Transform Method for Dielectric Periodic Interface Scattering

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Abstract **─** An extended transform method is developed for calculating the 2-D scattering problem from dielectric periodic interfaces. The method transforms the problem into scattering from two imaginary planes, one of which cuts across the maximum points and another across the minimum points of the periodic interface. The fields just above and below the periodic interface are expanded into Taylor series with respect to the two planes respectively. Then by satisfying boundary condition, the unknown coefficients can be determined. Comparing with T-Matrix and MoM, proposed method is simpler in formulation and less in computational time. Near scattered field distributions above and in the trough region of the periodic interface are calculated by proposed method. The results are in good agreements with those of T-Matrix and MoM respectively.

Index Terms **─** electromagnetic scattering, periodic surface, transform method.

I. INTRODUCTION

Periodic structures frequently appear in the applications such as antenna design, microwave systems, meta-materials etc [1-4] and corresponding problem of electromagnetic waves scattering from periodic surfaces [5-10] is of fundamental importance in science and engineering. Scattering from periodic rough interfaces has been investigated by integral methods such as MoM [6] and T-Matrix [6-8], Because of the slowly convergent integral kernel and the treatment of singularities [6, 11], MoM needs much computational time. While only considering the field points outside the trough region of the corrugation [8] to remove the absolute operation in the periodic Green's function in the spectral domain [6-8] and to speed up the convergence [6], T-Matrix method can not be used to evaluate the fields in the trough region of the corrugation. Those fields may be interested in certain cases [9, 10]. Therefore, more convenient and efficient methods for this kind of problem are always attractive in many applications. Transform method [11] is such a method, but its formulation is only available for perfectly conducting periodic surface scattering.

In this paper, transform method is extended to scattering from dielectric periodic interface. The fields just above and below the periodic interface are expanded into Taylor series with respect to two imaginary planes closest to the interface. By applying the boundary condition that both tangential electric and magnetic fields are continuous on the periodic interface to this form of the scattered and transmitted field, we are able to establish an infinite set of linear equations for the amplitudes of the scattered and the transmitted waves. In section II, the scattering problem is formulated. In Section III, several representative examples are given and compared with T-Matrix. Conclusions are given in Section IV.

II. FORMULATION

The geometry of considered 2-D problem and parameters used in the formulation are shown in Fig. 1. The periodic interface $y=f(x)=f(x+P)$, $x\in[0,P)$, P being period, is illuminated by a z-polarized plane wave with incident angle ϕ_0. The geometrical parameters and the fields do not depend on the z-coordinate. Therefore, the TE and TM problems can be considered independently. The only non-zero component of the total electric fields E_z for TE or that of the total magnetic fields

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H_z for TM mode are denoted by ψ with different script for different regions respectively.

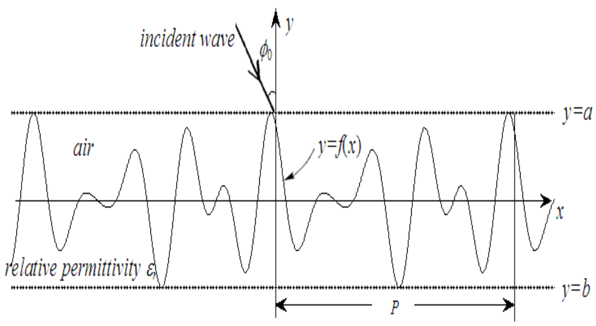


Fig. 1. The geometry of the problem.

Consider two imaginary planes $y=a$ and $y=b$ as shown in Fig. 1, where $a=\max[f(x)]$, $x\in[0,P)$ and $b=\min[f(x)]$, $x\in[0,P)$. These planes are the planes nearest to the rough interface $y=f(x)$ which is sandwiched by them. First, ψ^+, the total field just above the periodic interface and ψ^-, the total field just below the interface, are expanded into Taylor series around planes $y=a$ and $y=b$ respectively [11]

$$\psi^+(x,f(x))=\sum_{m=0}^{\infty}\frac{1}{m!}\frac{\partial^m\psi}{\partial y^m}|_{y=a}\,(f(x)-a)^m, \quad (1)$$

$$\psi^-(x,f(x))=\sum_{m=0}^{\infty}\frac{1}{m!}\frac{\partial^m\psi_1}{\partial y^m}|_{y=b}\,(f(x)-b)^m, \quad (2)$$

In the mean time, ψ is considered as the summation of incident and scattered field, namely

$$\psi=\psi^i+\psi^s, \quad (3)$$

$$\psi^i=e^{ik(x\sin\phi_0-y\cos\phi_0)}. \quad (4)$$

As widely accepted [6-9] that the scattered field on and above the top point imaginary plane and the transmitted field on and below the bottom point imaginary plane can be represented by a summation of a discrete set of planar propagating and evanescent harmonics as

$$\psi^s=\sum_{n=-\infty}^{\infty}R_n e^{i\beta_n x}e^{iq_n y},\ y\ge a \quad (5)$$

and transmitted field

$$\psi_1=\sum_{n=-\infty}^{\infty}T_n e^{i\beta_n x}e^{-iq_{1n}y},\ y\le b \quad (6)$$

where

$$\beta_n=k\sin\phi_0+n\frac{2\pi}{P}.$$

and

$$q_n=\begin{cases}(k^2-\beta_n^2)^{1/2},\ \beta_n^2\le k^2\\ i(\beta_n^2-k^2)^{1/2},\ \beta_n^2>k^2\end{cases}.$$

$$q_{1n}=\begin{cases}(k^2\varepsilon_r-\beta_n^2)^{1/2},\ \beta_n^2\le k^2\varepsilon_r\\ i(\beta_n^2-k^2\varepsilon_r)^{1/2},\ \beta_n^2>k^2\varepsilon_r\end{cases}.$$

when $\beta_n^2\le k^2$ then the harmonic is propagating, otherwise it is evanescent. The assumed time factor $\exp\{-i\omega t\}$ is omitted here and here after. Substituting (4), (5) and (3) into (1) and substituting (6) into (2), it gets that

$$\psi^+(x,f(x))=\sum_{m=0}^{\infty}\frac{1}{m!}e^{ikx\sin\phi_0}[e^{-ika\cos\phi_0}(-ik\cos\phi_0)^m$$

$$+\sum_{n=-\infty}^{\infty}R_n(iq_n)^m e^{i(2\pi n/P)x}e^{iq_na}](f(x)-a)^m. \quad (7)$$

$$\psi^-(x,f(x))=\sum_{m=0}^{\infty}\frac{1}{m!}e^{ikx\sin\phi_0}\sum_{n=-\infty}^{\infty}T_n(-iq_{1n})^m$$

$$e^{-iq_{1n}b}e^{i(2\pi n/P)x}(f(x)-b)^m. \quad (8)$$

The boundary conditions state that the tangential components of electric and magnetic field are continuous across the boundary

$$\psi^+(x,f(x))=\psi^-(x,f(x)). \quad (9)$$

$$\hat{n}\times(\nabla\times\hat{z}\psi^+(x,f(x)))=$$

$$c_1\hat{n}\times(\nabla\times\hat{z}\psi^-(x,f(x))). \quad (10)$$

where $c_1=\mu_0/\mu_1=1$ for TE polarization and $c_1=\varepsilon_0/\varepsilon_1$ for TM polarization [8]. Substituting (7) and (8) into (9) and (10), the continuity of tangential electric and magnetic fields on the periodic rough interface gives

$$\sum_{m=0}^{\infty}e^{-ika\cos\phi_0}(-ik\cos\phi_0)^m Z_m(x)+$$

$$\sum_{n=-\infty}^{\infty}R_n e^{i(2\pi n/P)x}e^{iq_na}\sum_{m=0}^{\infty}(iq_n)^m Z_m(x)=$$

$$\sum_{n=-\infty}^{\infty}T_n e^{i(2\pi n/P)x}e^{-iq_{1n}b}\sum_{m=0}^{\infty}(-iq_{1n})^m Z_{1m}(x). \quad (11)$$

$$\sum_{m=0}^{\infty} e^{-ika\cos\phi_0}(-ik\cos\phi_0)^m Z_m(x)(f'(x)k\sin\phi_0$$
$$+k\cos\phi_0)+\sum_{n=-\infty}^{\infty} R_n e^{i(2\pi n/P)x} e^{iq_n a}\sum_{m=0}^{\infty}(iq_n)^m$$
$$Z_m(x)(f'(x)\beta_n - q_n) = c_1\sum_{n=-\infty}^{\infty} T_n e^{i(2\pi n/P)x} e^{-iq_{1n}b}$$
$$\sum_{m=0}^{\infty}(-iq_{1n})^m Z_{1m}(x)(f'(x)\beta_n + q_{1n}). \qquad (12)$$

in which two periodic function sequences denote that

$$Z_m(x) = Z_m(x+P) = \frac{1}{m!}(f(x)-a)^m,$$
$$m = 0,1,2\ldots \qquad (13)$$
$$Z_{1m}(x) = Z_{1m}(x+P) = \frac{1}{m!}(f(x)-b)^m,$$
$$m = 0,1,2\ldots \qquad (14)$$

Both (11) and (12) are periodic function of x so domain $[0,P]$ can be discretized into L intervals each with width P/L. The center of p-th interval is denoted by $x=x_p$. Point matching the two identities at $x=x_p$, $p=1,2,\ldots L$ and truncating terms in Taylor series to M and reducing the other infinite summation from $-N$ to N, (11) and (12) can be written in the matrix-vector form as

$$F + C\cdot R = D\cdot T, \qquad (15)$$
$$Fh + Ch\cdot R = Dh\cdot T, \qquad (16)$$

in which unknown coefficient vectors

$$R = [R_{-N}\cdots R_{-1}R_0R_1\cdots R_N]^T. \qquad (17)$$
$$T = [T_{-N}\cdots T_{-1}T_0T_1\cdots T_N]^T. \qquad (18)$$

C, D, Ch and Dh are matrices, F and Fh are column vectors and their elements with respect to element index are given by,

$$C_{pn} = e^{i(2\pi n/P)x_p} e^{iq_n a}\sum_{m=0}^{M}(iq_n)^m Z_m(x_p). \qquad (19)$$
$$D_{pn} = e^{i(2\pi n/P)x_p} e^{-iq_{1n}b}\sum_{m=0}^{M}(-iq_{1n})^m Z_{1m}(x_p). \qquad (20)$$
$$Ch_{pn} = e^{i(2\pi n/P)x_p} e^{iq_n a}\sum_{m=0}^{M}(iq_n)^m Z_m(x_p)$$
$$(f'(x_p)\beta_n - q_n). \qquad (21)$$
$$Dh_{pn} = c_1 e^{i(2\pi n/P)x_p} e^{-iq_{1n}b}\sum_{m=0}^{M}(-iq_{1n})^m Z_{1m}(x_p)$$
$$(f'(x_p)\beta_n + q_{1n}). \qquad (22)$$
$$F_p = \sum_{m=0}^{M} e^{-ika\cos\phi_0}(-ik\cos\phi_0)^m Z_m(x_p). \qquad (23)$$
$$Fh_p = \sum_{m=0}^{M} e^{-ika\cos\phi_0}(-ik\cos\phi_0)^m Z_m(x_p)$$
$$(f'(x_p)k\sin\phi_0 + k\cos\phi_0). \qquad (24)$$

(15) and (16) are linear system of equations and can be solved straightforwardly. Then substituting R into (5), scattered field on and above the top point of the surface can be calculated. In the trough region of the corrugation the scattered field can be calculated by

$$\psi^s(x,y) = \sum_{m=0}^{\infty}\frac{1}{m!}\frac{\partial^m\psi^s}{\partial y^m}\Big|_{y=a}(y-a)^m,\ f(x)\le y<a \qquad (25)$$

Also reducing the harmonic summation from $-N$ to N and truncating terms in Taylor series to M, the scattered field is expressed as

$$\psi^s(x,y) = \sum_{n=-N}^{N} R_n e^{i\beta_n x} e^{iq_n a}\sum_{m=0}^{M}\frac{1}{m!}(iq_n)^m (y-a)^m,$$
$$f(x)\le y<a \qquad (26)$$

III. NUMERICAL RESULTS

A. Diffracted efficiency

The proposed method is applied to the silver grating described in paper [8]. The grating is made of silver with 830 lines/mm and a sinusoidal profile $f(x)=-h\cos(2\pi x/P)$, P=1205nm, h=100nm. The relative complex permittivity of the silver is −7.29135+i0.294387, the incident wavelength is 482nm. Sum all the propagating powers in the air, the diffracted efficiency is given by

$$P_r = \sum\frac{|R_n|^2 q_n}{k\cos\phi_0}. \qquad (27)$$

The sum of the diffracted energy for TM polarization is a severe test of the method since anomaly is generally observed and a significant fraction of the incident light does not reappear in the diffracted orders. The obtained results are shown in Fig. 2 and compared with those by T-Matrix. We can see from the figure that the efficiency curves of the two methods are matched well. Calculation times of proposed and T-Matrix method for this example are 206.3 s and 1797.8 s, respectively under the same soft and hardware conditions.

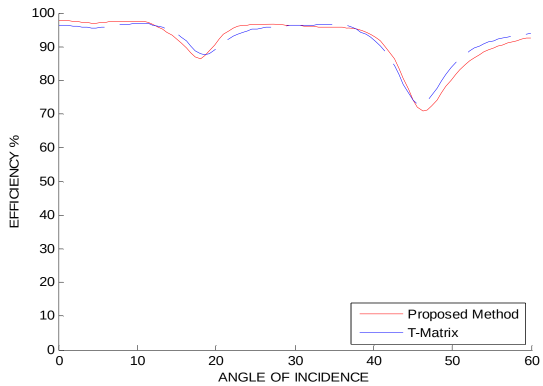
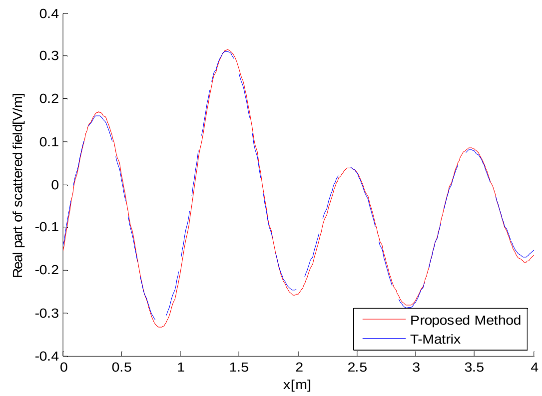


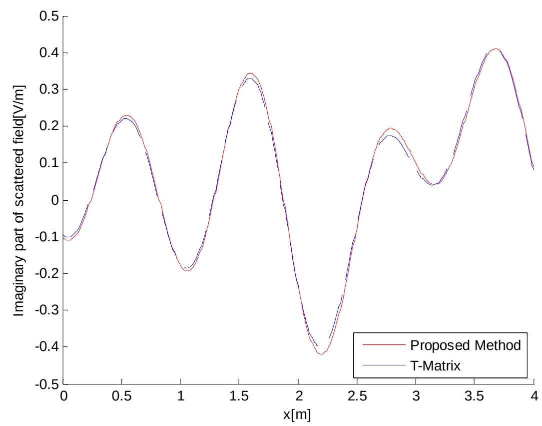
Fig. 2. Diffracted efficiency of the silver grating for TM polarized waves.

B. Scattered field

Proposed method is applied to two representative interfaces for TE polarization, one is sinusoidal $f(x)=0.5\sin(\pi x/2)$ and the other is triangle $f(x)=0.5|x-2|-0.5$. The obtained results are also compared with T-Matrix results. The illuminating plane wave is of frequency 300 MHz, and the relative permittivity of the substrate is 2.25, period P=4 m and the near scattered fields are calculated on y_0=3 m plane within one period. Incident angles are chosen as 45° for sinusoidal and 60° for triangle interface. Number of harmonics, terms in Taylor series and point matching number of transformed problem for calculation in (15) and (16) are chosen as N=12, M=20, L=80 respectively. The parameter N is so selected that all the propagating harmonics and part of evanescent harmonics are included for the given truncation errors of (5) (6). The selection of M is related to truncation errors of Taylor series. We choose the biggest M for given accuracy remainder term of equations (19)-(24). As the corrugation depth increases, more harmonics and more Taylor series terms need to be taken into account. Real and Imaginary parts of scattered field are depicted in Figs. 3 and 4, respectively. As seen from figures, results of proposed and T-Matrix method are in satisfactory agreement. Calculation times of the first and second example for proposed method are 5.312 s and 5.469 s, those for T-Matrix method are 47.125s and 47.687 s, respectively.



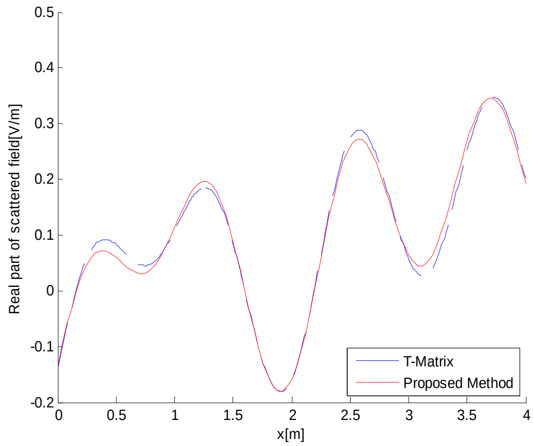
(a)



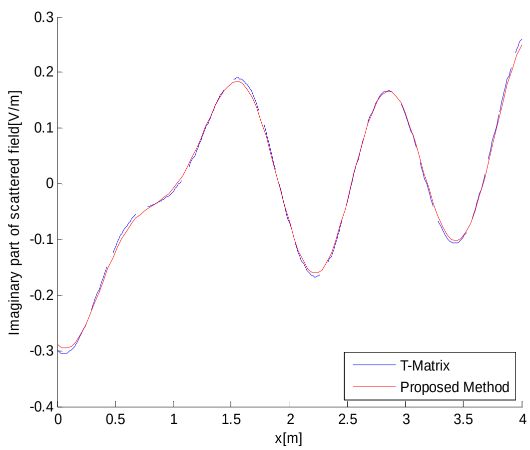
(b)

Fig. 3. Scattered field above sinusoidal interface within one period (a) Real part, (b) Imaginary part.

Then the scattered field within the groove of the first example are calculated on y_0=0 m , $x\in(P/2, P)$. The parameters are the same as the first example. Since the T-Matrix can not calculate the scattered field in the trough region of corrugation, the results of proposed method are compared with MoM [6]. Real and Imaginary parts of the scattered field are depicted in Fig. 5. Calculation times of this example for proposed and MoM are 3.844s and 541.391s respectively. Numerical results show good agreements.

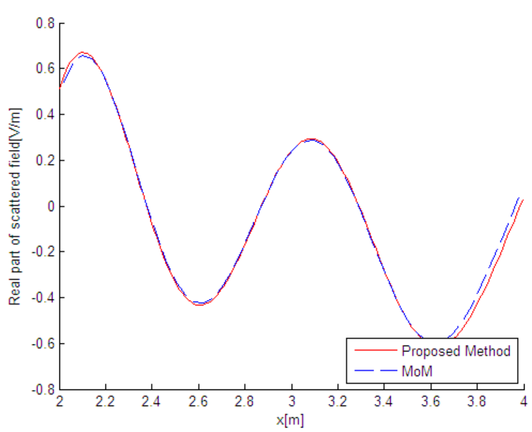


(a)

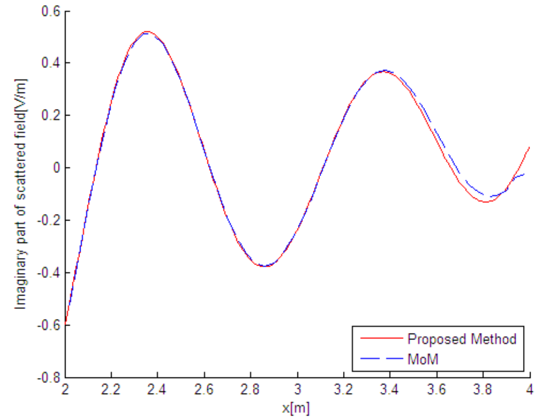


(b)

Fig. 4. Scattered field above triangle interface within one period (a) Real part, (b) Imaginary part.



(a)



(b)

Fig. 5. Scattered field in the trough region of sinusoidal interface (a) Real part, (b) Imaginary part.

IV. DISCUSSION AND CONCLUSION

The scattering problem from periodic interface is transformed to scattering from two imaginary planes, and then the fields just above and below the rough interface are represented by Taylor expansions at the two planes respectively. Applying the boundary conditions to these fields the scattering problem is solved. Comparing with T-Matrix, the proposed method is of simple formulation and it can calculate the field in the trough region of the corrugation. Obtained numerical results of near field distribution are in good agreements with T-Matrix or MoM and the proposed method is with much higher computation efficiency. Besides, for some particular conditions (such as f=300 MHz, ε_r=2.25, P=4 and ϕ_0=30º), one of the y-components of propagation constants becomes zero, T-Matrix cannot work because its formulation has a factor $(q_n)^{-1/2}$ [7] while proposed method does not have this difficulty. Compare with MoM which is based on the slowly convergent periodic Green's function, the proposed method is much more efficient in the calculation of the field within the grooves.

It is the limitation of proposed method that the matrices C, D, Ch and Dh may become ill-conditioned when the surface corrugation is deep.

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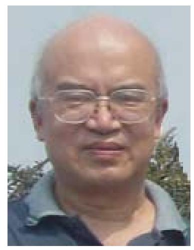

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