Fast Analysis of Electromagnetic Scattering from a Coated Conductor with the Parabolic Equation

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Abstract **─** In recent years, the electromagnetic scattering from the coated conductors has been paid more and more attention by many scholars. The parabolic equation (PE) method is firstly utilized to analyze electrically large conductors coated with lossy medium in this paper. The impedance boundary condition is implemented to analyze the lossy medium and the implicit finite difference method of Crank–Nicolson scheme is implemented to solve the parabolic equation. As a result, the computations can be taken in each two-dimensional transverse plane. By this means, both the CPU time and memory requirement are reduced greatly. Numerical results are given to demonstrate the accuracy and efficiency of the proposed method.

Index Terms **─** electromagnetic scattering, finite difference scheme, impedance boundary condition, parabolic equation method,

I. INTRODUCTION

The electromagnetic (EM) scattering analysis of the conductor has become a research hot due to its wide application in military area. There are a lot of rigorous numerical methods to solve this problem, such as the finite difference time domain (FDTD) [1-3], method of moment (MoM) [4-6], time domain integral equation (TDIE) method [7-8] and so on. In order to reduce the computational requirement, many acceleration techniques were implemented. Firstly, the fast Fourier transform (FFT) has been used to obtain the scattering characteristics [9]. Then the adaptive cross approximate (ACA) algorithm was proposed in [10] to accelerate the surface integral equation-based MoM. Besides, some novel techniques have been applied to reduce the computational complexity, such as adaptive integral method (AIM) [11], equivalent dipole moment (EDM) method [12], thin dielectric sheet (TDS) approximation [13-14]. However, for the rigorous numerical methods, solving electrically large problems takes a great deal of computational resources. As a result, it is necessary to develop the approximation methods to efficiently compute the EM scattering properties of coated conductors.

Parabolic equation (PE) method bridges the rigorous and high frequency methods. It can provide encouraging accuracy along the paraxial direction with limited computational resources. It should be noted that the energy is supposed to propagate in a cone. The PE method was firstly used to analyze the underwater acoustics [15]. Then it was widely applied to calculate the long-range propagation problems of radio wave [16-18]. Many early works have been done by using the split-step Fourier-based PE (SSPE) method. When compared with other PE solvers, the computational efficiency of SSPE is extremely high. However, for the complicated targets, it is not flexible to model the boundary. Therefore, the finite difference (FD) schemes are good choices to deal with targets with complicated boundaries. In recent years, the PE is also introduced to the EM scattering from electrically large conducting targets [19–25]. The Crank-Nicolson-based implicit FD method can be used as an efficient solver for the PE. It should be noted that the rectangular meshes with the mesh size of 1/10th of a wavelength are applied. Nevertheless, when analyzing electrically large targets, there is a supersized sparse matrix equation should be solved in each transverse plane. In recent years, some novel FD schemes are proposed to accelerate the calculation, such as alternating direction implicit (ADI) [17, 22-23, 26], alternating group explicit (AGE) [24, 27-28] schemes. In this way, both the efficiency and the accuracy can be guaranteed. However, there is no report of scattering characteristics for coated targets by using the PE method.

In this paper, the impedance boundary condition is integrated into parabolic equation to fast analyze the electromagnetic scattering from coated conductors. Firstly, the parabolic equation is constructed and the waves in each transverse plane are absorbed with the help of the perfect matching layer (PML). Then Leontovich

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boundary condition is applied on the boundary of perfectly electrical conductor (PEC). It can be found that the error becomes larger with the thickness of the coating material increasing. In other words, the proposed PE method can be used to analyze thin medium-coated conductors. By adopting the Crank-Nicolson scheme to the paraxial direction, PE is implemented in a marching manner. In addition, the bistatic RCS results can be fully obtained by rolling the paraxial direction of PE. Some complex structures, such as missile and plane, are modeled to validate the proposed method.

II. THEORY

A. The standard PE method

Assume x axis as the paraxial direction of PE. Then the standard PE in free space can be written as:

$$\frac{\partial u}{\partial x} = \frac{i}{2k}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right), \tag{1}$$

where u is the reduced scattered field and can be written as:

$$u(x,y,z) = e^{-ikx}\psi(x,y,z), \tag{2}$$

where ψ represents the scattered field component.

The Crank-Nicolson scheme is introduced to equation (1), and the discretized formula for (1) can be obtained:

$$\left(\frac{i\Delta x}{k\Delta y^2} + \frac{i\Delta x}{k\Delta z^2} + 1\right)u_{p,q}^{m+1} - \frac{i\Delta x}{2k\Delta y^2}u_{p-1,q}^{m+1} - \frac{i\Delta x}{2k\Delta z^2}u_{p,q-1}^{m+1} - \frac{i\Delta x}{2k\Delta y^2}u_{p+1,q}^{m+1} - \frac{i\Delta x}{2k\Delta z^2}u_{p,q+1}^{m+1} = u_{p,q}^{m}, \tag{3}$$

in which, Δx denotes the range step along the paraxial direction, $\Delta y, \Delta z$ are the mesh sizes along the y, z directions, $u_{p,q}^{m}$ is the reduced scattered fields for $(m\Delta x, p\Delta y, q\Delta z)$.

B. Leontovich impedance boundary condition (IBC)

The electromagnetic field components of u_x, u_y, u_z are coupled by adding the proper boundary conditions on PEC surface. The Leontovich impedance boundary condition is given in terms of surface impedance and can be expressed in the following form

$$\hat{n}\times\mathbf{E}(P) = Z\left[\hat{n}\times\left(\hat{n}\times\mathbf{H}(P)\right)\right], \tag{4}$$

in which $\hat{n}$ is the unit normal vector on the surface of the scattering target, Z is the impedance of the object at point P and it is defined as:

$$Z = -iZ_0\sqrt{\frac{\mu_r}{\varepsilon_r}}\tan(Nkd), \tag{5}$$

where Z_0 denotes the wave impedance in free space, d is the thickness of the coated medium, $N = \sqrt{\mu_r \varepsilon_r}$, μ_r, ε_r represent the permeability and permittivity of the coated medium.

By eliminating the magnetic field, the equation (4) can be rewritten as:

$$\hat{n}\times\mathbf{E}(P) = \frac{Z}{ikZ_0}\left[\hat{n}\cdot\left(\nabla\times\mathbf{E}(P)\right)\hat{n} - \nabla\times\mathbf{E}(P)\right]. \tag{6}$$

The equation (4) can be expressed in terms of the electric fields. Then the magnetic fields can be calculated by taking advantage of the curl equation:

$$n_y u_z - n_z u_y - \frac{Z}{ikZ_0}\begin{bmatrix} n_x n_z\left(iku_y - \frac{1}{2ik}\left(\frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) - \frac{\partial u_x}{\partial y}\right) + \\ (n_x^2 - 1)\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) + \\ n_x n_y\left(\frac{\partial u_x}{\partial z} + \frac{1}{2ik}\left(\frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) - iku_z\right) \end{bmatrix},$$

$$= (-n_y E_z^i + n_z E_y^i)e^{-ikx} + \frac{Ze^{-ikx}}{ikZ_0}\begin{bmatrix} (n_x^2 - 1)\left(\frac{\partial E_z^i}{\partial y} - \frac{\partial E_y^i}{\partial z}\right) + \\ n_x n_y\left(\frac{\partial E_x^i}{\partial z} - \frac{\partial E_z^i}{\partial x}\right) + \\ n_x n_z\left(\frac{\partial E_y^i}{\partial x} - \frac{\partial E_x^i}{\partial y}\right) \end{bmatrix} \tag{7}$$

$$n_z u_x - n_x u_z - \frac{Z}{ikZ_0}\begin{bmatrix} n_x n_y\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) + \\ (n_y^2 - 1)\left(\frac{\partial u_x}{\partial z} + \frac{1}{2ik}\left(\frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) - iku_z\right) + \\ n_y n_z\left(iku_y - \frac{1}{2ik}\left(\frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) - \frac{\partial u_x}{\partial y}\right) \end{bmatrix},$$

$$= (-n_z E_x^i + n_x E_z^i)e^{-ikx} + \frac{Ze^{-ikx}}{ikZ_0}\begin{bmatrix} n_x n_y\left(\frac{\partial E_z^i}{\partial y} - \frac{\partial E_y^i}{\partial z}\right) + \\ (n_y^2 - 1)\left(\frac{\partial E_x^i}{\partial z} - \frac{\partial E_z^i}{\partial x}\right) + \\ n_y n_z\left(\frac{\partial E_y^i}{\partial x} - \frac{\partial E_x^i}{\partial y}\right) \end{bmatrix} \tag{8}$$

$$n_x u_y - n_y u_x - \frac{Z}{ikZ_0}\begin{bmatrix} n_x n_z\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) + \\ n_y n_z\left(\frac{\partial u_x}{\partial z} + \frac{1}{2ik}\left(\frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) - iku_z\right) + \\ (n_z^2 - 1)\left(iku_y - \frac{1}{2ik}\left(\frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) - \frac{\partial u_x}{\partial y}\right) \end{bmatrix},$$

$$= (-n_x E_y^i + n_y E_x^i)e^{-ikx} + \frac{Ze^{-ikx}}{ikZ_0}\begin{bmatrix} n_x n_z\left(\frac{\partial E_z^i}{\partial y} - \frac{\partial E_y^i}{\partial z}\right) + \\ n_y n_z\left(\frac{\partial E_x^i}{\partial z} - \frac{\partial E_z^i}{\partial x}\right) + \\ (n_z^2 - 1)\left(\frac{\partial E_y^i}{\partial x} - \frac{\partial E_x^i}{\partial y}\right) \end{bmatrix} \tag{9}$$

where (E_x^i, E_y^i, E_z^i) is the incident plane wave.

At last, the discretized form can be obtained by using the FD scheme, which can be derived as:

$$
\begin{aligned}
&\left(\frac{n_x n_z}{ik\Delta y}-\frac{n_x n_y}{ik\Delta z}\right)u^m_{x,p,q}-\frac{n_x n_z}{ik\Delta y}u^m_{x,p+1,q}+\frac{n_x n_y}{k^2\Delta z^2}u^m_{z,p,q+1}+\\
&\frac{n_x n_y}{ik\Delta z}u^m_{x,p,q+1}-\frac{n_x n_z}{k^2\Delta y^2}u^m_{y,p+1,q}+\left(\frac{n_x n_y}{k^2\Delta y^2}+\frac{n_x^2-1}{ik\Delta z}\right)u^m_{z,p+1,q}\\
&+\left(\begin{array}{l}\dfrac{n_x n_z}{2k^2\Delta y^2}+\dfrac{n_x n_z}{2k^2\Delta z^2}+\\ \dfrac{n_x^2-1}{ik\Delta z}+n_x n_z+\dfrac{Z_0}{Z}n_z\end{array}\right)u^m_{y,p,q}-\left(\frac{n_x n_z}{k^2\Delta z^2}+\frac{n_x^2-1}{ik\Delta z}\right)u^m_{y,p,q+1}\\
&-\left(\begin{array}{l}\dfrac{n_x n_y}{2k^2\Delta y^2}+\dfrac{n_x n_y}{2k^2\Delta z^2}\\ +\dfrac{n_x^2-1}{ik\Delta y}+n_x n_y+n_y\dfrac{Z_0}{Z}\end{array}\right)u^m_{z,p,q}-\frac{n_x n_y}{2k^2\Delta y^2}u^m_{z,p+2,q}\\
&-\frac{n_x n_y}{2k^2\Delta z^2}u^m_{z,p,q+2}+\frac{n_x n_z}{2k^2\Delta y^2}u^m_{y,p+2,q}+\frac{n_x n_z}{2k^2\Delta z^2}u^m_{y,p,q+2}\\
&=n_y\frac{Z_0}{Z}+n_x n_y
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&\left(\frac{n_y n_z}{ik\Delta y}-\frac{n_y^2-1}{ik\Delta z}-n_z\frac{Z_0}{Z}\right)u^m_{x,p,q}-\frac{n_y n_z}{ik\Delta y}u^m_{x,p+1,q}\\
&+\frac{n_y^2-1}{ik\Delta z}u^m_{x,p,q+1}+\left(\frac{n_y n_z}{2k^2\Delta y^2}+\frac{n_y n_z}{2k^2\Delta z^2}+\frac{n_x n_y}{ik\Delta z}+n_y n_z\right)u^m_{y,p,q}\\
&-\frac{n_y n_z}{k^2\Delta y^2}u^m_{y,p+1,q}-\left(\frac{n_y n_z}{k^2\Delta z^2}+\frac{n_x n_y}{ik\Delta z}\right)u^m_{y,p,q+1}+\frac{n_y n_z}{2k^2\Delta y^2}u^m_{y,p+2,q}\\
&-\left(\frac{n_y^2-1}{2k^2\Delta y^2}+\frac{n_y^2-1}{2k^2\Delta z^2}+\frac{n_x n_y}{ik\Delta y}-n_x\frac{Z_0}{Z}+n_y^2-1\right)u^m_{z,p,q}\\
&+\frac{n_y n_z}{2k^2\Delta z^2}u^m_{y,p,q+2}+\left(\frac{n_y^2-1}{k^2\Delta y^2}+\frac{n_x n_y}{ik\Delta y}\right)u^m_{z,p+1,q}\\
&-\frac{n_y^2-1}{2k^2\Delta y^2}u^m_{z,p+2,q}-\frac{n_y^2-1}{2k^2\Delta z^2}u^m_{z,p,q+2}+\frac{n_y^2-1}{k^2\Delta z^2}u^m_{z,p,q+1}\\
&=n_y^2-1-n_x\frac{Z_0}{Z}
\end{aligned}
,\tag{11}
$$

$$
\begin{aligned}
&\left(\frac{n_z^2-1}{ik\Delta y}-\frac{n_y n_z}{ik\Delta z}+n_y\frac{Z_0}{Z}\right)u^m_{x,p,q}-\frac{n_z^2-1}{ik\Delta y}u^m_{x,p+1,q}+\frac{n_y n_z}{ik\Delta z}u^m_{x,p,q+1}\\
&+\left(\frac{n_z^2-1}{2k^2\Delta y^2}+\frac{n_z^2-1}{2k^2\Delta z^2}+\frac{n_x n_z}{ik\Delta z}+n_z^2-1-n_x\frac{Z_0}{Z}\right)u^m_{y,p,q}\\
&-\frac{n_z^2-1}{k^2\Delta y^2}u^m_{y,p+1,q}-\left(\frac{n_z^2-1}{k^2\Delta z^2}+\frac{n_x n_z}{ik\Delta z}\right)u^m_{y,p,q+1}+\frac{n_z^2-1}{2k^2\Delta y^2}u^m_{y,p+2,q}\\
&-\left(\frac{n_y n_z}{2k^2\Delta y^2}+\frac{n_y n_z}{2k^2\Delta z^2}+\frac{n_x n_z}{ik\Delta y}+n_y n_z\right)u^m_{z,p,q}+\frac{n_y n_z}{k^2\Delta z^2}u^m_{z,p,q+1}\\
&+\left(\frac{n_y n_z}{k^2\Delta y^2}+\frac{n_x n_z}{ik\Delta y}\right)u^m_{z,p+1,q}+\frac{n_z^2-1}{2k^2\Delta z^2}u^m_{y,p,q+2}\\
&-\frac{n_y n_z}{2k^2\Delta y^2}u^m_{z,p+2,q}-\frac{n_y n_z}{2k^2\Delta z^2}u^m_{z,p,q+2}=n_y n_z
\end{aligned}
.\tag{12}
$$

It can be seen from equation (3) that the computation is taken one by one in each transverse plane. Moreover, the inhomogeneous boundary conditions of equations (10-12) are introduced on the surface of PEC.

III. NUMERICAL EXAMPLES

In this part, all the numerical results are tested on the computer of Intel Xeon E7-4850 CPU with 8GB RAM. The plane wave with the incident angle of $\theta_{inc}=90^\circ$ $\phi_{inc}=0^\circ$ is used as the incidence source, and the whole simulation system are working at 300 MHz. It should be noted that the IBC-based combined field integral equation (IBC-CFIE) is set to be the rigorous solution for comparison.

At first, we consider a sphere with the radius of 5λ and the relative permittivity of $\varepsilon_r = 3.84 - j1.6$. In this numerical example, the mesh size is 0.05 *m*. Therefore, the calculation is split into 200 transverse planes to the paraxial direction. The full bistatic RCS result for the horizontal plane pattern of the proposed method is given and compared with that of Mie Series in Fig. 1. For the proposed method, the full bistatic RCS result is achieved by rotating the paraxial direction of PE. As shown in Fig. 1, it coincides well for these two methods.

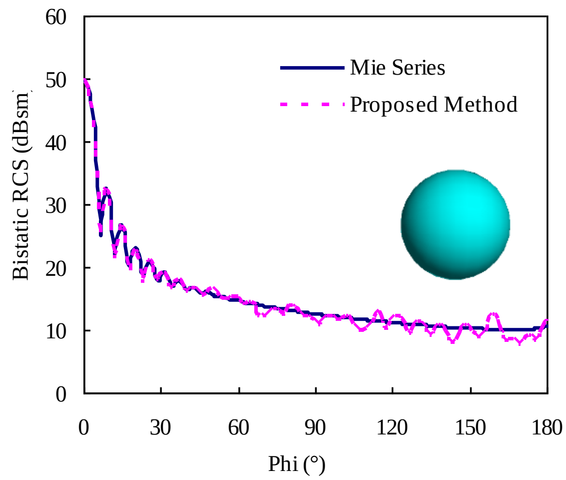

Fig. 1. Comparison of the RCS between the Mie Series and the proposed method for a sphere of the horizontal plane pattern.

Secondly, the EM scattering from a coated cone is considered with the coating thickness of 0.1*m* and the permittivity of $3.5 - j1.5$. Its radius and height are 1*m* and 3*m*, respectively. The mesh size is set to be 0.05 *m* and the calculation is split into 60 transverse planes to the paraxial direction. The bistatic RCS of both the PEC and coated cones are compared in Fig. 2. It can be found that the bistatic RCS is decreased greatly at some angles for the coated cone when compared with the PEC cone with no coating. Therefore, the proposed method can be used as an efficient tool to design RCS reduction.

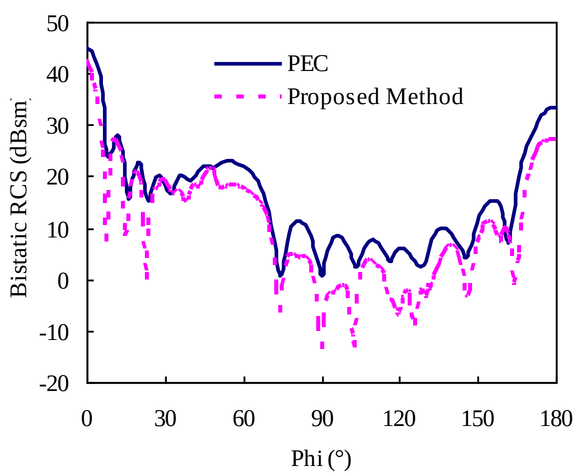

Fig. 2. Comparisons of the bistatic RCS between a coated cone and a PEC cone.

Thirdly, the EM scattering from a PEC missile coated with lossy medium is considered. Its coating thickness is 0.05*m* and the permittivity of the coating medium is $3-j$. As shown in Fig. 3, the size of this PEC missile model is given. The full bistatic RCS results of both the IBC-CFIE and the proposed method are shown and compared in Fig. 4. In this numerical example, there are seven rotating PE runs are applied. More specifically, the generalized minimal residual (GMRES) method is used as the solver of IBC-CFIE and the convergence precision is set to be $1e-3$. Moreover, the computational resources of these methods are also listed in Table 1. It can be concluded that both the memory requirement and the total CPU time of the proposed method can be reduced greatly when compared with the IBC-CFIE. Therefore, the proposed method can be used to fast analyze the electromagnetic scattering from electrically large coated targets with encouraging accuracy.

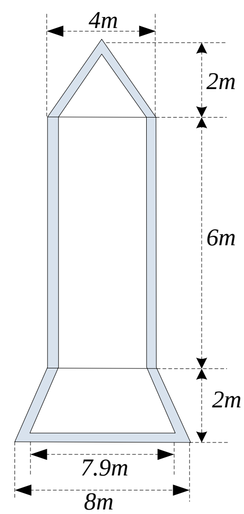

Fig. 3. Geometry structure of a PEC missile model.

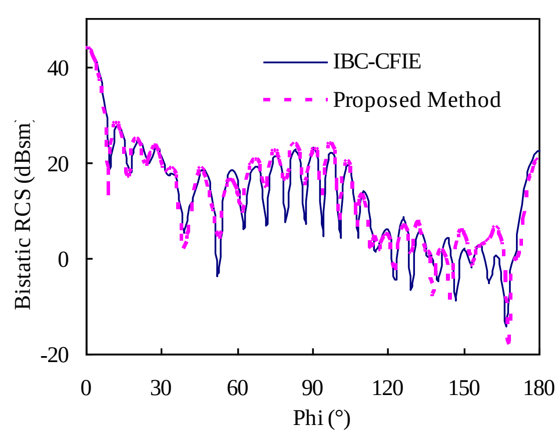

Fig. 4. Bistatic RCS of a coated missile model at the frequency of 300MHz.

Table 1: Comparisons of memory requirement and CPU time between the IBC-CFIE and the proposed method for the coated PEC missile at the frequency of 300 MHz

Method	Memory Requirement (MB)	CPU Time (s)
IBC-CFIE	904	5060
Proposed method	155	15

Finally, a coated PEC aircraft is modeled and its maximum size along x, y, and z directions are 12m, 9m, and 4.8m, respectively. Its coating thickness is 0.01m and the permittivity of the coating medium is $1.5-j1.3$. Seven rotating PE runs are applied in this numerical example. The bistatic RCS is given in Fig. 5 and the computational resources of the coated PEC aircraft is also shown in Table 2.

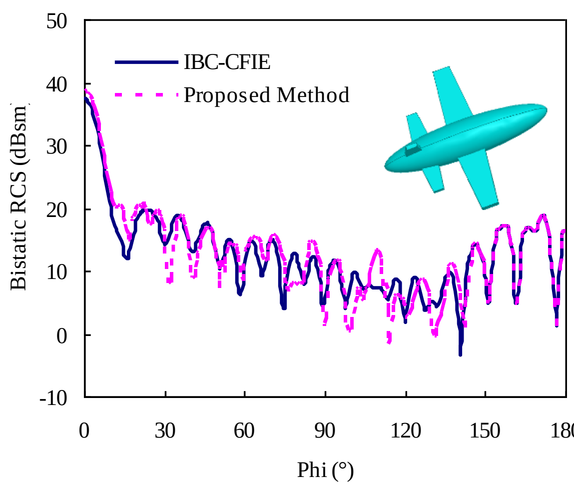

Fig. 5. Bistatic RCS of a coated aircraft model at the frequency of 300MHz.

Table 2: Comparisons of memory requirement and CPU time between the IBC-CFIE and the proposed method for the coated PEC aircraft at the frequency of 300 MHz

Method	Memory Requirement (MB)	CPU Time (s)
IBC-CFIE	823	1814
Proposed method	183	16

IV. CONCLUSION

An efficient solver based on PE is proposed to analyze the EM scattering from electrically large PEC which is coated with thin lossy medium. The Crank-Nicolson scheme is implemented for its accuracy and unconditional stability. In this way, the computations are taken plane by plane. More general boundary conditions are applied to the scattering targets. The first numerical example of a dielectric sphere is given to validate the accuracy of the proposed method. Furthermore, more complicated structures, such as coated missile and plane, are modeled to show the efficiency.

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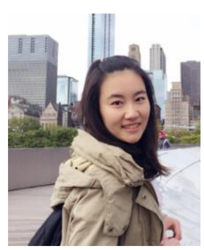

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