Norm Constrained Noise-free Algorithm for Sparse Adaptive Array Beamforming

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Abstract **─** In this paper, a reweighted l_1-norm constrained noise-free normalized least mean square (NLMS) (RL$_1$-CNFLMS) algorithm is proposed for dealing with sparse adaptive array beamforming. The proposed RL$_1$-CNFLMS algorithm integrates a reweighted l_1-norm penalty into the traditional objective function of constrained least mean square least mean square (LMS) (CLMS) algorithm to drive the weighted coefficient vector to sparsity. Besides, the Lagrange multiplier (LM) method and the gradient descent principle are employed during the derivation procedure for getting the update equation. Additionally, we utilize the l_1-l_2 optimization method to acquire the noise-free a posteriori error signal in normalizing process to achieve a quicker convergence speed, a better signal to interference plus noise ratio (SINR) performance as well as a higher array sparsity with an acceptable computational complexity. Simulation results turn out that by using the noise-free and norm constraint techniques, a fairly comparable beampattern is achieved by using only 38.4%, 39.4% and 69.4% antenna elements in contrast to the constrained NLMS (CNLMS), reweighted l_1-norm constrained LMS (RL$_1$-CLMS) and reweighted l_1-norm constrained normalized LMS (RL$_1$-CNLMS) algorithms, respectively.

Index Terms **─** Array beamforming, constrained LMS algorithm, l_1-norm constraint, noise-free normalizing, sparse adaptive beamforming.

I. INTRODUCTION

In the development of antenna array theory, adaptive beamforming algorithms have been recognized as a critical role in array signal processing. Based on the high capacity, adaptive beamforming algorithms have drawn significantly concern and widely applied to modern telecommunication systems, medical, radar, sonar and other areas [1].

Extensive studies have been reported that adaptive beamformers can create ideal beampatterns toward the direction of signal of interest (SOI) to keep a high gain and give nulls to prevent the influence of interferences [2-6]. Meanwhile, in this way the SINR is enhanced [2].

The existing researches suggest that adaptive beamforming algorithms can use the array weight vector to increase the gain of SOI and attenuate the interferences. The linearly constrained minimum variance (LCMV) algorithm was developed by Frost [2], and then, plenty of adaptive algorithms based on least mean square (LMS) principle have been devised for adaptive beamforming [3-6]. Many researches focus on the resolution, robustness and other properties rather than the sparsity of the antenna array [2, 3, 7]. To pursuit high performance, large arrays are always crucial in practice applications, especially in radar and satellite communication [4]. Thus, an antenna array utilizing less antenna elements to generate the beampattern without sacrificing performance is amazing technique.

Recently, a considerable method has grown up around the theme of sparse signal processing [8-21]. Inspired by sparse signal processing in system identification, channel estimation and other fields [8-19], the sparse adaptive beamforming algorithms have been proposed to exploit the sparsity of the corresponding antenna arrays [4-6]. However, new techniques are still needed to further improve these existing algorithms to achieve higher performance, e.g., low sidelobe level

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(SLL) and fast convergence.

The specific objective of this study is to develop a normalizing approach for enhancing sparse adaptive beamforming algorithms to accelerate the convergence. In [22, 23], the l_1-l_2 minimization method is discussed, in which the noise-free error signal is obtained and then applied to other adaptive-filtering algorithms [24]. In this paper, we aim to develop a new variable convergence factor to promote the conventional constrained LMS (CLMS) algorithm for sparse adaptive beamforming by minimizing the noise-free a posteriori error signal. Simulation results indicate that a similar beampattern performance is obtained with less antenna elements, and a faster convergence speed as well as a better SINR are achieved for a circular antenna array beamforming.

II. MATHEMATICAL MODEL

As is shown in Fig. 1, a narrowband beamformer with N omnidirectional antennas is considered, and the output signal at time index k is formulated by:

$$\mathrm{y}_k=\mathbf{w}^{\mathrm{H}}\mathbf{x}_k, \tag{1}$$

where $\mathbf{w}_k=[w_1,\ldots,w_N]^{\mathrm{T}}$ is the weighted coefficient vector, while $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ represent the transpose and Hermitian operators. The (M+1) input signal which composed of directed SOI (θ_{s}, φ_{s}), and interferences with direction of (θ_{i}, φ_{i}) (i=1,2,…M) is given by:

$$\mathbf{x}_k=\mathbf{A}_s\mathbf{E}_k+\mathbf{A}_i\mathbf{i}_k+\mathbf{n}_k, \tag{2}$$

where $\mathbf{A}_s$ and $\mathbf{A}_i$ are the steering matrices corresponding to the SOI and interferences, $\mathbf{E}_k$ as well as $\mathbf{i}_k$ are the complex SOI and interferences envelope vectors. $\mathbf{n}_k$ denotes the zero-mean white Gaussian noise vector.

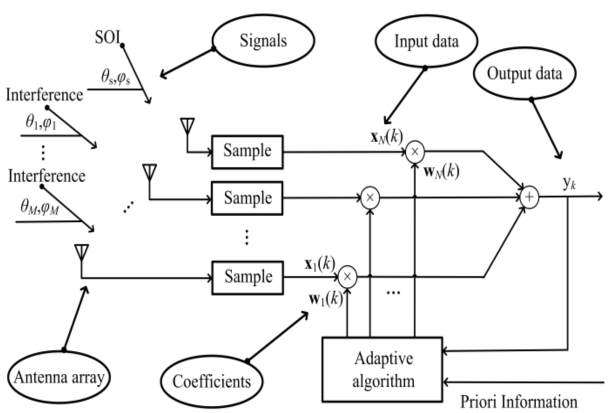

Fig. 1. The narrowband beamformer model.

Then, the beampattern for a given direction (θ, φ) is:

$$B(\theta,\phi)=\mathbf{w}^{\mathrm{H}}\exp\left\{\frac{-j2\pi\mathbf{Ap}}{\lambda}\right\}. \tag{3}$$

Herein, $\mathbf{A}$ is the steering matrix consists of $\mathbf{A}_s$ and $\mathbf{A}_i$, λ is the transmission wavelength, and $\mathbf{p}$ is the positions of antenna elements. The output SINR of a beamformer is expressed as [8]:

$$\mathrm{SINR}=\frac{\sigma_s^2|\mathbf{w}^{\mathrm{H}}\mathbf{A}_s|^2}{\mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{n+i}}\mathbf{w}}, \tag{4}$$

where σ_s^2 is the SOI power, and $\mathbf{R}_{\mathrm{n+i}}$ is the covariance matrix of the interference-plus-noise which is depicted as:

$$\mathbf{R}_{\mathrm{n+i}}=E\left\{(\mathbf{i}_k+\mathbf{N}_k)(\mathbf{i}_k+\mathbf{N}_k)^{\mathrm{H}}\right\}, \tag{5}$$

where $E\{\cdot\}$ represents the expectation operator.

III. THE PROPOSED RL1-CNFLMS ALGORITHM

A. Review of the CLMS and CNLMS algorithms

The CLMS algorithm exerts a constraint condition on the cost function of LMS algorithm to solve the following problem [25]:

$$\min_{\mathbf{w}} E\left\lfloor |e_k|^2 \right\rfloor \quad \text{subject to} \quad \mathbf{C}^{\mathbf{H}}\mathbf{w}=\mathbf{f}, \tag{6}$$

where e_k=d_k-$\mathbf{w}^{\mathrm{H}}\mathbf{x}_k$ represents the estimation error, and d_k is the desired output, while $\mathbf{C}$ and $\mathbf{f}$ are constrained matrix and vector which relates to the SOI and interferences.

The solution is then found out by using the LM method, and then, we obtain the following cost function:

$$L_{clms}(k)=E\left[|e_k|^2\right]+\boldsymbol{\lambda}_1^{\mathrm{H}}(\mathbf{C}^{\mathrm{H}}\mathbf{w}_k-\mathbf{f}). \tag{7}$$

In (7), $\boldsymbol{\lambda}_1$ is regarded as the LM vector. The gradient descent principle is also used for carrying out the solution of (6). In this case, the update equation is expressed as:

$$\mathbf{w}_{k+1}=\mathbf{w}_k\text{-}\mu\mathbf{g}_{\mathbf{w}}L_{clms}(k), \tag{8}$$

where μ acts as the convergence factor and $\mathbf{g}_{\mathbf{w}}L_{clms}(k)$ is gradient vector.

The instantaneous estimate is used to calculate the gradient vector $\mathbf{g}_{\mathbf{w}}L_{clms}(k)$, and we have:

$$\mathbf{g}_{\mathbf{w}}L_{clms}(k)=\text{-}2e_k^*\mathbf{x}_k+\mathbf{C}\boldsymbol{\lambda}_1. \tag{9}$$

Then, the final updating function is obtained and given by:

$$\mathbf{w}_{k+1}=\mathbf{P}\left[\mathbf{w}_k+\mu e_k^*\mathbf{x}_k\right]+\mathbf{f}_c, \tag{10}$$

with

$$\begin{cases}\mathbf{P}=\mathbf{I}_{N\times N}-\mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathbf{H}},\\ \mathbf{f}_c=\mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{f}.\end{cases} \tag{11}$$

Then, the CNLMS algorithm is proposed to speed up the convergence process, which is the normalized version of CLMS algorithm [4, 25]. In the CLMS, the step size is fixed. If we use a variable step size μ_k that can minimize the instantaneous posteriori squared error, the convergence process will be greatly accelerated. That's to say, the CLMS algorithm can be normalized by letting the derivative in terms of μ be 0, which is to calculate [25]:

$$\frac{\partial[|e_{ap}(k)|^2]}{\partial\mu_k^*}=\frac{\partial\left[e_{ap}(k)e_{ap}^*(k)\right]}{\partial\mu_k^*}=0, \tag{12}$$

where

$$e_{ap}(k) = e_k\left(1-\mu_k \mathbf{x}_k^{\mathrm{H}}\mathbf{P}\mathbf{x}_k\right). \tag{13}$$

Thus, we have:

$$\mu_k = \frac{\mu_0}{\mathbf{x}_k^{\mathrm{H}}\mathbf{P}\mathbf{x}_k + \xi_c}. \tag{14}$$

where ξ_c>0 is to avoid excessively large convergence factor, and μ_0 is the initial convergence factor.

Then, taking (14) into (10), we get the iteration equation for the CNLMS algorithm:

$$\mathbf{w}_{k+1} = \mathbf{P}[\mathbf{w}_k + \mu_0 \frac{e_k \mathbf{x}_k}{\mathbf{x}_k^{\mathrm{H}}\mathbf{P}\mathbf{x}_k + \xi_c}] + \mathbf{f}_c. \tag{15}$$

B. The proposed RL$_1$-CNFLMS algorithm

In this paper, a RL$_1$-CNFLMS algorithm for sparse adaptive beamforming is proposed, which uses the norm constraint technique to solve:

$$\min_{\mathbf{w}} E\left[|e_k|^2\right] \quad \text{subject to} \quad \begin{cases}\mathbf{C}^{\mathbf{H}}\mathbf{w}_k = \mathbf{f};\\ \|\mathbf{s}_k\mathbf{w}_k\|_1 = t,\end{cases} \tag{16}$$

where t is the sparseness constraint factor which is to force the small coefficients to zero, and $\mathbf{s}_k$ is presented as [8]:

$$[\mathbf{s}_k]_i = \frac{1}{\xi_{rl1} + |\mathbf{w}_{k-1}|_i}, i = 1,\ldots,N, \tag{17}$$

and ξ_{rl1}>0 is a constant analogous to ξ_c in (14).

The first step in finding out the solution is to utilize the LM method, and the corresponding cost function is:

$$\begin{aligned} L_{rl1}(k) = & E\left[|e_k|^2\right] + \boldsymbol{\lambda}_1^{\mathbf{H}}(\mathbf{C}^{\mathbf{H}}\mathbf{w}_k - \mathbf{f}) \\ & + \lambda_{rl1}[\|\mathbf{s}_k\mathbf{w}_k\|_1 \text{-} t], \end{aligned} \tag{18}$$

where $\boldsymbol{\lambda}_1$ and λ_{rl1} are the LMs.

Then, the instantaneous gradient estimation of (18) is given by:

$$\mathbf{g}_{\mathbf{w}} L_{rl1}(k) = -2e_k^*\mathbf{x}_k + \mathbf{C}\boldsymbol{\lambda}_1 + \lambda_{rl1}\mathbf{B}_{rl1}(k), \tag{19}$$

with

$$\mathbf{B}_{rl1}(k) = \frac{\operatorname{sgn}(\mathbf{w}_k)}{\xi_{rl1} + |\mathbf{w}_{k-1}|}, \tag{20}$$

where sgn(·) is the sign function which transforms each element in the vector as -1 for x<0, 0 for x=0 and 1 for x>0, respectively.

The gradient descent concept is used to address (16), resulting in the following update equation:

$$\mathbf{w}_{k+1} = \mathbf{w}_k \text{-} \mu \mathbf{g}_{\mathbf{w}} L_{rl1}(k). \tag{21}$$

The LMs $\boldsymbol{\lambda}_1$ and λ_{rl1} are assessable by using the constraints in (16). An approximating approach which considers the iteration process at the steady-state (i.e., $\mathbf{w}_{k+1}$=$\mathbf{w}_k$) is chosen to help to obtain the following equation [4, 6]:

$$\begin{cases}\mathbf{C}^{\mathrm{H}}\mathbf{w}_{k+1} = \mathbf{f},\\ \mathbf{B}_{rl1}(k)\mathbf{w}_{k+1} = t.\end{cases} \tag{22}$$

Based on (16), (19), (21) and (22), we finally acquire the equations below, for $\boldsymbol{\lambda}_1$ and λ_{rl1}, respectively:

$$\begin{cases}\boldsymbol{\lambda}_1 = \mathbf{D}(2\mathrm{e}_k^*\mathbf{x}_k - \gamma_{rl1}\mathbf{B}_{rl1}(k)),\\ \lambda_{rl1} = (\frac{-2}{n\mu})(t - \mathbf{B}_{rl1}^{\mathrm{H}}(k)\mathbf{w}_k) + \frac{2e_k^*\mathbf{B}_{rl1}^{\mathrm{H}}(k)\mathbf{P}\mathbf{x}_k}{n},\end{cases} \tag{23}$$

with

$$\begin{cases}\mathbf{D} = (\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathrm{H}},\\ n = \|\mathbf{P}\mathbf{B}_{rl1}(k)\|_2^2.\end{cases} \tag{24}$$

Then, substituting $\boldsymbol{\lambda}_1$ and λ_{rl1} into (19) and (21), we can derive the updating equation:

$$\mathbf{w}_{\boldsymbol{k}+1} = \mathbf{w}_{\boldsymbol{k}} + \mu_0 e_k^* \mathbf{U} + \mathbf{f}_{rl1}(k), \tag{25}$$

where

$$\begin{cases} q = \mathbf{B}_{rl1}^{\mathrm{H}}(k)\mathbf{P}\mathbf{x}_k,\\ m = \mathbf{B}_{rl1}^{\mathrm{H}}(k)\mathbf{P}\mathbf{B}_{rl1}(k),\\ \mathbf{P} = \mathbf{I}_{N\times N} - \mathbf{C}(\mathbf{C}^{\mathbf{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathbf{H}},\\ \mathbf{U} = \mathbf{P}(\mathbf{x}_k - \frac{q\mathbf{B}_{rl1}(k)}{m}),\\ \mathbf{f}_{rl1}(k) = (t - \mathbf{B}_{rl1}^{\mathrm{H}}(k)\mathbf{w}_k)(\frac{\mathbf{P}\mathbf{B}_{rl1}(k)}{m}).\end{cases} \tag{26}$$

To enable the algorithm to reach the steady-state more fleetly, the noise-free normalizing procedure is adopted in our proposed algorithm. The noise-free error signal obtained via the l_1-l_2 minimization method is introduced to many adaptive-filtering algorithms [24].

First, we rewrite the a priori error signal as:

$$e_k = e_{nf,k} + n(k), \tag{27}$$

where $n(k)$ is the noise-component and $e_{nf,k}$ is the expression of the a priori noise-free error signal, which is given by:

$$e_{nf,k} = d_k - \mathbf{x}_k\mathbf{w}_k. \tag{28}$$

Similarly, we use the a posteriori error signal:

$$\varepsilon_{nf,k} = d_k - \mathbf{x}_k\mathbf{w}_{k+1} = (1\text{-}\mu_k\mathbf{U}^{\mathrm{H}}\mathbf{x}_k)e_k \text{-} \mathbf{f}_{rl1}^{\mathrm{H}}\mathbf{x}_k, \tag{29}$$

Taking (27) into (29), and using straight-forward calculations, we have:

$$\varepsilon_{nf,k} = (1\text{-}\mu_k\mathbf{U}^{\mathrm{H}}\mathbf{x}_k)e_{nf,k} + n(k) \text{-} \mu_k\mathbf{U}^{\mathrm{H}}\mathbf{x}_k n(k) - \mathbf{f}_{rl1}^{\mathrm{H}}\mathbf{x}_k. \tag{30}$$

Then, take the expectation of the a posteriori squared error at time index k:

$$\begin{aligned} E[\varepsilon_{nf,k}^2] = & E[(1\text{-}\mu_k\mathbf{U}^{\mathrm{H}}\mathbf{x}_k)^2 e_{nf,k}^2] + E[n^2(k)] \\ & \text{-}E[(\mu_k\mathbf{U}^{\mathrm{H}}\mathbf{x}_k n^2(k))^2] \\ & - E[2(1\text{-}\mu_k\mathbf{U}^{\mathrm{H}}\mathbf{x}_k)e_{nf,k}\mathbf{f}_{rl1}^{\mathrm{H}}\mathbf{x}_k] \\ & + E[(\mathbf{f}_{rl1}^{\mathrm{H}}\mathbf{x}_k)^2]. \end{aligned} \tag{31}$$

In the formulations given above, $n(k)$ is a statistic independence and identically distributed white Gaussian signal, and $e_{nf,k}$ is very small when the algorithm converges so that its dependence is ignored. To keep the simplicity of the equation, as $\mathbf{U}^{\mathrm{H}}\mathbf{x}_k$ and $\mathbf{f}_{rl1}^{\mathrm{H}}\mathbf{x}_k$ are scalars, their instantaneous estimation is used in our derivation. With these assumptions, by minimizing $E[\varepsilon_{nf,k}^2]$ with respect to μ_k, yields:

$$\mu_k = \frac{\beta\left\{E[e_{nf,k}^2] - \mathbf{f}_{rl1}^{\mathrm{H}}\mathbf{x}_k E[e_{nf,k}]\right\}}{\mathbf{U}^{\mathrm{H}}\mathbf{x}_k\left\{E[e_{nf,k}^2] + E[n^2(k)]\right\}}, \tag{32}$$

where $0<\beta<1$ is a constant and $E[n^2(k)]=\sigma_n^2$ is the variance of noise.

Then, we will capture $E[e_{nf,k}]$ and $E[e_{nf,k}^2]$. The latter term $E[e_{nf,k}^2]$ can be approximated by the squared time average of $e_{nf,k}$ writing as:

$$E[e_{nf,k}^2] = \alpha E[e_{nf,k\text{-}1}^2]\mathrm{sign}[e_k] + (1\text{-}\alpha)e_{nf,k}^2. \tag{33}$$

The parameter α acts as the forgetting factor within (0, 1). In this case, the major task for us is to get the expression for $e_{nf,k}$ so as to address $E[e_{nf,k}]$ and $E[e_{nf,k}^2]$.

According to maximum a posteriori probability (MAP) [23], $e_{nf,k}$ can be recovered form e_k via the optimization problem:

$$f[e_{nf,k}] = 0.5\,|\,e_k - e_{nf,k}\,|^2 + \gamma\,|\,e_{nf,k}\,|, \tag{34}$$

where γ is a threshold parameter which balances the representation error and the sparsity. The optimal estimation of $e_{nf,k}$ is then calculated by minimizing $f\,[e_{nf,k}]$ with respect to $e_{nf,k}$, which is given by [23, 24]:

$$\hat{e}_{nf,k} = \mathrm{sign}[e_k]\max(|\,e_k\,| - \gamma, 0). \tag{35}$$

The threshold parameter γ is chosen as [22-24]:

$$\gamma = \sqrt{Q\sigma_n^2}, \tag{36}$$

with $1<Q<4$.

Until now, we have essentially studied all the expressions for μ_k which are summarized as follows:

$$\mu_k = \frac{\beta\left\{E[e_{nf,k}^2] - \mathbf{f}_{rl1}^{\mathrm{H}}\mathbf{x}_k E[e_{nf,k}]\right\}}{\mathbf{U}^{\mathrm{H}}\mathbf{x}_k\left\{E[e_{nf,k}^2] + E[n^2(k)]\right\}}, \tag{37}$$

where

$$\begin{cases} E[e_{nf,k}^2] = \alpha E[e_{nf,k\text{-}1}^2]\mathrm{sign}[e_k] + (1\text{-}\alpha)e_{nf,k}^2, \\ \hat{e}_{nf,k} = \mathrm{sign}[e_k]\max(|\,e_k\,| - \gamma, 0), \\ \gamma = \sqrt{Q\sigma_n^2}. \end{cases} \tag{38}$$

Finally, replacing μ_0 with μ_k in (25), we can acquire the updating equation for RL$_1$-CNFLMS algorithm which is omitted here for brief. It is found that the difference between the two algorithms, namely the RL$_1$-CNLMS in [6] and our proposed RL$_1$-CNFLMS is the calculation of step size, which requires a little computational complexity. That is to say, our proposed algorithm provides an acceptable computational complexity comparing to the existing algorithm.

IV. SIMULATIONS

In our experiments, a circular array is used to receive five QPSK narrowband signals. The signals, composing of an SOI and four interferences, come from the azimuths of 90°, 30°, 58°, 127°, 163°, respectively, with a uniform elevation of 45°. The interference to noise ratio is set to 30 dB. Other parameters are given in Table 1 in detail.

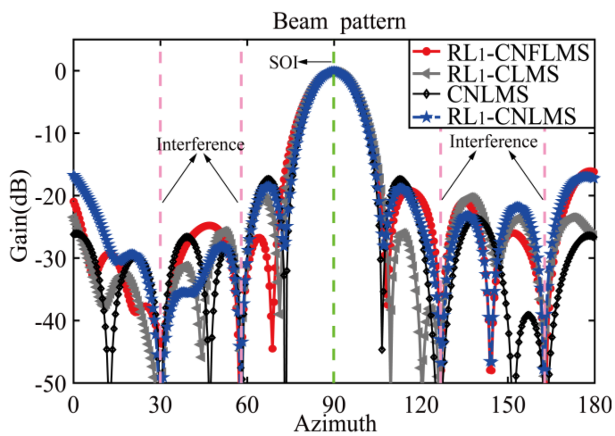

Fig. 2. Beam patterns of the proposed algorithm versus the CNLMS algorithm and the RL$_1$-CNLMS algorithm in [6].

Table 1: Parameters in simulations

Parameters	CNLMS	RL$_1$-CLMS	RL$_1$-CNLMS	RL$_1$-CNFLMS
Step-size (μ)	5×10^{-3}	5×10^{-9}	2×10^{-2}	-
Elements' interval	$2/\lambda$	$2/\lambda$	$2/\lambda$	$2/\lambda$
l_1-norm constraint	0.93	0.93	0.93	0.93
Signal frequencies	8GHz	8GHz	8GHz	8GHz
ε	5×10^{-3}	5×10^{-3}	5×10^{-3}	5×10^{-3}
α	-	-	-	0.52
δ	-	-	-	0.12
Q	-	-	-	1

The first set of experiment examined the impact of the developed RL$_1$-CNFLMS algorithm on beampatterns. The performance comparison of the proposed RL$_1$-CNFLMS, CNLMS, RL$_1$-CLMS and RL$_1$-CNLMS algorithms [6] are presented in Fig. 2. It can be seen that these algorithms are able to give resistant to the interferences by forming nulls and they can still remain high gain against the SOI around main lobe. As for our introduced algorithm, it has a quite similar main lobe with other algorithms as well as the same level of SLL.

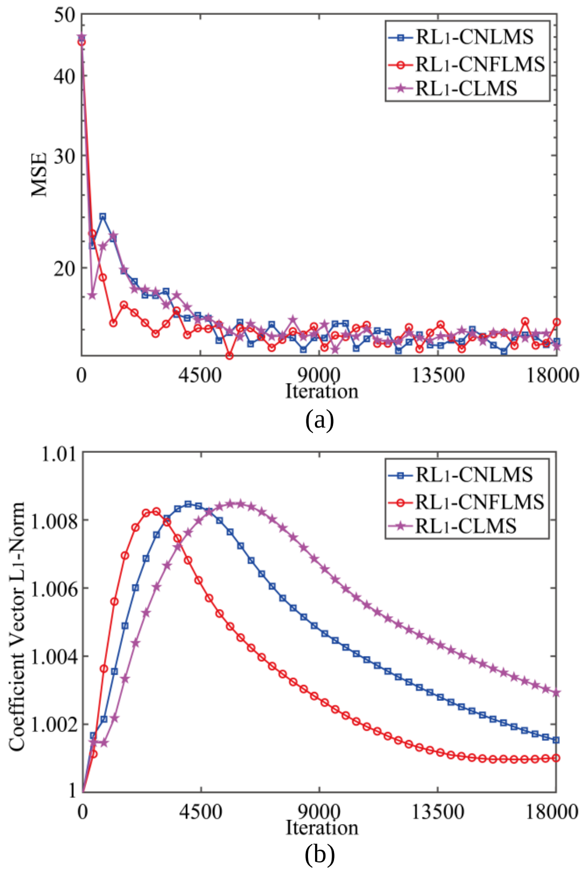

Fig. 3. The Mean square error (MSE) performance and the l_1-norm of coefficients for the proposed algorithm versus other related algorithms. (a) MSE performance, (b) l_1-norm.

Figure 3 shows the MSE performance and the coefficients' l_1-norm of the proposed RL$_1$-CNFLMS algorithm in contrast to other related beamforming algorithms. From Fig. 3, we can find that the proposed RL$_1$-CNFLMS have a superior convergence speed which validates the results in adaptive filtering and other fields, and verifies the effectiveness of our algorithm.

Figure 4 presents the beamformed antenna arrays. The sparse ratio, which is the proportion of the active antenna elements to the entire array, are set to be 65.2%, 55.4% and 38.4%, for RL$_1$-CLMS, RL$_1$-CNLMS and RL$_1$-CNFLMS algorithms. Considering Fig. 4 and referring to Fig. 2 and Fig. 3, it is obvious that our proposed algorithm has the ability to enhance the array sparsity and accelerate the convergence procedure with good beamforming properties.

In Fig. 5, the output SINR for different beamforming algorithms is discussed. We can see that the proposed RL$_1$-CNFLMS algorithm achieves better SINR for various SNRs in comparison with the related sparse adaptive beamformers.

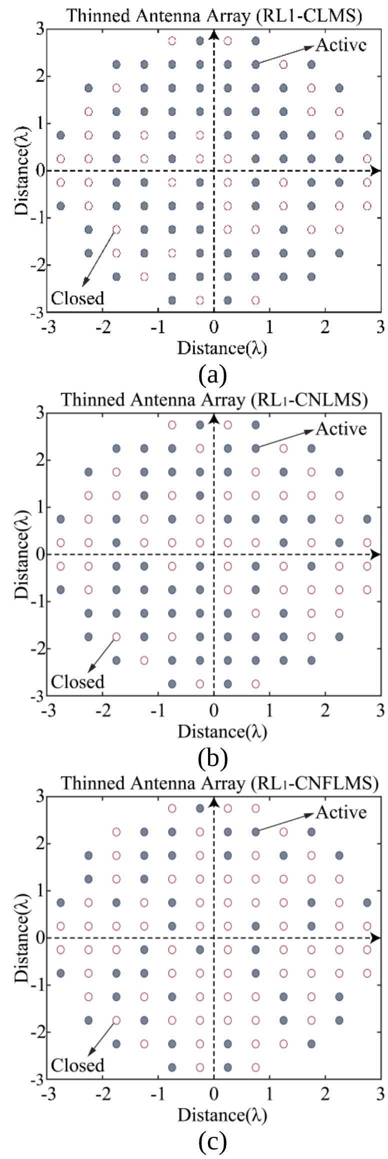

Fig. 4. Sparse arrays shrinked by RL$_1$-CNFLMS (proposed), RL$_1$-CLMS and RL$_1$-CNLMS [6].

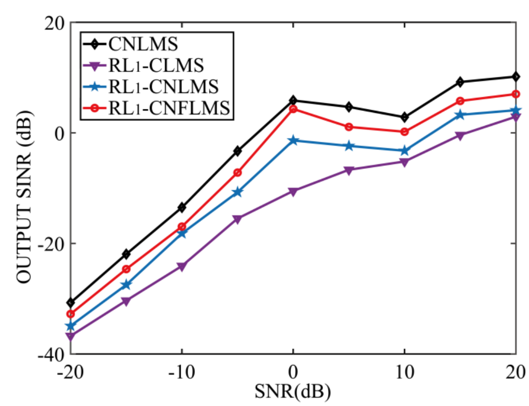

Fig. 5. Output SINR versus the input SNR.

From the discussions above, our developed RL_1-CNFLMS algorithm achieves a better capacity in sparsity, convergence process and output SINR with a comparable beampattern compared with other relevant sparse adaptive beamformers. As a consequence, the proposed RL_1-CNFLMS is worthwhile for practical applications.

V. CONCLUSION

In this paper, a RL_1-CNFLMS algorithm for sparse adaptive beamforming has been proposed and analyzed in detail. By means of the l_1-l_2 minimization method, the noise-free a posteriori error signal is obtained and adopted to normalize the sparse CLMS algorithm. The proposed RL_1-CNFLMS algorithm generates the desired beampattern while achieving a better performance in sparsity, convergence speed and output SINR along with an acceptable computational complexity. However, during the derivation procedure, as we use lots of approximations, the parameters maybe difficult to adjust. Also, the coupling in the array is neglected in our mathematical model, which may cause estimate error. All in all, we still have much more efforts to do in the future study.

In the future, we will consider the subarray method to construct the sparse blocked array beamforming using the technique in [26-28]. In addition, we will develop a platform to verify the proposed algorithms in a MIMO antenna array beamforming to analyze the effects of the mutual coupling in the array [29-31].

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