Efficient Marching-on-in-Degree Solver of Time Domain Integral Equation with Adaptive Cross Approximation Algorithm-Singular Value Decomposition

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Abstract — Adaptive cross approximation algorithm with singular value decomposition postcompression (ACA-SVD) is introduced into the marching-on-in-degree solver of time domain integral equation for the analysis of transient electromagnetic scattering from perfect electric conductor (PEC). The computational domain is divided into multilevel groups based on octree. ACA-SVD algorithm is utilized to compute the impedance matrices associated with the well-separated groups at each level. Whereas, the impedance matrices formed by self and neighboring groups are calculated entirely in the traditional manner. Numerical results demonstrate that the proposed method can greatly reduce the memory requirement and matrix-vector product (MVP) time per iteration.

Index Terms — Time domain integral equation, marching-on-in-degree, transient scattering, adaptive cross approximation algorithm, singular value decomposition.

I. INTRODUCTION

In recent years, the direct time domain methods has attracted extensive attention in calculating transient electromagnetic problems, which is due to the increasing interest in target identification, short pulse radar design, or other related applications. Several common time domain methods are finite-difference time domain (FDTD) method, time domain finite-element (TDFE) method, finite-volume time domain (FVTD) method and time domain integral equation (TDIE) method, among which the TDIE method is more suitable for analysis of electromagnetic scattering and radiation problems in homogeneous medium because it only needs surface meshing and does not need absorbing boundary condition. There are two popular approaches to solve TDIE. One is the marching-on-in-time (MOT) method [1], [2], and the other is the marching-on-in-degree (MOD) method [3]-[6]. The proposed method in this paper is based on the MOD solver of TDIE.

The MOD method uses causal weighted Laguerre polynomials as temporal basis and testing functions. Due to the property of weighted Laguerre polynomials [7], [8], this method does not involve late-time instability. However, the conventional MOD method requires much more memory and CPU time than that of MOT method [9], which precludes its application in large scale problems. Moreover, the plane wave time domain (PWTD) algorithm, developed at Michelssen's group at Urbana-Champaign, can reduce the CPU time and memory requirements of MOT to $O(N_t N_s log^2 N_s)$ and $O(N_t N_s)$ respectively [10]. In order to improve the capability of MOD method, several accelerating techniques have been applied, such as fast Fourier transform (FFT) [11], UV method [12], and so on. The FFT-based MOD utilizes the spatial translational invariance nature of the Green's function and reduces the computational cost and the storage requirements respectively to $O(N_t^2 N_s log N_s)$ and $O(N_t N_s)$, where N_s and N_t denote the number of spatial and temporal basis functions. But the FFT method applied in [11] requires uniform mesh of the object. The UV method is utilized in [12] to reduce both the memory requirement and CPU time per interaction to $O(N_s^{4/3} log N_s)$.

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The adaptive cross approximation (ACA) algorithm was developed by Bebendorf [13] and is widely used to solve electromagnetic wave problems with moderate electric size [14-16]. It is a purely algebraic and therefore, kernel independent algorithm. In this paper, ACA algorithm with singular value decomposition (SVD) postcompression [17] is applied to the MOD solver of TDIE. Numerical results show that the proposed method can greatly reduce the memory requirement and matrix-vector product (MVP) time per iteration.

The remainder of this paper is organized as follows. Section II describes the marching-on-in-degree solver of time domain combined field integral equation (TD-CFIE). Section III gives the details about the acceleration of MOD with ACA-SVD algorithm. Section IV presents validations and numerical experiments. Section V gives some conclusions.

II. MARCHING-ON-IN-DEGREE SOLVER OF TD-CFIE

A. TD-CFIE

Considering that a PEC scatterer is illuminated by a transient electromagnetic field. The induced current on the conducting surface is denoted as $\mathbf{J}(\mathbf{r},t)$, which satisfies the time domain electric field integral equation (TD-EFIE) and magnetic field integral equation (TD-MFIE):

$$\begin{aligned}\hat{n}\times\hat{n}\times\mathbf{E}^i(\mathbf{r},t) &= \hat{n}\times\hat{n}\times\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_s \frac{\mathbf{J}(\mathbf{r}',\tau)}{R}dS' \\ &-\hat{n}\times\hat{n}\times\frac{\nabla}{4\pi\varepsilon_0}\int_s\int_{-\infty}^{\tau}\frac{\nabla'\bullet\mathbf{J}(\mathbf{r}',t)}{R}dtdS',\end{aligned} \tag{1}$$

$$\hat{n}\times\mathbf{H}^i(\mathbf{r},t) = \frac{\mathbf{J}(\mathbf{r},t)}{2} - \hat{n}\times\nabla\times\int_{s_0}\frac{\mathbf{J}(\mathbf{r}',\tau)}{4\pi R}dS', \tag{2}$$

where $\tau = t - R/c$ is the retarded time, $R = |\mathbf{r}-\mathbf{r}'|$, $\mathbf{r}$ and $\mathbf{r}'$ refer to the position vectors of observation and source point respectively, S_0 denotes the surface without the singularity point at $\mathbf{r}=\mathbf{r}'$, $\hat{n}$ is the unit normal vector outward to the conducting surface S.

Using a combination factor α ranging from 0 to 1, we can get the TD-CFIE:

$$\alpha(\textit{TD-EFIE}) + (1-\alpha)\eta(\textit{TD-MFIE}), \tag{3}$$

where η is the wave impedance of free space.

B. Spatial and temporal discretization of TD-CFIE

For the expanding of TD-CFIE, we choose RWG basis functions [18] and weighted Laguerre polynomials as spatial and temporal basis functions, respectively. Thus, the surface current density can be discretized as

$$\mathbf{J}(\mathbf{r},t) = \sum_{n=1}^{N_s}\sum_{j=0}^{N_t} \mathrm{J}_{n,j}\mathbf{S}_n(\mathbf{r})\varphi_j(\bar{t}), \tag{4}$$

where $\mathbf{S}_n(\mathbf{r})$ represents the n*th* RWG basis function, $\varphi_j(\bar{t})$ is the j*th* degree weighted Laguerre polynomial

$$\varphi_j(\bar{t}) = e^{-\bar{t}/2}L_j(\bar{t}), \tag{5}$$

$\bar{t} = st$, s is the temporal scaling factor and L_j is the j*th* degree Laguerre polynomial

$$L_j(t) = \frac{e^t}{j!}\frac{d^j}{dt^j}\left(t^j e^{-t}\right),\ 0 \le t < \infty, \tag{6}$$

N_s is the number of spatial basis functions, N_t is the number of temporal basis functions and it is related to time duration T and frequency bandwidth B of the incident wave [3]

$$N_t > 2BT + 1. \tag{7}$$

Taking (4) into (3) and making the spatial and temporal testing with Galerkin's method, we can obtain

$$\begin{aligned}\sum_{n=1}^{N_s}\sum_{j=0}^{i}\left[sJ_{n,j}^D A_{mn} + \frac{2}{s}J_{n,j}^I B_{mn} - J_{n,j}D_{mn}\right]\varphi_{i,j}(sR/c) \\ +\sum_{n=1}^{N_s}J_{n,i}C_{mn} = \int_0^\infty \varphi_i(\bar{t})R_m d\bar{t},\end{aligned} \tag{8}$$

where

$$\begin{aligned}A_{mn} &= \frac{\alpha\mu_0}{4\pi}\int_s\int_{s'}\frac{S_m(\mathbf{r})\cdot S_n(\mathbf{r}')}{R}dS'dS \\ &-\frac{(1-\alpha)\eta}{4\pi c}\int_s S_m(\mathbf{r})\cdot\hat{n}\times\int_{S_0}S_n(\mathbf{r}')\times\frac{\hat{\mathbf{R}}}{R}dS'dS,\end{aligned} \tag{9}$$

$$B_{mn} = \frac{\alpha}{4\pi\varepsilon_0}\int_s\int_{s'}\frac{\nabla\cdot S_m(\mathbf{r})\nabla'\cdot S_n(\mathbf{r}')}{R}dS'dS, \tag{10}$$

$$C_{mn} = \frac{(1-\alpha)\eta}{2}\int_s S_m(\mathbf{r})\cdot S_n(\mathbf{r}')dS, \tag{11}$$

$$D_{mn} = \frac{(1-\alpha)\eta}{4\pi}\int_s S_m(\mathbf{r})\cdot\hat{n}\times\int_{S_0}S_n(\mathbf{r}')\times\frac{\hat{\mathbf{R}}}{R^2}dS'dS, \tag{12}$$

$$\begin{aligned}R_m &= \alpha\int_s S_m(\mathbf{r})\cdot\mathbf{E}^i(\mathbf{r},t)dS \\ &+(1-\alpha)\eta\int_s S_m(\mathbf{r})\cdot\hat{n}\times\mathbf{H}^i(\mathbf{r},t)dS,\end{aligned} \tag{13}$$

$$\varphi_{i,j}(sR/c) = \varphi_{i-j}(sR/c) - \varphi_{i-j-1}(sR/c), \tag{14}$$

$$J_{n,j}^{D} = 0.5J_{n,j} + \sum_{k=0}^{j-1} J_{n,k}, \tag{15}$$

$$J_{n,j}^{I} = J_{n,j} + 2\sum_{k=0}^{j-1} J_{n,k}\left(-1\right)^{j+k}. \tag{16}$$

Making the $i=j$ terms at left and the $i<j$ terms at right, we can rewrite (8) in matrix form

$$\left[\mathbf{Z}_{mn}\right]\left\{J_{n,i}\right\} = \left\{\mathbf{V}_{m,i}\right\} + \left\{\mathbf{P}_{m,i}\right\}, \tag{17}$$

where

$$\mathbf{Z}_{mn} = \left[0.5sA_{mn} + \frac{2}{s}B_{mn} - D_{mn}\right]e^{\frac{-sR}{2c}} + C_{mn}, \tag{18}$$

$$\mathbf{V}_{m,i} = \int_0^\infty \varphi_i\left(\bar{t}\right) R_m d\bar{t}, \tag{19}$$

$$\begin{aligned}\mathbf{P}_{m,i} = &-\sum_{j=0}^{i-1}\left[sJ_{n,j}^{D}A_{mn} + \frac{2}{s}J_{n,j}^{I}B_{mn} - J_{n,j}D_{mn}\right]\varphi_{i,j}\left(sR/c\right) \\ &-\left[s\sum_{k=0}^{i-1}J_{n,k}A_{mn} + \frac{2}{s}\left(2\sum_{k=0}^{i-1}J_{n,k}\left(-1\right)^{j+k}\right)B_{mn}\right]e^{\frac{-sR}{2c}}.\end{aligned} \tag{20}$$

Equation (17) is a recursion equation and can be solved degree by degree to obtain the current coefficients $\left\{J_{n,i}\right\}$.

III. ACCELERATION OF MOD WITH ACA-SVD ALGORITHM

Based on the knowledge of Section II, it can be discovered that four kinds of matrices $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, $\mathbf{M}_4$ need to be constructed and stored for the implementation of MOD-TDCFIE, where

$$\mathbf{M}_{1,mn} = \mathbf{Z}_{mn} = \left[0.5sA_{mn} + \frac{2}{s}B_{mn} - D_{mn}\right]e^{\frac{-sR}{2c}} + C_{mn}, \tag{21}$$

$$\mathbf{M}_{2,mn,k} = sA_{mn}\phi_{k,k-1}\left(sR/c\right), \quad k = 0,1,2,\cdots,N_t \tag{22}$$

$$\mathbf{M}_{3,mn,k} = \frac{2}{s}B_{mn}\phi_{k,k-1}\left(sR/c\right), \quad k = 0,1,2,\cdots,N_t \tag{23}$$

$$\mathbf{M}_{4,mn,k} = D_{mn}\phi_{k,k-1}\left(sR/c\right), \quad k = 0,1,2,\cdots,N_t \tag{24}$$

$$\phi_{k,k-1}\left(sR/c\right) = \begin{cases} e^{\frac{-sR}{2c}} & k = 0 \\ \varphi_k\left(sR/c\right) - \varphi_{k-1}\left(sR/c\right) & k \in \left[1, N_t\right]. \end{cases} \tag{25}$$

$\mathbf{M}_1$ refers to the impedance matrix at present degree. $\mathbf{M}_2$, $\mathbf{M}_3$, $\mathbf{M}_4$ represent the differential, integral and normal term of each degree, respectively. There are totally $N_s \times N_s \times \left[1 + 3\times\left(1 + N_t\right)\right]$ matrices to be stored. Because each of these matrices is handled in the same manner, we only take $\mathbf{M}_{1,mn}$ as an example in the rest of this section to introduce the combination of ACA-SVD with MOD method.

The ACA-SVD algorithm needs a multilevel grouping of the computational domain. The grouping pattern based on octree, which is popularly used in the multilevel fast multipole algorithm [19-21] is adopted in this paper. The coupling of self and neighboring groups at some level are computed directly and the whole submatrices with elements (21) are stored. Whereas, the submatrices associated with two well-separated groups are evaluated and stored with ACA-SVD algorithm. Considering two well-separated groups, the interaction between them will lead to a rank-deficient submatrix $\mathbf{M}_1^{p\times q}$, where p and q are the number of basis functions in the two groups, the superscript $p\times q$ represents the size of the submatrix, the digit 1 of the subscript refers to the kind of matrix $\mathbf{M}_1$. We firstly use ACA algorithm to approximate submatrix $\mathbf{M}_1^{p\times q}$ with $\left[\mathbf{U}_1^{p\times r_1}\right]\left[\mathbf{W}_1^{q\times r_1}\right]^T$, where r_1 is the rank of matrix $\mathbf{M}_1^{p\times q}$, $[\ ]$ notation is used to represent a column matrix. With moderately grouping in the application, the rank r_1 is always smaller than p and q [14]. Because the columns of the matrices $\left[\mathbf{U}_1^{p\times r_1}\right]$ and $\left[\mathbf{W}_1^{q\times r_1}\right]$ generated by ACA are usually not orthogonal, we can use SVD algorithm to further remove the redundancies contained in them. Assume that the QR decompositions of them are

$$\left[\mathbf{U}_1^{p\times r_1}\right] = \left[\mathbf{Q}_u^{p\times r_1}\right]\left[\mathbf{R}_u^{r_1\times r_1}\right], \tag{26}$$

$$\left[\mathbf{W}_1^{q\times r_1}\right] = \left[\mathbf{Q}_w^{q\times r_1}\right]\left[\mathbf{R}_w^{r_1\times r_1}\right]. \tag{27}$$

The product of matrices $\left[\mathbf{R}_u^{r_1\times r_1}\right]$ and $\left[\mathbf{R}_w^{r_1\times r_1}\right]^T$ is then decomposed by SVD algorithm:

$$\left[\mathbf{R}_u^{r_1\times r_1}\right]\left[\mathbf{R}_w^{r_1\times r_1}\right]^T = \left[\tilde{\mathbf{U}}^{r_1\times r_1}\right]\left[\tilde{\Sigma}^{r_1\times r_1}\right]\left[\tilde{\mathbf{V}}^{r_1\times r_1}\right]^T \tag{28}$$

Discarding the columns of $\left[\tilde{\mathbf{U}}^{r_1\times r_1}\right]$ and $\left[\tilde{\mathbf{V}}^{r_1\times r_1}\right]^T$ corresponding to negligible singular values, we can obtain

$$\left[\mathbf{R}_u^{r_1\times r_1}\right]\left[\mathbf{R}_w^{r_1\times r_1}\right]^T = \left[\bar{\mathbf{U}}^{r_1\times \bar{r}_1}\right]\left[\bar{\Sigma}^{\bar{r}_1\times \bar{r}_1}\right]\left[\bar{\mathbf{V}}^{r_1\times \bar{r}_1}\right]^T, \tag{29}$$

where the upper horizontal bar denotes an approximate version of the corresponding matrix.

Finally, the decomposition of matrix $\left[\mathbf{M}_1^{p\times q}\right]$ can be rewritten as

$$\begin{aligned}\left[\mathbf{M}_1^{p\times q}\right] &= \left[\mathbf{U}_1^{p\times r_1}\right]\left[\mathbf{W}_1^{q\times r_1}\right]^T \\ &= \left[\mathbf{Q}_u^{p\times r_1}\right]\left[\bar{\mathbf{U}}^{r_1\times\bar{r}_1}\right]\left[\bar{\Sigma}^{\bar{r}_1\times\bar{r}_1}\right]\left[\bar{\mathbf{V}}^{r_1\times\bar{r}_1}\right]^T\left[\mathbf{Q}_{\mathrm{w}}^{q\times r_1}\right]^T \\ &= \left[\mathbf{X}_1^{p\times\bar{r}_1}\right]\left[\mathbf{Y}_1^{\bar{r}_1\times q}\right],\end{aligned} \tag{30}$$

where

$$\left[\mathbf{X}_1^{p\times\bar{r}_1}\right] = \left[\mathbf{Q}_u^{p\times r_1}\right]\left[\bar{\mathbf{U}}^{r_1\times\bar{r}_1}\right]\left[\bar{\Sigma}^{\bar{r}_1\times\bar{r}_1}\right], \tag{31}$$

$$\left[\mathbf{Y}_1^{\bar{r}_1\times q}\right] = \left[\bar{\mathbf{V}}^{r_1\times\bar{r}_1}\right]^T\left[\mathbf{Q}_{\mathrm{w}}^{q\times r_1}\right]^T. \tag{32}$$

Because $\bar{r}_1 < r_1$, the storage requirement of $\left[\mathbf{X}_1^{p\times\bar{r}_1}\right]$ and $\left[\mathbf{Y}_1^{\bar{r}_1\times q}\right]$ is smaller than $\left[\mathbf{U}_1^{p\times r_1}\right]$ and $\left[\mathbf{W}_1^{q\times r_1}\right]^T$.

It is obvious that if we compute the submatrices $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, $\mathbf{M}_4$ of two well-separated groups directly at each degree, we will need to store $\mathrm{p}\times\mathrm{q}\times\left[1+3\times\left(1+N_t\right)\right]$ elements. However, if we evaluate the submatrix degree by degree with ACA-SVD algorithm, we only need to store $(p+q)\bar{r}_1+\sum_{m=2}^{4}\sum_{k=0}^{N_t}(p+q)\bar{r}_{\mathrm{m},k}$ elements. $\bar{r}_{\mathrm{m},k}$ is usually smaller than p and q, so the application of ACA-SVD to MOD can greatly reduce the memory requirement.

IV. NUMERICAL RESULTS

Several numerical experiments are carried out to validate the accuracy and efficiency of the proposed method. The combination factor α of TD-CFIE is set to be 0.5 for closed bodies. ACA terminating tolerance is set to be 10^{-3} unless noted otherwise. The temporal scaling factor of the weighted Laguerre polynomials is 1.2×10^9. All experiments are performed on 2.67GHz CPU and 48 GB RAM.

The incident pulse used in all the following examples is a modulated Gaussian pulse which is defined as

$$\mathbf{E}^i\left(\mathbf{r},t\right) = \hat{e}_x\cos\left(2\pi f_0\tau\right)\exp\left[-\left(\tau-t_p\right)^2\big/2\sigma^2\right], \tag{33}$$

where f_0 is the central frequency, $\tau = t-\hat{k}\bullet\mathbf{r}/c$, $\hat{k}$ refers to the propagation direction of incident wave and is along z direction in our examples, $\hat{e}_x$ is the unit vector along x axis and represents the polarization of the incident wave, $t_p = 3.5\sigma$, $\sigma = 6/\left(2\pi f_{bw}\right)$, f_{bw} denotes the bandwidth of the incident pulse.

A. Accuracy

Three examples are given to show the accuracy of the proposed method. As the first example, we consider a PEC plate with 1.4m side length, which lies in the *xoy* plane and is centered at the origin. The problem is discretized into 1044 edges. 50 temporal basis functions and single level ACA-SVD are used. The modulated Gaussian pulse parameter is chosen as $f_0 = 150$MHz and $f_{bw} = 300$MHz. Based on the proposed method, equation (17) is solved to obtain the current coefficients $\left\{J_{n,i}\right\}$. Then, the current at the n*th* edge in time domain can be calculated as

$$\mathrm{J}_n\left(t\right) = \sum_{j=0}^{N_t}\mathrm{J}_{n,j}\varphi_j\left(\bar{t}\right). \tag{34}$$

Finally, current at a randomly chosen inner edge is compared with the results obtained by inverse discrete Fourier transform (IDFT) of the frequency domain data as shown in Fig. 1. The frequency domain data is computed by method of moments (MoM). The two endpoints of the inner edge are (0.1499, -0.0037, 0) and (0.1531, -0.0856, 0).

A PEC cylinder with radius of 0.5m and height of 3m is analyzed as the second example. The problem is discretized into 5856 edges. 90 temporal basis functions and two levels ACA-SVD are adopted. The modulated Gaussian pulse parameter is the same as that of the first example. After the current coefficients are obtained, the time domain far-field data is computed and transformed into frequency domain. Then it is normalized by the incident wave. Finally, the wideband bistatic RCS can be obtained. Results at several frequencies are given and compared with that of MoM. It can be observed in Fig. 2 that RCS data obtained from MOD-ACA-SVD agrees well with that of MoM. In order to present the influence of terminating tolerance to the accuracy of proposed method, the relative error of wideband bistatic RCS with the terminating tolerance of 10^{-1}, 10^{-2}, 10^{-3} is shown in Fig. 3. The relative error of RCS at certain observation angle is defined as

$$\text{Relative error} = \left|\frac{RCS^{MOD-ACA-SVD}-RCS^{\mathrm{Ref}}}{RCS^{\mathrm{Ref}}}\right|\times100\%,$$

where $RCS^{MOD-ACA-SVD}$ is the RCS obtained by the proposed method and RCS^{Ref} is the reference results obtained by traditional MOD method. It is acceptable to set the terminating tolerance to be

10^{-3} where the maximum relative error is below 5%.

The third example is a PEC ogive modeled with 8463 edges. The maximum size in the x, y and z directions are 3.81m, 0.76m and 0.76m. Please refer to [22] for detail information about this model. 125 temporal basis functions and three levels ACA-SVD are employed. The modulated Gaussian pulse parameter is chosen as f_0 = 225MHz and f_{bw} = 450MHz in this example. Results at several frequencies are given and compared with that of MoM. Good agreement can be achieved as shown in Fig. 4.

The memory requirement of the proposed method is given in Table 1 and compared with that of the traditional MOD method, which is computed directly by using the formula $N_s \times N_s \times \left[1+3\times\left(1+N_t\right)\right]\times 4/1024^3$. The total solution time of the proposed method and MOD method are shown in Table 2. For the example of PEC ogive, the memory requirement of MOD exceeds the available memory and the total solution time can not be obtained. But it can still be computed by the proposed method. So it can be concluded that the traditional MOD method is less useful for large problems though it spends less time than proposed method for small problems.

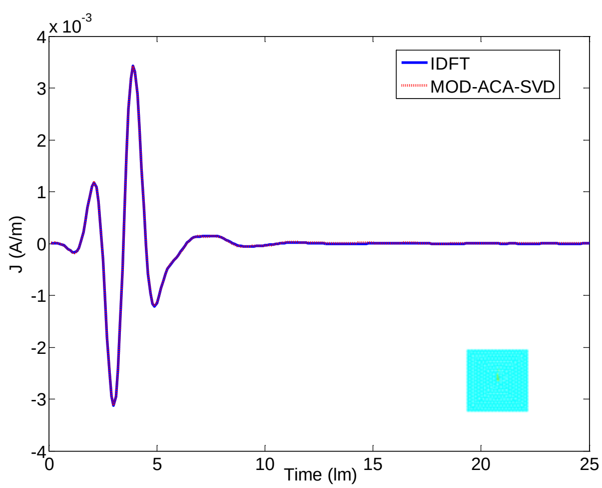

Fig. 1. The current at a randomly chosen inner edge compared with the results obtained by IDFT of the frequency domain data. The unit lm represents light meter and 1 (lm) = 1/light speed in free space (s).

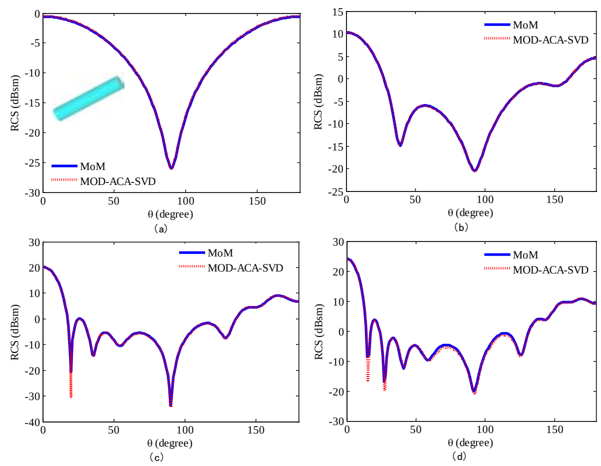

Fig. 2. Bistatic RCS results of a PEC cylinder when Φ = 0: (a) f = 40MHz, (b) f = 110MHz, (c) f = 180MHz, (d) f = 260MHz.

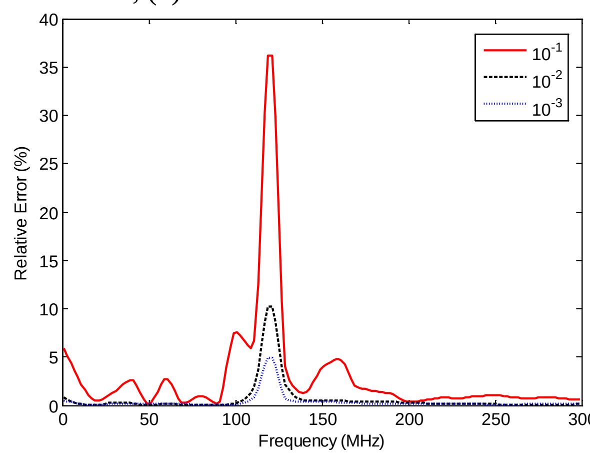

Fig. 3. Relative error of Bistatic RCS with three sets of terminating tolerance when Φ = 0, θ = 0.

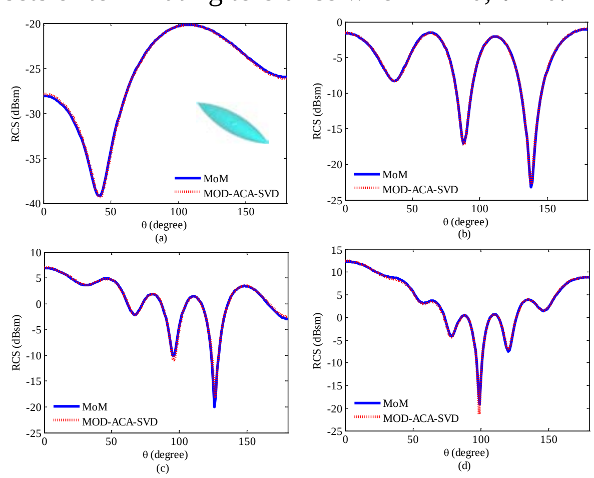

Fig. 4. Bistatic RCS results of a PEC ogive when Φ = 0: (a) f = 45MHz, (b) f = 150MHz, (c) f = 300MHz, (d) f = 390MHz.

Table 1: Memory requirement of three examples

Examples	Memory Requirement (GB)	
	MOD-ACA-SVD	MOD
Plate	0.288	0.428
Cylinder	14.85	35.84
Ogive	32.66	103.55

Table 2: Total solution time of three examples

Examples	Total Solution Time (s)	
	MOD-ACA-SVD	MOD
Plate	410	39
Cylinder	46,040	5,242
Ogive	148,737	–

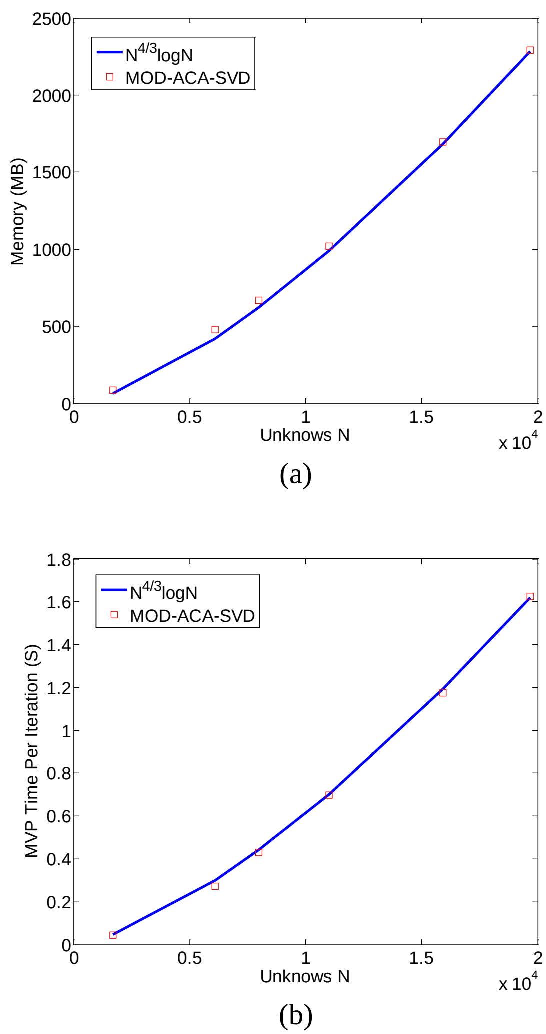

Fig. 5. Complexity of MOD-ACA-SVD algorithm for a PEC sphere example: (a) memory, (b) MVP time per iteration.

B. Efficiency

In this section, the numerical complexity of the proposed method is explored. A metallic sphere of radius 1 meter centered at the origin and is meshed with different number of edges according to different frequency band of the modulated Gaussian pulse. The highest frequency of the frequency band is increased from 200MHz to 667MHz. The degree of temporal basis functions is chosen to be 0*th* to 3*th* for the sake of available memory. Both the memory requirement and MVP time per iteration with 1692, 6102, 7989, 10998, 15918, and 19674 unknowns are shown in Fig. 5. It can be observed that the complexity of proposed method scales as $N_s^{4/3} \log N_s$ for moderate sized problems.

V. CONCLUSION

The combination of marching-on-in-degree solver of time domain integral equation and adaptive cross approximation algorithm with singular value decomposition postcompression is achieved in this paper. The impedance matrices of each degree related to well-separated groups are compressed by ACA-SVD algorithm. Numerical results show that the method proposed in this paper is very stable and accurate. Moreover, it can greatly reduce the memory requirement and matrix-vector product (MVP) time per iteration of MOD method.

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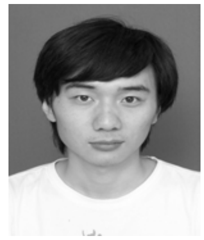

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