Computer Simulations of Microwave Heating with Coupled Electromagnetic, Thermal, and Kinetic Phenomena

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Abstract — A new functionality for the hybrid electromagnetic – thermodynamic simulations of microwave heating scenarios in the time domain is presented. The new functionality consists in the modelling of the heated load movement throughout the simulated process. Various types of the movement trajectories have been allowed for, including load rotation and load translation along user-defined piecewise-linear paths. Key aspects of this novel approach are addressed and justified from the computational electromagnetics perspective. Multiphysics simulation results are presented for a fictitious and a commercially available microwave oven.

Index Terms — Coupled electromagnetic and thermal simulation, FDTD methods, hybrid modelling, microwave heating, multiphysics modelling.

I. INTRODUCTION

Over the last decade, electromagnetic simulation tools originally designed for telecommunication purposes have made their way into the microwave power industry. The fundamental relevance of those tools for microwave heating applications has been confirmed [1]. Moreover, the purely electromagnetic approach has been extended with other features adequate for the high-power processing, such as automatic variation of material parameters versus temperature [2] or bilateral coupling with the heat transfer equations [3].

A well known problem of microwave heating is spatial non-uniformity of final temperature patterns, which results from non-uniform modal electromagnetic field patterns characteristic for most applicators. In many low-cost domestic microwave ovens, one may observe so-called "hot spots", where the food is overcooked hence losing its nutritional and sensory qualities, as well as "cold spots", where the temperature of bacterial inactivation is not reached. A popular solution for fighting those problems and homogenising the temperature patterns consists in the movement of the heated objects during the microwave process. Examples include linear translation on conveyor belts in industrial tunnel ovens or rotation on the glass plate in household ovens.

In early works on computer simulations of microwave heating, e.g. [4], the effect of load rotation was studied at the post-processing stage. First, independent electromagnetic simulations were run for several angular positions of the load.
Subsequently, the dissipated power patterns for those positions were superimposed and averaged. On their basis, the final temperature was approximated. Such an approach is rigorous if the load parameters remain constant; it provides a physically consistent approximation if the load parameters vary slowly with temperature. However, dielectric and thermal parameters of typical foods are strongly temperature dependent and undergo drastic changes around phase change points. For example, relative permittivity and electric conductivity of beef change from 4.9 and 0.064 S/m, respectively, in the frozen state, to 48.2 and 2.194 S/m at the room temperature. The post-processing approach to the load movement misses the non-linear effects due to the material changes and may end up with physically inconsistent results.

In a recent work [5], the effect of load rotation has been embedded into the electromagnetic-thermal simulation loop. Material parameters are automatically updated at each consecutive position, as a function of the current temperature pattern. This facilitates accurate prediction of the final temperature, even for strongly non-linear problems.

In this work, we extend the methodology of [5] to arbitrary movements of the load. Specifically, the load is translated along piecewise-linear trajectories. For full flexibility, consecutive positions along the trajectory as well as residual times at each position are defined by the user in external text-files. At the conceptual level, the algorithm has been presented at the recent ACES Symposium [6].

In further extension to [5] and [6], which focus on predicting temperature patterns in domestic oven scenarios, our focus herein is on the computational electromagnetics aspects. In particular, we show how a continuous non-linear process is approximated with a discrete parametric one. We also demonstrate a speed-up of the electromagnetic calculations part achieved by resuming those calculations from the previous steady state, rather than the initial conditions. The elaborated algorithms are implemented in the finite-difference time-domain (FDTD) electromagnetic software environment of [7].

Due to the lack of household ovens enabling translation of the object along an arbitrary trajectory, those studies are limited to numerical investigations without further comparison with real measurements.

II. FDTD ALGORITHM OF COUPLED ELECTROMAGNETIC AND THERMAL SIMULATIONS WITH LOAD MOVEMENT

Figure 1 shows the flow chart of the hybrid algorithm, which bilaterally couples the electromagnetic and thermodynamic computations. Strictly speaking, the electromagnetic process is non-linear, as the media
parametres continuously change in response to the dissipated power and increasing temperature. However, we can parameterise the problem taking advantage of the different time-scales characteristic for the electromagnetic and thermal phenomena.

Imagine we wish to raise temperature of the heated beef load by a small but observable amount, e.g. 5 °C. According to Table 1, this requires delivering approximately 15 kJ/kg at the frozen stage (-20°C to -15°C) or approximately 24 kJ/kg at the thawed stage (50°C to 55°C). For a typical load that weighs, for example, 500 g, we would need to deliver about 8-12 kJ. Since domestic ovens operate at below 1 kW, this entails at least 8-12 s of processing.

Industrial installations operate at a few hundred kilowatts but their loads are proportionally bigger, tens or hundreds of kilograms. Thus in any case, significant changes of thermally-dependent material parametres occur in a matter of seconds or tens of seconds. Heat flow effects proceed on a similar time scale.

On the other hand, the operating frequency of domestic ovens is 2.45 GHz, while industrial appliances work at 2.45 GHz or 915 MHz. This corresponds to the period of 0.4 ns or 1.1 ns, respectively, and the EM FDTD analysis proceeds with the time step not exceeding 50 ps. In view of the above contrast in the time scales, there is no need to solve a non-linear problem at each FDTD time-step. Instead, the continuous problem can be discretised into a sequence of discrete heating steps. We shall use the symbol $\Delta \tau$ to denote the duration of each such heating time step, $\Delta \tau$ being of the order of seconds.

Each heating step starts with the EM FDTD analysis, with load parametres adjusted to the previously reached temperature patterns - hence spatially inhomogeneous but constant in time during this EM analysis. It is important to stress that each consecutive EM analysis starts from the previous EM steady state, and not from the initial conditions of zero EM fields. As computational experiments show, such an approach allows reducing the number of FDTD iterations needed to reach a new steady state by at least a factor of 10. Since the EM FDTD analysis is the most time-consuming part of the developed hybrid system, overall efficiency is also improved by nearly a factor of 10.

When the EM steady state is reached, the time-averaged value of power dissipated within each FDTD cell over the volume of the heated object is calculated. Temperature rise is then predicted under the assumption that the heating is purely electromagnetic (with no heat flow effects) and proceeds with the constant dissipated power pattern for the $\Delta \tau$ duration. The enthalpy density is thus updated by the following predictor equation:

$$H_p^{n+1}(x,y,z) = H_p^n(x,y,z) + \frac{P_v^n(x,y,z) \cdot \Delta \tau}{W(x,y,z)}, \quad (1)$$

where $H_p$ is enthalpy density in J/m$^3$ at the previous heating time step, $H_p^{n+1}$ - predicted enthalpy density at the current step, $P_v^{n+1}$ - time-averaged dissipated power in each cell in W, $W$ - volume of the cell in m$^3$, and $\Delta \tau$ in s. A new temperature pattern $T_p^{n+1}$ is now also predicted, based on the enthalpy pattern and the material parametres provided by the user for each temperature-dependent medium (see Table 1).

The new geometry due to the load movement is then read. It is assumed that the temperature and enthalpy patterns follow the load, while the electromagnetic field patterns remain bound to the cavity. This assumption is physically viable since enthalpy is bound to materials. On the other hand, microwave power applicators are designed so as to work with various types of loads. This requires that their modal patterns be cavity-controlled, with load-focussing effects suppressed as much as possible.

Now heat diffusion effects are considered. The enthalpy $H$ and temperature $T$ patterns are corrected by solving the heat transfer equation:

$$\frac{\partial H(x,y,z)}{\partial t} = \nabla (k(T) \nabla T), \quad (2)$$

where $k$ is thermal conductivity expressed in W/(m°C). The dependence between temperature and enthalpy can be non-linear and it is read from the material parametre files (see Table 1). The initial conditions for eq.(2) are $H_p^{n+1}$ and $T_p^{n+1}$ predicted by eq.(1). The diffusion time is $\Delta \tau$. The solution is accomplished by the thermal FDTD algorithm operating on the same mesh as the EM FDTD algorithm. Such an approach avoids numerical diffusion errors, which occur if additional interpolations between the EM and thermal meshes are required [8]. The results of solving eq.(2) are the corrected enthalpy $H^{n+1}$ and...
temperature $T^{n+1}$ patterns, fed back to the EM solver.

Before the EM FDTD algorithm proceeds to calculate the next EM steady state, it updates the dielectric properties of the heated load (or more precisely, of each FDTD cell within the load) based on the temperature-dependent characteristics provided in the material files (see Table 1). At this point, a numerical challenge arises. Since the material parameters change between the two EM analyses, care must be taken not to induce numerical parasites violating the Gauss law. It has been shown [9] that such modes with non-zero divergence can exist on the FDTD mesh and remain static. They do not interact with the propagating microwave solutions but they would contaminate the calculated power envelopes and thus the predicted temperature. The procedure proposed in [10] and based on the $E, H$ to $D, B$ conversion is therefore adapted to our coupled system before each media parameters update, in order to suppress such numerical artifacts.

**III. NUMERICAL MODELS OF THE INVESTIGATED CASES**

Two test cases are considered: one based on a fictitious oven cavity and the other being a numerical replica of the Whirlpool Max oven available on the market.

The first cavity (upper in Fig. 2) is 267 mm in width ($x$-direction in Fig. 2), 270 mm in length ($y$-direction in Fig. 2), and 188 mm in height ($z$-direction in Fig. 2). The feeding waveguide of 78 mm in width ($y$-direction in Fig. 2), 18 mm in height ($x$-direction in Fig. 2), and 80 mm in length ($z$-direction in Fig. 2) launches the fundamental TE01 waveguide mode, at the ISM frequency of 2.45 GHz.

The plate of 227 mm diametre and 6 mm height is located centrally in the cavity, 15 mm above its floor. The material of the plate is lossless and its relative permittivity is equal to 6. The waveguide is excited by the magnetron of 650 W time-averaged available power.

The other oven is shown in the lower part of Fig. 2. The main cavity is semi-cylindrical in shape. Its sophisticated feeding system including two apertures separated by a septum is modelled in detail.

The heated object of a cylindrical shape, 100 mm diametre and 30 mm height possesses electromagnetic and thermal properties of beef. Those properties are listed in Table 1 as a function of temperature. The values of enthalpy density (column 2) are taken from [11]. The dielectric properties (columns 3 and 4) are based on measurements published in [12]. Heat conductivity (column 5) is based on the measurements performed by the authors for raw beef at room temperature. Adiabatic boundary conditions (zero heat flux) are set at the boundaries between the heated element and air as well as the heated element and the plate.

Within this work we do not consider the influence of convection and radiation on temperature of the heated object. However, as shown in [13], the amount of emitted heat is proportional to the fourth power of temperature and thus it has a strong impact on thermal behaviour of the object only when heated to very high temperatures.

**IV. RESULTS OF MULTIPHYSICS COMPUTATIONS**

Three cases regarding the object’s kinetics during its electromagnetic heating are considered: rotation of the plate is blocked and the object
remains static at the centre of the plate (Fig. 3), the plate rotates at 5 r.p.m and the object moves along a circular trajectory of 120 mm diametre (Fig. 4), the object moves along a piecewise linear trajectory (Fig. 5).

Table 1: Dielectric and thermal properties of beef

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Enthalpy density [J/kg]</th>
<th>Relative permittivity [-]</th>
<th>Electric conductivity [S/m]</th>
<th>Heat conductivity [W/(m°C)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>0.0</td>
<td>4.9</td>
<td>0.064</td>
<td>0.69</td>
</tr>
<tr>
<td>-15</td>
<td>14840.0</td>
<td>5.5</td>
<td>0.093</td>
<td>0.69</td>
</tr>
<tr>
<td>-10</td>
<td>36464.0</td>
<td>6.1</td>
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<td>0.69</td>
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<tr>
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</tr>
<tr>
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<td>2.440</td>
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</tr>
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<tr>
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<td>80</td>
<td>692074.0</td>
<td>41.7</td>
<td>1.908</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Fig. 3. Static object located at the centre of the plate.

Fig. 4. Object located off-centre, rotating with constant speed.

Fig. 5. Object moving along a piecewise-linear trajectory.

The total heating time for each of the cases is set to 180 sec. In the case with rotation (Fig. 4), the continuous motion of the object is approximated by a set of 9 angular positions, with 40 degree step. In the case with the piecewise linear trajectory (Fig. 5), the heating sub-time at each position is 5 sec. and the trip along the trajectory is repeated until the total heating time is reached.
Figure 7 shows evolution of temperature over time at the centre of the load (Point 2 in Fig. 6) in the fictitious oven. Results for other points have been reported in [6]. Changes of the object’s trajectory significantly influence the temperature at Point 2. When the load is static, maximum temperature at Point 2 is reached after 40 s. This maximum temperature is equal 128.5 °C and subsequently decreases due to heat diffusion.

When the object moves along the piecewise-linear trajectory, the maximum temperature at Point 2 is substantially lower (47.6 °C) and occurs later in time. When the object moves along the circular trajectory, the temperature at Point 2 increases very slowly and hardly exceeds 0 °C over the simulated 180 sec heating time.

Table 3 shows the final temperature patterns for the three investigated cases in the fictitious oven. The patterns were taken across the 2D cuts defined in the first row of Table 3. As expected, the highest non-uniformity of the final temperature across the load results from the heating of the static object. This is because the heat transfer capabilities of beef are too weak to compensate for the inherent non-uniformity of the microwave dissipated power patterns in the considered scenario. The non-uniformities decrease when the object rotates in the cavity, as it is the case in most domestic ovens on the European market. However, the central cold spot and edge overheating effects along the object circumference cannot be compensated by the rotation.

Table 2: Temperature patterns across the $xy$, $zx$, $yz$ planes after 180 sec of heating in Whirlpool Max

<table>
<thead>
<tr>
<th>Plane</th>
<th>Rotation</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZX-plane</td>
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<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>YZ-plane</td>
<td><img src="image3.png" alt="Image" /></td>
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<tr>
<td>XY-plane</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>160 °C</th>
<th>-10 °C</th>
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<td></td>
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<td></td>
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</table>
The most uniform final temperature pattern is produced when the object moves along the piecewise-linear trajectory. Simulations in the other oven (Table 2) also support this claim. While the rotating load suffers from the edge overheating, the translation leads to more uniform radial temperature distribution.

The translation trajectory in this experiment has been proposed *ad hoc* for the purpose of demonstrating the new computational feature and it does not necessarily give the best possible performance. However, the results confirm that introducing load movements other than rotation is a promising way towards designing new microwave cavities with enhanced heating uniformity.

**V. CONCLUSIONS**

The coupled electromagnetic-thermodynamic simulations toolkit based on the concepts summarized in [1-5] and implemented in [7] has been extended with new capabilities pertaining to different load kinetics during the heating process. In particular, load movement along piecewise-linear trajectories has been facilitated. For maximum flexibility, the trajectory as well as the speed of load movement along each segment are defined by the user in text files. Specific numerical challenges related to the parameterisation of the non-linear electromagnetic problem, suppressing numerical parasitic modes in the parameterised EM FDTD algorithm, and accelerating the overall analysis have been addressed.

Computational experiments with the *ad hoc* constructed piecewise-linear trajectory show its superiority in terms of the final temperature patterns. The resulting electromagnetic-thermodynamic-kinetic simulation methodology becomes a convenient tool for the design of new microwave heating appliances.

**ACKNOWLEDGEMENT**

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<table>
<thead>
<tr>
<th></th>
<th>$xy$-plane</th>
<th>$zx$-plane</th>
<th>$yz$-plane</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td>160 °C</td>
</tr>
<tr>
<td>Rotation</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td>-10 °C</td>
</tr>
<tr>
<td>Translation</td>
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<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
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REFERENCES


Malgorzata Celuch received the International Baccalaureate (honours) at the United World College of the Atlantic, UK. She then graduated from the Warsaw University of Technology, receiving M.Sc. (honours) and Ph.D. (honours) in 1988 and 1996, respectively. Since 1996 she has been Assistant Professor at the Warsaw University of Technology (www.ire.pw.edu.pl). Her main fields of research are electromagnetic modelling of microwave circuits and numerical methods for computational electromagnetics, including conformal FDTD methods, new applications of FDTD with enthalpy-dependent material parameters, and frequency-domain parameter extraction from FDTD simulations.

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