivalent magnetic current that represents the aperture located at the center of the y-z wall is analyzed here in detail. The magnetic current density of the equivalent magnetic current is:

$$M = \frac{\lambda}{2} j \mu \epsilon \eta \delta(x) \delta(y - \frac{b}{2}) \delta(z - \frac{d}{2}),$$

It can be seen from equation (11) that only the $y$ component of $H_{n,p}$ has a contribution to the coefficient $A_{n,p}$. The $y$ component of $H_{n,p}$ has a form of

$$(H_{n,p})_y = D \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d} e^{-j\beta z},$$

where $D$ is a constant. In order to ensure the coefficient $A_{n,p}$ of the $(n, p)$ mode does not vanish when $y$ is set to be $0.5b$ and $z$ is set to be $0.5d$ in evaluation of equation (11), $n$ must be an odd number and $p$ must be an even number.

In conclusion, for the original obliquely incident plane wave, its mode indexes need to satisfy the following conditions: $m$ and $n$ cannot be even numbers at the same time, and $p$ must be an even number. It can also be deduced that the resonant frequencies are larger than 2GHz when $p$ is greater than or equal to 2, so $p$ must be equal to 0, thus $m$ and $n$ must be nonzero. Consequently, the resonant modes must be TE$_x$ type or TE$_y$ type. The predicted resonant modes and resonant frequencies are indicated in Table 2 as gray cells. The simulated shielding effectiveness revealing resonances shown in Fig. 6 again confirms the prediction. It is worth noting that the simulation takes several hours on a desktop to predict the entire resonances.

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Fig. 6. The resonant frequencies and corresponding modes of the enclosure with aperture arrays residing on the front y-z wall and the side x-z wall.

V. CONCLUSION

In view of the importance of resonance prediction in evaluation of the shielding performance of a shielding enclosure with apertures, an analytical method to determine the resonant modes and resonant frequencies is proposed based on the specified knowledge of the excitation plane wave. This method outperforms the existing methods in several ways. Firstly, once the properties of the excitation plane wave are known, the resonances of the shielding enclosure can be readily
predicted through a simple theoretical derivation. Moreover, this method can predict all the resonant modes and resonant frequencies of shielding enclosures under plane wave excitation without omission of some potential resonances. Furthermore, compared with full-wave methods, the method presented here is easier to use. Although the studied frequency band is below 2GHz in this paper, the method is applicable to higher frequencies.

ACKNOWLEDGMENT
This project was supported in part by the National Natural Science Foundation of China (Grant No. 51705067), the Fundamental Research Funds for the Central Universities of China (Grant No. ZYGX2018046), and in part by the China Postdoctoral Science Foundation (Grant No. 2015M582534).

REFERENCES

Bao-Lin Nie was born in Shangluo, China, in 1985. He received the B.S. degree and Ph.D. degree in Mechatronics Engineering from the University of Electronic Science and Technology of China, Chengdu, China, in 2007 and 2014, respectively. From 2010 to 2012, he was a Visiting Student in the Center for Computational Electromagnetics, Department of Electrical and
Zhong Cao was born in Chongqing, China, in 1988. He received the Bachelor and Master degrees from the University of Electronic Science and Technology of China, Chengdu, China, in 2011 and 2014. His research interests include electromagnetic simulation, and EMC in shielding enclosures.

Ping-An Du received the M.S. and Ph.D degrees in Mechanical Engineering from Chongqing University, Chongqing, China, in 1989 and 1992, respectively. He is currently a Full Professor of Mechanical Engineering at the University of Electronic Science and Technology of China, Chengdu, China. His research interests include numerical simulation in EMI, vibration, temperature, and so on.