

# A Gaussian Modulated Sinusoidal Pulse for Circuit-Parameter Estimation of a Synchronous Generator Using a 2D-FE Field Model

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**Abstract** — This paper presents the parameter estimation of a synchronous generator model based on the Gaussian Modulated Sinusoidal Pulse (GMSP). A 2D Finite Element (FE) model is used to evaluate the proposed signal in the estimation of the equivalent-circuit parameters of the generator. The presented methodology is based on the application of a FE model to simulate a standstill test. The FE model is validated against GMSP experimental results whilst the generator is at standstill. Afterwards, the FE simulation results are used to estimate the equivalent circuit parameters using a genetic algorithm. Finally, the estimated parameters are validated by comparing the simulation results against experimental data of a sudden three-phase short-circuit fault. A synchronous generator of a 7 kVA, 220V, 60 Hz, 1800 rpm, four-pole was employed for validating the estimated model parameters.

**Index Terms** - Finite element method, genetic algorithm, parameter estimation, synchronous generator.

## I. INTRODUCTION

Equivalent electric circuits of electrical machines are a lumped representation of the complex electromagnetic behavior of these devices [1-3]. They have been used since several

decades ago for predicting the performance of synchronous generators, induction motors and transformers. The equivalent circuits have advantages such as simplicity, fast computation and good accuracy in predicting the dynamic behavior of electric machines. On the other hand, lumped parameter models can be represented by state-space equations. The parameters of these generator models have usually been obtained by standstill and online tests [4]. The standstill tests are attractive because a small perturbation signal can be applied to each magnetic axis and a possible damage to the generator is avoided. The step voltage, dc-flux decay, and frequency response are standstill tests that have been commonly applied in estimating the complete set of parameters. Recently, new signals have been explored using the Finite Element Method (FEM) in induction machines [5]. A Finite Element (FE) model can be employed for evaluating any excitation signal at the design stage of an electrical machine. This approach is attractive for manufacturers in characterizing their synchronous generators before they are built [6-8]. Moreover, the FE method has also been used to study other types of electrical machines, e.g. switched reluctance generators and helical motion induction motors [9-10]. However, there are excitation signals which may be of interest in the parameter estimation of electrical machines such as the

Gaussian Modulated Sinusoidal Pulse (GMSP). The GMSP has been successfully applied in the analysis of transient electromagnetic wave propagation but it has not been used in electrical machines [11].

This paper presents the standstill GMSP test for estimating the parameters of the  $d$ - $q$ -axis model of a synchronous generator. The proposed methodology uses a 2D FE model which is validated against standstill GMSP experimental tests. Afterwards, the obtained FE data is used to estimate the model parameters by applying a Genetic Algorithm (GA) and they are fine tuned by employing the quasi-Newton method. To validate the estimated parameters, a sudden three-phase short-circuit fault is simulated and it is compared against experimental results where a good agreement was found. A 7 kVA, 220V, 1800 rpm and 60 Hz salient-pole synchronous generator is used in the proposed approach.

## II. FINITE ELEMENT MODEL

The FEM has been used in many areas of knowledge where it is used to solve partial differential equations. In electrical engineering, the FEM has been used to solve the Maxwell equations that describe the electromagnetic dynamics of electrical machines. The 2D time-domain electromagnetic behavior of a synchronous machine is governed by the diffusion equation (1).

$$\frac{\partial}{\partial x} \left( \nu \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu \frac{\partial A}{\partial y} \right) + \sigma E - \sigma \frac{\partial A}{\partial t} = 0, \quad (1)$$

where  $\sigma$  is the electric conductivity,  $E$  is the electric field intensity,  $A$  is the magnetic vector potential and  $\nu$  is the magnetic reluctivity.

The FEM can be used to solve the diffusion equation (1) and this may be achieved by applying the Galerkin method. This method is based on the minimization of the residual by using a weighting function. Due that most electrical devices are fed by a voltage source, equation (1) needs to be modified to allow this source type instead injected currents. In this paper, the model formulation takes into account the electromagnetic field and circuit equations. The model formulation considers the winding currents as additional degrees of freedom and the related circuit equations are solved simultaneously with (1). The resulting FE formulation can be expressed by (2) and (3) [8].

$$\mathbf{S}\mathbf{A} + \mathbf{N} \frac{d\mathbf{A}}{dt} - \mathbf{P}\mathbf{I} = \mathbf{0}, \quad (2)$$

$$\mathbf{Q} \frac{d\mathbf{A}}{dt} + \mathbf{R}\mathbf{I} + \mathbf{L} \frac{d\mathbf{I}}{dt} = \mathbf{V}, \quad (3)$$

where  $\mathbf{S}$  is the stiffness matrix,  $\mathbf{N}$  represents eddy current regions,  $\mathbf{P}$  represents machine windings, and  $\mathbf{Q}$  stands for induced voltages. Matrix  $\mathbf{R}$  represents dc resistances, matrix  $\mathbf{L}$  stands for end-winding inductances, vector  $\mathbf{V}$  represents the source voltages,  $\mathbf{I}$  represents a vector of winding currents, and  $\mathbf{A}$  is the magnetic vector potential.

The matrices  $\mathbf{S}$ ,  $\mathbf{N}$ ,  $\mathbf{P}$  and  $\mathbf{Q}$  are obtained by assembling its corresponding elemental matrices which are defined by (4)-(7).

$$S_{kj} = \int_{S_i} \nabla \alpha_k' \nu \nabla \alpha_j ds, \quad (4)$$

$$N_{kj} = \int_{S_i} \sigma \alpha_k' \alpha_j ds, \quad (5)$$

$$P_{kj} = \int_{S_i} \frac{n_{cj}}{S_{fj}} \alpha_k ds, \quad (6)$$

$$Q_{kj} = \int_{S_i} \frac{n_{ck} \ell}{S_{fk}} \alpha_j ds, \quad (7)$$

where  $S_i$  indicates surface of the mesh element  $i$ ,  $\alpha$  is the shape function of first order finite elements and  $ds$  represents a differential surface.  $n_c$  represents the winding turns and  $S_f$  is the total surface of the coil. The axial length of the winding  $f$  is denoted by  $\ell$ .

By using the Euler scheme, the time-domain discretization of (2) and (3) can be expressed as (8).

$$\begin{bmatrix} \mathbf{S}(t + \Delta t) + \frac{1}{\Delta t} \mathbf{N} & -\mathbf{P} \\ \frac{1}{\Delta t} \mathbf{Q} & \mathbf{R} + \frac{1}{\Delta t} \mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{A}(t + \Delta t) \\ \mathbf{I}(t + \Delta t) \end{bmatrix} = \quad (8)$$

$$\begin{bmatrix} \frac{1}{\Delta t} \mathbf{N} & 0 \\ \frac{1}{\Delta t} \mathbf{Q} & \frac{1}{\Delta t} \mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{A}(t) \\ \mathbf{I}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{V} \end{bmatrix},$$

where  $\Delta t$  stands for the time step.

The GMSP voltage is applied to the stator with the field winding in short-circuit while the alternator is at standstill. This type of excitation has been applied to transient electromagnetic wave propagation and it is given by (9) [11]:

$$v(t) = V_m \exp\left(-\frac{(t-t_{delay})^2}{\beta}\right) \sin(\omega(t-t_{delay})), \quad (9)$$

where  $V_m$  is the amplitude of the signal,  $\omega=2\pi f$  is the operational angular frequency, and  $t$  is time. The parameters  $\beta$  and  $t_{delay}$  define the signal shape. The studied synchronous generator has solid poles where eddy currents are induced. A constant permeability of 200 and a first order FE mesh was used [12]. The magnetic flux distributions along the  $d$ - $q$  axis positions are shown in Figs. 1 and 2. The effect of the induced eddy currents in the flux distribution is clearly seen in the pole faces.

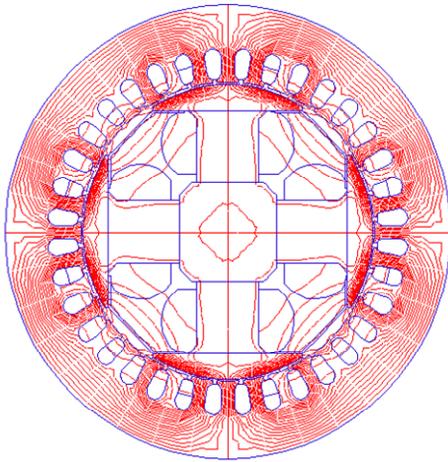


Fig. 1. D-axis field distribution with the GMSP standstill test.

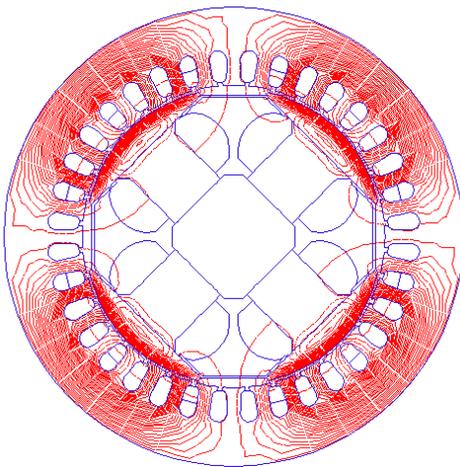


Fig. 2. Q-axis field distribution with the GMSP standstill test.

### III. FE-GMSP MODEL VALIDATION

The second step in the proposed approach is the validation of the developed FE model. The GMSP was applied to the machine  $d$ - $q$  magnetic axes, with the machine previously positioned in each magnetic axis at standstill. The GMSP test arrangement for the  $d$ -axis test is shown in Fig. 3. The signal generation and data acquisition were carried out by a computer and an acquisition card. A virtual instrument was developed in LabVIEW and allows the GMSP generation through a friendly user interface. A power amplifier was used to get the appropriate level of the testing current. The stator and field currents ( $i_a$  and  $i_f$ ) were measured with resistive shunts. The GMSP voltage and the generator currents can be visualized in the computer during the test.

The GMSP voltage applied to the  $d$ -axis is shown in Fig. 4. A comparison of the FE predicted currents against experimental data for the  $d$ -axis, field and  $q$ -axis currents are shown in Figs. 5-7 where a good accuracy was achieved. The small values of field current causes a noisy signal as it is seen in Fig. 6.

The use of a FE model for parameter estimation offers the advantage of having noise free signals that allows estimating a better set of parameters. The employed GMSP signal uses a shape parameter ( $\beta$ ) of 0.0311, a frequency of 60 Hz, and 0.5s for  $t_{delay}$ . The GMSP magnitude was adjusted to have approximately a stator current of 1 A in the  $d$ - $q$  axis tests. A small difference is observed between the FE model and test results for the  $q$ -axis test (Fig. 7).

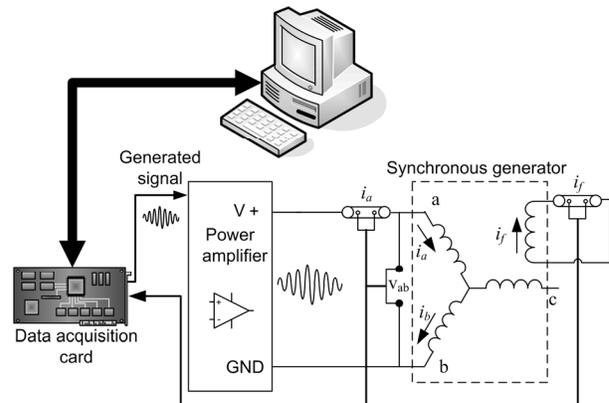


Fig. 3. Direct-axis standstill test with the GMSP excitation.

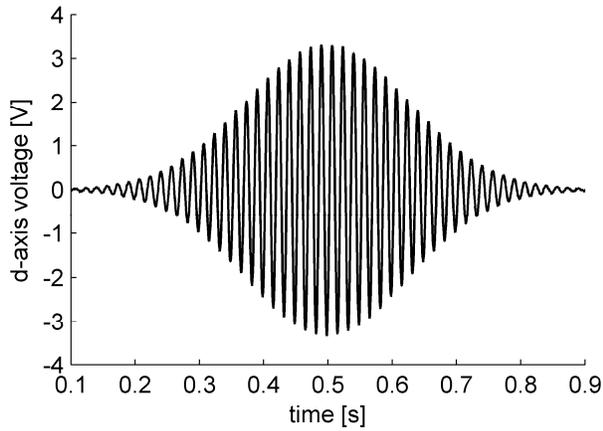
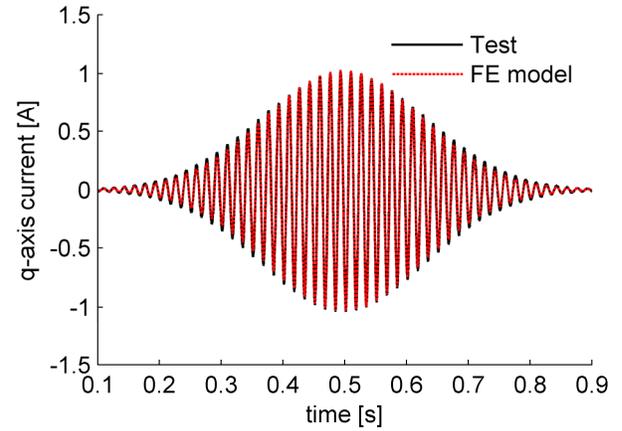
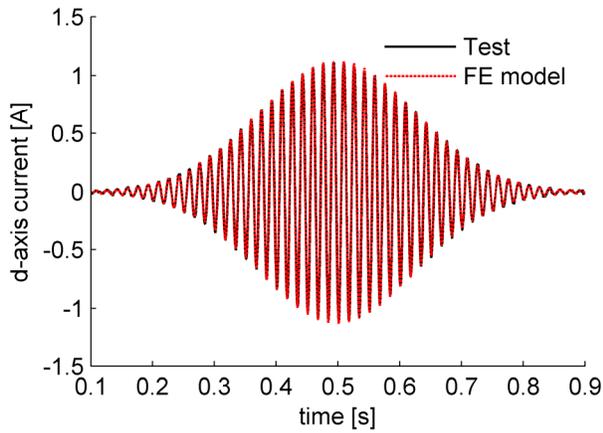
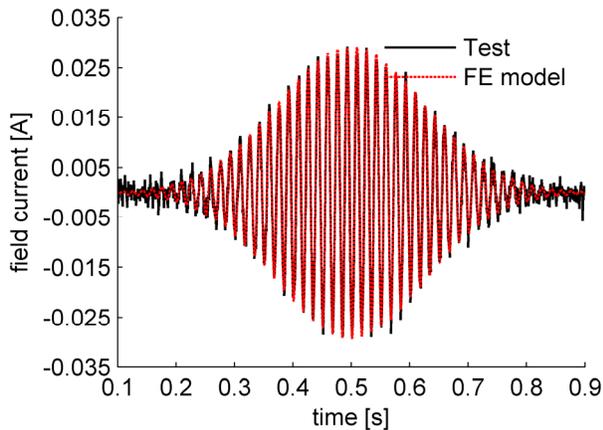

 Fig. 4. GMSP voltage applied to the  $d$ -axis.

 Fig. 7. FE model prediction and experimental results for the  $q$ -axis standstill test.

 Fig. 5. FE model prediction and standstill test results for the  $d$ -axis current.


Fig. 6. FE model prediction and standstill test results for the field current.

#### IV. PARAMETER ESTIMATION

The lumped model of the synchronous generator is represented with equivalent electric circuits and they are shown in Fig. 8. These circuit parameters need to be estimated and validated for its proper application in simulation studies. The machine voltage equations based on the two-axis theory are given by (10)-(14).

$$v_d = d\psi_d/dt + \omega\psi_q + r_a i_d, \quad (10)$$

$$v_f = d\psi_f/dt + r_f i_f, \quad (11)$$

$$0 = d\psi_{kd}/dt + r_{kd} i_{kd}, \quad (12)$$

$$v_q = d\psi_q/dt - \omega\psi_d + r_a i_q, \quad (13)$$

$$0 = d\psi_{kq}/dt + r_{kq} i_{kq}, \quad (14)$$

where  $v_d$ ,  $v_f$ ,  $v_q$ ,  $i_d$ ,  $i_f$ , and  $i_q$  are the  $d$ -axis, field, and  $q$ -axis voltages and currents respectively.  $\psi$  denotes flux linkages.

The set of state-equations can be derived using the equivalent circuits of the machine at standstill. A model structure with one damper winding and a differential leakage reactance ( $X_{kf}$ ) was selected for the  $d$ -axis. The currents were chosen as state variables and the resulting equations for the  $d$ -axis model are given by (15).

$$\dot{\mathbf{I}}_d = \omega_0 \mathbf{X}_d^{-1} \mathbf{V}_d - \omega_0 \mathbf{X}_d^{-1} \mathbf{R}_d \mathbf{I}_d, \quad (15)$$

where  $\mathbf{I}_d$ ,  $\mathbf{R}_d$ ,  $\mathbf{V}_d$ , and  $\mathbf{X}_d$  are the  $d$ -axis current vector, resistance matrix,  $d$ -axis voltage vector, and  $d$ -axis reactance matrix, respectively.  $\omega_0$  is the rated angular speed.

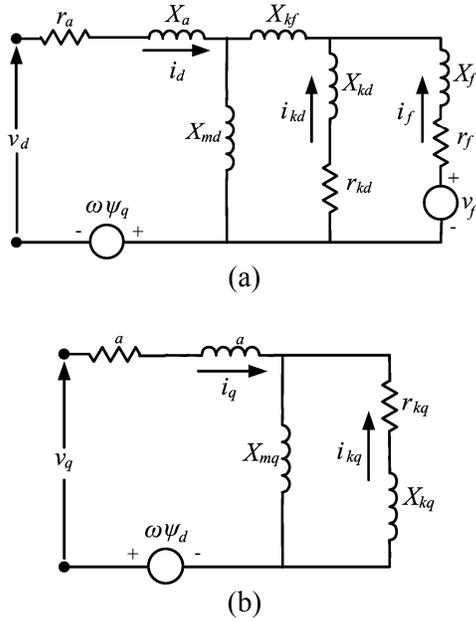


Fig. 8. Equivalent circuits. a)  $d$ -axis, b)  $q$ -axis.

In the  $q$ -axis model, one damper winding was considered and its state-space equations are given by (16).

$$\dot{\mathbf{I}}_q = \omega_0 \mathbf{X}_q^{-1} \mathbf{V}_q - \omega_0 \mathbf{X}_q^{-1} \mathbf{R}_q \mathbf{I}_q, \quad (16)$$

where  $\mathbf{I}_q$ ,  $\mathbf{R}_q$ ,  $\mathbf{V}_q$ , and  $\mathbf{X}_q$  stand for  $q$ -axis current vector, resistance matrix, voltage vector, and  $q$ -axis reactance matrix, respectively.

The equivalent-circuit parameter estimation was made using the least squares approach. The objective function can be expressed by (17) and it is minimized by a GA.

$$\min J(\theta) = \frac{1}{2} \sum_{i=1}^N (y_i - \hat{y}_i)^2, \quad (17)$$

where  $J(\theta)$  is the objective function,  $y_i$  is the set of experimental data, and  $\hat{y}_i$  is the set of estimated responses from the proposed model.  $N$  is the number of the experimental points, and  $\theta$  is the parameter vector to be determined.

The GA optimization is based on evolution of a population of individuals throughout generations. These individuals are randomly selected, recombined and mutated. A population of 5000 individuals represented with a 32-b floating point representation was used here. Each individual is composed by genes that represent the different variables of the search space. In this case, they correspond to the resistances and reactances of the equivalent circuits. The performance of each

individual is evaluated by employing the cost function defined by (17). In the evaluation of the objective function, the fitness proportional selection, along with an elitist succession, were applied to produce offsprings. Crossover and mutation values of 0.9 and 0.001 were used, respectively [13]. The final model parameters were determined using the quasi-Newton algorithm [14] and the estimated set of parameters is shown in Table 1. The reactance  $X_{kf}$  has a negative value which means that the magnetic flux linkage between the field and  $d$ -axis damper windings is smaller than the magnetic flux linkage between stator and  $d$ -axis damper windings.

Table 1: Set of estimated parameters.

Parameter	Value (pu)
direct axis magnetizing reactance ( $X_{md}$ )	1.43299600
armature leakage reactance ( $X_a$ )	0.12460000
field leakage reactance ( $X_f$ )	0.17413213
direct-axis damper leakage reactance ( $X_{kd}$ )	0.02386047
direct-axis damper-field leakage reactance ( $X_{kf}$ )	-0.10320315
direct-axis damper resistance ( $r_{kd}$ )	0.30042507
quadrature axis magnetizing reactance ( $X_{mq}$ )	0.98989783
quadrature-axis damper leakage reactance ( $X_{kq}$ )	0.20369278
quadrature-axis damper resistance ( $r_{kq}$ )	1.59745263
armature resistance ( $r_a$ )	0.08568540
field resistance ( $r_f$ )	0.01113500

The fitting of (15) for the  $d$ -axis test is shown in Figs. 9-10. The comparison of the  $d$ -axis current between the FE model and the  $d$ -axis equivalent circuit is illustrated in Fig. 9. The resulting fitting for the field current is shown in Fig. 10. Good estimation results were also obtained for the  $q$ -axis GMSP test. A previous research was carried out by the authors to estimate the synchronous machine parameters with the step voltage and the sine cardinal excitations [15-16]. Although these excitation signals are different, the results were similar to the ones obtained with the proposed GMSP test approach. However, the proposed methodology based on the usage of a FE model,

represents an advantage because it does not depend on the availability of high current power amplifiers. In addition, this approach can be applied at the design state of a large generator that allows having the model parameters of the machine before it is built.

## V. LUMPED MODEL VALIDATION

The parameters were estimated using data from the GMSP-FE model simulations of the generator at standstill. However, these model parameters need to be validated. This was achieved by applying a sudden three-phase short-circuit fault to the synchronous generator. The fault was carried out at 70% of the rated terminal voltage. The generator was properly instrumented with current and voltage Hall effect sensors. The

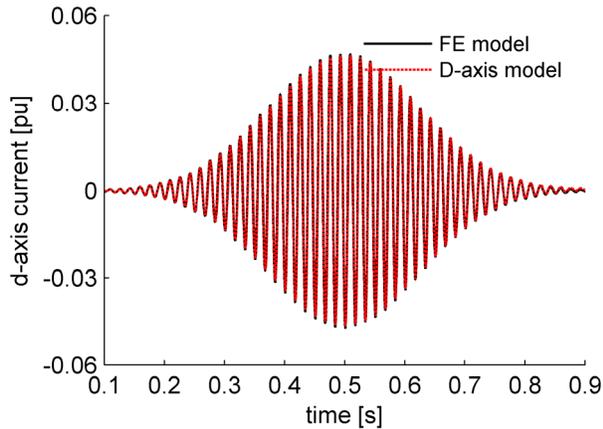


Fig. 9. FE and estimated lumped model results for the d-axis current at standstill.

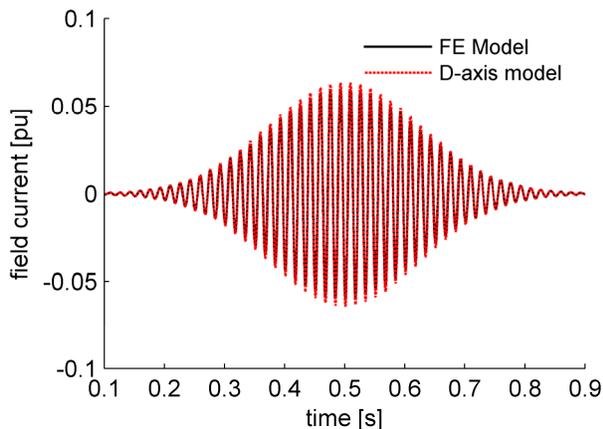


Fig. 10. FE and estimated lumped model results for the field current at standstill.

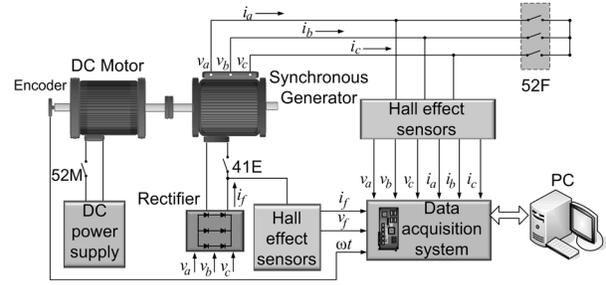


Fig. 11. Setup for the sudden short-circuit fault.

which has its own breaker (51M). An acquisition rotor position was measured with an encoder and the electrical operation was performed with the field winding breaker 41E, and a fault breaker 52F. The generator was driven with a dc motor system was used to collect all data as it is shown in Fig. 11. The 52F breaker was triggered at 0.3s causing the sudden short-circuit fault and the stator currents ( $i_{abc}$ ), phase voltages ( $v_{abc}$ ), rotor position, field voltage and field current were recorded.

The two-axis non-linear model of the synchronous generator can be represented by (18), and it can be derived from (10)-(14) [1].

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{F}(\mathbf{x}) + \mathbf{B}\mathbf{z}, \quad (18)$$

where  $\mathbf{x}$  denotes the flux linkage state-variables,  $\mathbf{F}$  is a function of nonlinear terms,  $\mathbf{A}$  is the system matrix,  $\mathbf{B}$  is the input vector and  $\mathbf{z}$  is the input variable vector.

The comparison of the lumped model prediction against test results for the phase current in the short-circuit fault is shown in Fig. 12. It can be seen that the phase current is predicted with an accuracy of 6.5% at the first peak current swing. The field winding current is also predicted with an accuracy of 1.8% in the first current oscillation as it can be seen in Fig. 13. This demonstrates the validity of the estimated parameters. On the other hand, it was found that the peak stator current was predicted with the same accuracy as in [16] and the peak field current was reproduced more accurately than in [15]. However, in the second and third swings of the field current the three methods need improvement.

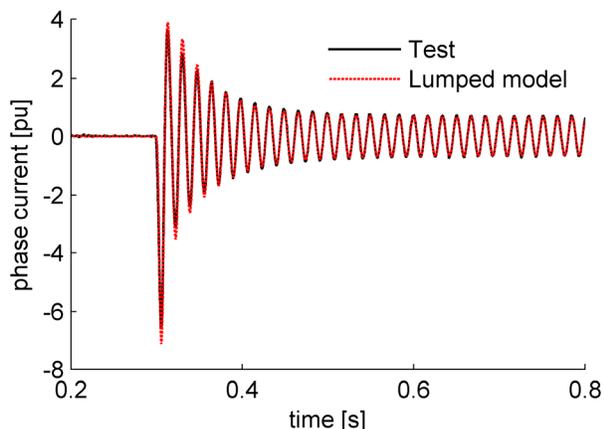


Fig. 12. Phase current in the sudden three-phase short-circuit test using the estimated set of parameters from the GMSP-FE model.

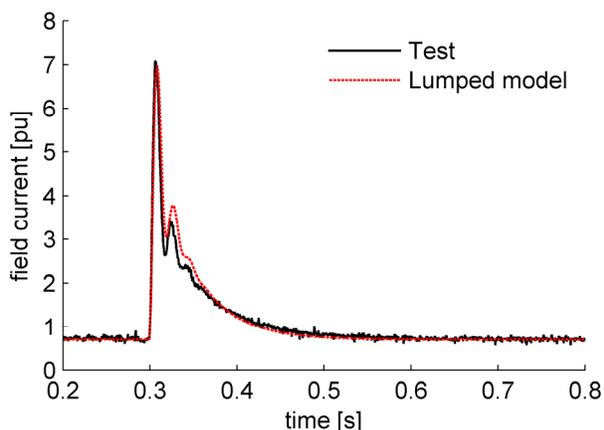


Fig. 13. Field current in the sudden three-phase short-circuit test using the estimated set of parameters from the GMSP-FE model.

## VI. CONCLUSION

This paper has presented the parameter estimation of a synchronous generator model based on the use of a new excitation signal. The Gaussian Modulated Sinusoidal Pulse voltage was used to excite the generator at standstill. A FE model was developed and validated against standstill experiments and it was used to simulate the GMSP test. Afterwards, the FE simulation results were used to estimate the parameters of the  $d$ - $q$  equivalent circuits of the synchronous generator. The Genetic and quasi-Newton algorithms were used to obtain the equivalent circuit parameters. Finally, the estimated parameters were employed in a non-linear lumped model to simulate the sudden three-phase short-circuit fault. The simulation results of the lumped

model were compared against test data and a good agreement was achieved which demonstrates the validity of the proposed approach.

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