Null Broadening and Sidelobe Control Algorithm via Multi-Parametric Quadratic Programming for Robust Adaptive Beamforming

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Abstract **─** Adaptive beamforming algorithm can automatically optimize the array pattern by adjusting the elemental control weights until a prescribed objective function is satisfied. Unfortunately, it is possible that the mismatch occurs between adaptive weights and data, due to the perturbation of the interference location when the antenna platform vibrates or interference moves quickly. Besides, the traditional beamformers may have unacceptably high sidelobes when few samples are available. To solve these problems, an effective robust adaptive beamforming method is presented. In the proposed method, firstly, a tapered covariance matrix is constructed to broaden the width of nulls for interference signal sources. Secondly, multiple additional quadratic inequality constraints outside the mainlobe beampattern area are used to guarantee that the sidelobe level is strictly lower than the prescribed threshold value. Finally, the beamforming optimization problem is formulated as a multi-parametric quadratic programming problem, such that the optimal weight vector can be easily obtained by real-valued computation. Simulation results are shown to demonstrate the efficiency of the proposed approach.

Index Terms **─** Covariance Matrix Taper (CMT), multi-parametric Quadratic Programming (mp-QP), null broadening, robust adaptive beamforming and sidelobe control.

I. INTRODUCTION

Adaptive beamforming has been found in numerous applications from radar, sonar, wireless communications, seismology and microphone arrays. One of the most popular approaches to adaptive beamforming is the so-called Minimum Variance Distortionless Response (MVDR) processor, which minimizes the array output power while maintaining a distortionless mainlobe response toward the desired signal [1]. However, most of the conventional adaptive beamformers, such as MVDR, etc., may have unacceptably low and narrow null level in the interference direction or high sidelobes in the case of low sample support. In adaptive array systems, these may lead to significant performance degradation in the case of unexpected interference signals.

Several approaches of null broadening technique have been proposed. For example, the null broadening technique originally [2-3] is developed for robust beamforming. It is generalized by the concept of a "Covariance Matrix Taper (CMT)" [4]. A multi-parametric quadratic programming method is presented to control the null level of adaptive antenna array [5]. In the proposed method, the optimal weight vector can be easily obtained by real-valued computation. Unfortunately the sidelobe level is not controlled efficiently. A null broadening technique based on reduced rank conjugate gradient algorithm is proposed in [6]. It can obtain better performance

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even if few samples are available. The space-time averaging techniques and rotation techniques of the steering vectors are utilized in [7] to improve the performance of null level, so that it can provide increased robustness against the mismatch problem, as well as control over the sidelobe level. A robust beamforming control method based on semidefinite programming is presented to broaden null of adaptive antenna array [8]. The presented method can provide an improved robustness against the interference angle shaking and suppress the interference signals. An effective approach based on genetic algorithm is presented in [9]. In order to minimize the total output power and place nulls in the jammers, the presented approach exploits genetic algorithm to adjust some of the least significant bits of the beam steering phase shifters and amplitude weights. In [10], particle swarm optimization technique is used to obtain optimal null levels for the symmetric linear antenna array. By the linearization of the transmit model with Taylor expansion, a fast broad null beamforming technology [11] is presented and the application of the broad null transmitting beamforming technology for the radar in anti-ARM battle is studied. A null steering beamforming algorithm is proposed to cancel unwanted signals by steering nulls of the pattern in the direction of high interference without affecting the main beam [12].

To control sidelobe, several approaches have been proposed [13-16]. A modified MVDR beamformer is presented by multiple additional quadratic inequality constraints outside the mainlobe beampattern area [13]. These constraints can guarantee that the beampattern sidelobe level remains lower than a certain prescribed value. An improved adaptive beamforming technique [14] is proposed by an adaptive dispersion invasive weed optimization. It can not only provide sufficient steering ability regarding the mainlobe and the nulls, but also work faster than the particle swarm optimization. An iterative beamforming method is proposed for sensor array with arbitrary geometry and element directivity [15]. By solving the linearly constrained least squares problem, it can effectively control the sidelobe level. Making use of multi-linear constrained minimum variance repetitiously, a low sidelobe beamforming method is presented in [16]. By searching the previously formed beampattern, the location of the highest sidelobe is found and the corresponding direction vector is added to the constrained conditions of multi-linear constrained minimum variance algorithm to receive the new weight value.

In this paper, an effective adaptive beamforming method is proposed to resolve null broadening and sidelobe control problem. Firstly, an approximation of sinc function is used to broaden null. It can be regarded as adding the coherent signals to each signal source but with two different directions, and it can place nulls in a certain range of angles instead of certain points. Secondly, multiple quadratic inequality constraints outside the mainlobe beampattern area are used to control sidelobe. These constraints can guarantee that the beampattern sidelobe level remains lower than a certain prescribed value. Thirdly, the beamforming control problem is formulated as a multi-parametric Quadratic Programming (mp-QP) problem, such that the optimal weight vector can be obtained by real-valued computation.

This paper is organized as follows. section 2 briefly introduces the signal model and presents the MVDR solution. The methods of null broadening and sidelobe control are addressed in section 3. Section 4 gives the algorithm formulation. In section 5, simulation results are presented to verify the performance of the proposed approach. Section 6 concludes the paper.

II. BACKGROUND

Consider a Uniform Linear Array (ULA), which consists of M elements. The beamformer output of the ULA at time t is given by:

$$y(t) = \boldsymbol{w}^{\mathrm{H}} \boldsymbol{x}(t),$$

where $\boldsymbol{w} = [w_1, w_2, \ldots, w_M]^{\mathrm{T}}$ is the complex-valued weight vector. The superscripts $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote the transpose and conjugate transpose of a matrix, respectively. The $M \times 1$ vector of array observations $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^{\mathrm{T}}$ is given by:

$$\begin{aligned}\boldsymbol{x}(t) &= \boldsymbol{s}(t) + \boldsymbol{i}(t) + \boldsymbol{n}(t) \\ &= s(t)\boldsymbol{a}(\theta_0) + \sum_{j=1}^{J} i_j(t)\boldsymbol{a}(\theta_j) + \boldsymbol{n}(t),\end{aligned}$$

where J is the number of interference signals. $s(t)$ and $i_j(t)$ stand for the signal and interference, respectively. The signal and interference Directions of Arrival (DOAs) are θ_0 and

θ_j $(j = 1,\ldots,J)$, respectively, with corresponding steering vectors $\boldsymbol{a}(\theta_0)$ and $\boldsymbol{a}(\theta_j)$. $\boldsymbol{n}(t) = [n_1(t),\ldots,n_M(t)]^{\mathrm{T}}$ with $n_i(t)$ denoting the additive noise of the ith sensor.

Let $\boldsymbol{R}$ denote the $M \times M$ theoretical covariance matrix of the array snapshot vector. Assume that $\boldsymbol{R}$ is a positive definite matrix with the following form:

$$\boldsymbol{R} = \sigma_0^2 \boldsymbol{a}(\theta_0)\boldsymbol{a}^{\mathrm{H}}(\theta_0) + \sum_{j=1}^{J} \sigma_j^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^{\mathrm{H}}(\theta_j) + \sigma_n^2 \boldsymbol{I}_M,$$

where σ_0^2, σ_j^2 $(j = 1,\ldots,J)$ and σ_n^2 are the powers of the uncorrelated impinging signals $s(t)$, $i_j(t)$ and noise, respectively. $\boldsymbol{I}_M$ is the $M \times M$ identity matrix. The common formulation of the beamforming problem that leads to the MVDR beamformer is described below. First determine the $M \times 1$ vector $\boldsymbol{w}_0$ as the solution to the following linearly constrained quadratic problem,

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}} \boldsymbol{R} \boldsymbol{w} \quad \text{subject to } \boldsymbol{w}^{\mathrm{H}} \boldsymbol{a}(\theta_0) = 1. \tag{1}$$

Then the solution of equation (1) for this particular case can be given as:

$$w_{\mathrm{MVDR}} = \frac{\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_0)}{\boldsymbol{a}^{\mathrm{H}}(\theta_0)\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_0)}. \tag{2}$$

In practice, the exact covariance matrix is not available and is replaced by the sample covariance matrix $\hat{\boldsymbol{R}}$,

$$\hat{\boldsymbol{R}} = \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{x}(k)\boldsymbol{x}^{\mathrm{H}}(k), \tag{3}$$

where N denotes the number of snapshots.

III. ADAPTIVE BEAMFORMING WITH NULL BROADENING AND SIDELOBE CONTROL

In this section, we firstly introduce the null broadening method to widen the nulling extent and control the nulling level. Secondly, multiple quadratic inequality constraints are derived to control sidelobe level. Finally, the modified CMT-MVDR problem is given.

A. Null broadening

1. Null extent control: Assume that the narrowband interference signals impinging on the array are uncorrelated with each other as well as with the spatially white noise. According to [2], the terms in the covariance matrix $\boldsymbol{R}$ for a one-dimensional array are given as:

$$R_{mn} = N_n \delta(m,n) + \sum_j \sigma_j^2 e^{j\frac{2\pi}{\lambda}(x_m - x_n)u_j},$$

where x_m is the location of the mth element. N_n is the noise power in the nth channel and $\delta(m,n)$ is a Kronecker delta function. The sum is performed over all interference signals with averaged power σ_j^2 and direction cosines $u_j = \sin\theta_j$, for θ_j measured from the broadside. According to [2], we construct a cluster of q equal-power incoherent signals around each original interfering signal to produce a notch of width W in each of the interference directions. In this case, the additional sources can be summed in closed form, as geometric sum and can be written as:

$$\sum_{k=1}^{q} (\sigma_j^2 / q) e^{j\frac{2\pi}{\lambda}(x_m - x_n)(u_j + k\delta)} = \frac{\sin(q\Lambda_{mn})}{q\sin(\Lambda_{mn})} \sigma_j^2 e^{j\frac{2\pi}{\lambda}(x_m - x_n)u_j},$$

where $\Lambda_{mn} = \dfrac{\pi(x_m - x_n)\delta}{\lambda}$ and $\delta = \dfrac{W}{q-1}$. Since there is no angle dependence in the sinc function. A new covariance matrix term is given as $\tilde{R}_{mn} = R_{mn} \dfrac{\sin(q\Lambda_{mn})}{q\sin(\Lambda_{mn})}$. In matrix form, the CMT can be expressed as:

$$\tilde{\boldsymbol{R}} = \boldsymbol{R} \circ \boldsymbol{T}_{Mai}, \tag{4}$$

where "$\circ$" represents Hadamard product, that is multiplying the corresponding elements of the two matrixes and the form of matrix $\boldsymbol{T}_{Mai}$ is:

$$T_{mn} = (\sin(q\pi\delta(m-n)/2)) / (q\sin(\pi\delta(m-n)/2)).$$

2. Null level control: Assume that the interference signal arrivals the received array from the angle of incident $\theta_p, (p = 1,\ldots,P)$. When the interference moves quickly, it is possible that the mismatch occurs for adaptive weight and data, due to the perturbation of the interference location. Let $\Delta\theta$ denote the angle spread for the interference signal, which comes from θ_p. Let $\theta_k \in [\theta_p - \Delta\theta,$ $\theta_p + \Delta\theta]$ $(k = 1,\cdots,K)$ be chosen grid that approximates the angle spread area. To control the null level for the angle spread area $[\theta_p - \Delta\theta, \theta_p + \Delta\theta]$, we use the following multiple quadratic inequality constraints inside the angle spread area:

$$\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_k)\right|^2 \le \xi^2,\ \ k=1,\ldots,K, \tag{5}$$

where ξ^2 is the prescribed null level.

B. Sidelobe control

Let $\theta_l \in \Theta(l=1,\ldots,L)$ be a chosen grid that approximates the sidelobe beampattern areas Θ using a finite number of angles. To control the sidelobe level, we use the following multiple quadratic inequality constraints outside the mainlobe beampattern area:

$$\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_l)\right|^2 \le \varepsilon^2,\ \ k=1,\ldots,L, \tag{6}$$

where ε^2 is the prescribed sidelobe level.

C. The modified CMT-MVDR

Adding the constraints (5) and (6) to the MVDR beamforming problem (1) and using the new tapered covariance matrix (4) instead of the sample covariance matrix (3), we obtain the following modified CMT-MVDR problem:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w} \text{ subject to } \quad \boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_0)=1$$
$$\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_k)\right|^2 \le \xi^2 \tag{7}$$
$$\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_l)\right|^2 \le \varepsilon^2,$$

where $k=1,\ldots,K$ and $l=1,\ldots,L$.

Notice that $\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_k)\right|^2$ can directly determine the output power of antenna array at the interference direction θ_k (refer to equation (17)), thus, it can be viewed as the "directional gain" of the antenna array.

In the next section, we will convert this problem (7) to an mp-QP problem, such that the optimal weight vector is estimated by the real-valued computation.

IV. ALGORITHM FORMULATION

A. CMT-mp-QP MVDR

In this section, we present the multi-parametric programming problem for covariance matrix taper MVDR beamformer, named as CMT-mp-QP MVDR. As seen in problem (7), the data is in general complex valued. However, for convenience, we will work with real-valued data. To do so, a pre-processing path is taken prior to the beamforming operation.

Let

$$\boldsymbol{R}_1=\mathrm{Real}\{\tilde{\boldsymbol{R}}\},\quad \boldsymbol{R}_2=\mathrm{Imag}\{\tilde{\boldsymbol{R}}\},$$
$$\boldsymbol{w}_1=\mathrm{Real}\{\boldsymbol{w}\},\quad \boldsymbol{w}_2=\mathrm{Imag}\{\boldsymbol{w}\},$$
$$\boldsymbol{a}_{01}(\theta_0)=\mathrm{Real}\{\boldsymbol{a}(\theta_0)\},\quad \boldsymbol{a}_{02}(\theta_0)=\mathrm{Imag}\{\boldsymbol{a}(\theta_0)\},$$
$$\boldsymbol{a}_{k1}(\theta_k)=\mathrm{Real}\{\boldsymbol{a}(\theta_k)\},\quad \boldsymbol{a}_{k2}(\theta_k)=\mathrm{Imag}\{\boldsymbol{a}(\theta_k)\},$$
$$\boldsymbol{a}_{l1}(\theta_l)=\mathrm{Real}\{\boldsymbol{a}(\theta_l)\},\quad \boldsymbol{a}_{l2}(\theta_l)=\mathrm{Imag}\{\boldsymbol{a}(\theta_l)\},$$

where $k=1,\ldots,K$ and $l=1,\ldots,L$. $\mathrm{Real}\{\cdot\}$ and $\mathrm{Imag}\{\cdot\}$ stand for the real and imaginary part of a complex matrix or vector, respectively.

By simple algebra, the cost function $\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w}$ can be rewritten as:

$$\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w}=\mathrm{Real}\{\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w}\}+j\mathrm{Imge}\{\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w}\}$$
$$=\begin{pmatrix}\boldsymbol{w}_1\\ \boldsymbol{w}_2\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{R}_1 & -\boldsymbol{R}_2\\ \boldsymbol{R}_2 & \boldsymbol{R}_1\end{pmatrix}\begin{pmatrix}\boldsymbol{w}_1\\ \boldsymbol{w}_2\end{pmatrix}+$$
$$j\begin{pmatrix}\boldsymbol{w}_1\\ \boldsymbol{w}_2\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{R}_2 & \boldsymbol{R}_1\\ -\boldsymbol{R}_1 & \boldsymbol{R}_2\end{pmatrix}\begin{pmatrix}\boldsymbol{w}_1\\ \boldsymbol{w}_2\end{pmatrix}.$$

It is easy to know that $\mathrm{Imag}\{\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w}\}=0$ since $(\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w})^{\mathrm{H}}=\boldsymbol{w}^{\mathrm{H}}\tilde{\boldsymbol{R}}\boldsymbol{w}\in\mathbb{R}$ for $\forall\boldsymbol{w}\in\mathbb{C}^M$. Thus, the modified MVDR problem (7) can be reformulated as the following mp-QP problem:

$$\min_{\boldsymbol{z}}\frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{z} \ \text{ subject to } \boldsymbol{G}\boldsymbol{z}\le\boldsymbol{b}, \tag{8}$$

where the matrices and vectors in equation (8) have the following forms:

$$\boldsymbol{z}=[\boldsymbol{w}_1^{\mathrm{T}},\boldsymbol{w}_2^{\mathrm{T}}]\in\mathbb{R}^{2M},$$
$$\boldsymbol{H}=\begin{pmatrix}\boldsymbol{R}_1 & -\boldsymbol{R}_2\\ \boldsymbol{R}_2 & \boldsymbol{R}_1\end{pmatrix}\in\mathbb{R}^{2M\times 2M},$$
$$\boldsymbol{G}=[\boldsymbol{G}_0^{\mathrm{T}},\boldsymbol{G}_1^{\mathrm{T}},\ldots,\boldsymbol{G}_K^{\mathrm{T}},\boldsymbol{G}_1^{\mathrm{T}},\ldots,\boldsymbol{G}_L^{\mathrm{T}}]^{\mathrm{T}},$$
$$\boldsymbol{G}_k=[\boldsymbol{B}_k^T,-\boldsymbol{B}_k^T]^T,\quad \boldsymbol{G}_l=[\boldsymbol{B}_l^T,-\boldsymbol{B}_l^T]^T,$$
$$\boldsymbol{B}_k=\begin{bmatrix}\boldsymbol{a}_{k_1}^{\mathrm{T}}(\theta_k) & \boldsymbol{a}_{k_2}^{\mathrm{T}}(\theta_k)\\ \boldsymbol{a}_{k_2}^{\mathrm{T}}(\theta_k) & -\boldsymbol{a}_{k_1}^{\mathrm{T}}(\theta_k)\end{bmatrix},$$
$$\boldsymbol{B}_l=\begin{bmatrix}\boldsymbol{a}_{l_1}^{\mathrm{T}}(\theta_l) & \boldsymbol{a}_{l_2}^{\mathrm{T}}(\theta_l)\\ \boldsymbol{a}_{l_2}^{\mathrm{T}}(\theta_l) & -\boldsymbol{a}_{l_1}^{\mathrm{T}}(\theta_l)\end{bmatrix},$$
$$\boldsymbol{b}=[\boldsymbol{b}_0^{\mathrm{T}},\boldsymbol{b}_1^{\mathrm{T}},\ldots,\boldsymbol{b}_K^{\mathrm{T}},\boldsymbol{b}_1^{\mathrm{T}},\ldots,\boldsymbol{b}_L^{\mathrm{T}}]^{\mathrm{T}},\ \boldsymbol{b}_0=[1,0,-1,0]^{\mathrm{T}},$$
$$\boldsymbol{b}_k=[\sqrt{\lambda_\xi}\xi,\sqrt{1-\lambda_\xi}\xi,-\sqrt{\lambda_\xi}\xi,-\sqrt{1-\lambda_\xi}\xi]^{\mathrm{T}},$$
$$\boldsymbol{b}_l=[\sqrt{\lambda_\varepsilon}\varepsilon,\sqrt{1-\lambda_\varepsilon}\varepsilon,-\sqrt{\lambda_\varepsilon}\varepsilon,-\sqrt{1-\lambda_\varepsilon}\varepsilon]^{\mathrm{T}},$$

where $k = 1,\ldots,K$ and $l = 1,\ldots,L$. λ_ξ and $\lambda_\varepsilon \in [0,1]$. The matrix $\boldsymbol{H}$ is a positive definite matrix.

B. The optimal solution

As shown in [17] and [18], CMT-mp-QP problem (8) can be solved by applying the Karush-Kuhn-Tucker (KKT) conditions,

$$\boldsymbol{Hz} + \boldsymbol{G}^{\mathrm{T}}\boldsymbol{\lambda} = \boldsymbol{0}, \quad \boldsymbol{\lambda} \in \mathbb{R}^{4(K+1)}, \tag{9}$$

$$\lambda_i \boldsymbol{G}^i \boldsymbol{z} - \boldsymbol{b}^i = 0, \quad i = 1,\ldots,4(K+1), \tag{10}$$

$$\boldsymbol{\lambda} \geq \boldsymbol{0}, \tag{11}$$

$$\boldsymbol{Gz} - \boldsymbol{b} \leq \boldsymbol{0}. \tag{12}$$

In the sequel, let the superscript index denote a subset of the rows of a matrix or vector. Since $\boldsymbol{H}$ has full rank, equation (9) gives:

$$\boldsymbol{z} = -\boldsymbol{H}^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{\lambda}. \tag{13}$$

Definition 1: Let $\boldsymbol{z}^*$ be the optimal solution to problem (8). We define active constraints the constraints with $\boldsymbol{G}^i\boldsymbol{z} - \boldsymbol{b}^i = 0$ and inactive constraints the constraints with $\boldsymbol{G}^i\boldsymbol{z} - \boldsymbol{b}^i < 0$. The optimal active set $A^* = \left\{ i \middle| \boldsymbol{G}^i\boldsymbol{z}^* = \boldsymbol{b}^i \right\}$.

Definition 2: For an active set, we say that the Linear Independence Constraint Qualification (LICQ) holds if the set of active constraint gradients are linearly independent, i.e., $\boldsymbol{G}^A$ has full row rank.

Assuming that LICQ holds, equation (10) and equation (13) lead to:

$$\boldsymbol{\lambda}^A = -(\boldsymbol{G}^A\boldsymbol{H}^{-1}(\boldsymbol{G}^A)^{\mathrm{T}})^{-1}\boldsymbol{b}^A. \tag{14}$$

Equation (14) can now be substituted into equation (13) to obtain:

$$\boldsymbol{z} = \boldsymbol{H}^{-1}(\boldsymbol{G}^A)^{\mathrm{T}}(\boldsymbol{G}^A\boldsymbol{H}^{-1}(\boldsymbol{G}^A)^{\mathrm{T}})^{-1}\boldsymbol{b}^A. \tag{15}$$

Partition, now, the vector $\boldsymbol{z}$ into $\boldsymbol{z}_1, \boldsymbol{z}_2 \in \mathbb{R}^M$, by $\boldsymbol{z} = [\boldsymbol{z}_1 \ \boldsymbol{z}_2]^{\mathrm{T}}$ and define $\boldsymbol{w}_0$ as follows:

$$\boldsymbol{w}_0 = \boldsymbol{z}_1 + j\boldsymbol{z}_2 \in \mathbb{C}^M. \tag{16}$$

It is clear that the optimal solution of problem (7) for this particular case is $\boldsymbol{w}_0$.

Summary of the proposed algorithm

1. Collect the sample data and estimate the covariance matrix $\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{k=1}^{N}\boldsymbol{x}(k)\boldsymbol{x}^{\mathrm{H}}(k)$.

2. Modify the sample covariance matrix using the CMT matrix $\boldsymbol{T}$, $\hat{\boldsymbol{R}} = \hat{\boldsymbol{R}} \circ \boldsymbol{T}$.

3. Add the constrains of null level control $\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_k)\right|^2 \leq \xi^2, (k = 1,\ldots,K)$ and sidelobe level control $\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_l)\right|^2 \leq \varepsilon^2, (l = 1,\ldots,L)$ to the MVDR beamformer (1).

4. Calculate the modified CMT-MVDR beamforming optimization problem (7) using multi-parametric quadratic programming method.

C. Output SINR and array gain

To investigate the performance of the proposed method, this section gives the definitions of Signal-to-Interference-and-Noise Ratio (SINR) and array gain for an adaptive antenna array system. From $y(k) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(k)$, the mean square power output of the beamformer can be expressed as:

$$\begin{aligned} P &= \mathrm{E}\left\{\left|y(k)\right|^2\right\} = \sigma_0^2\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_0)\right|^2 + \boldsymbol{w}^{\mathrm{H}}\mathbf{R}_{j+n}\boldsymbol{w} \\ &= \sigma_0^2\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_0)\right|^2 + \sum_{j=1}^{J}\sigma_j^2\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}(\theta_j)\right|^2 + \sigma_n^2\boldsymbol{w}^{\mathrm{H}}\boldsymbol{\rho}_n\boldsymbol{w}, \end{aligned} \tag{17}$$

where $\mathrm{E}\{\cdot\}$ denotes the statistical expectation. The $M \times M$ interference-plus-noise covariance matrix $\boldsymbol{R}_{j+n}$ is expressed as:

$$R_{j+n} = \mathrm{E}\{\boldsymbol{pp}^{\mathrm{H}}\} = \sum_{j=1}^{J}\sigma_j^2\boldsymbol{a}(\theta_j)\boldsymbol{a}^{\mathrm{H}}(\theta_j) + \sigma_n^2\boldsymbol{\rho}_n,$$

where $\boldsymbol{p} = \sum_{j=1}^{J}\boldsymbol{i}_j(k)\boldsymbol{a}(\theta_j) + \boldsymbol{n}(k)$ and σ_0^2 is the desired signal power. σ_j^2 $(j = 1,\ldots J)$ and σ_n^2 are the interference signal power and noise power, respectively. $\boldsymbol{\rho_n}$ is the Hermitian cross-spectral density matrix of the noise normalized to have its trace equals to M.

SINR is defined as follows:

Definition 3: For an adaptive antenna array system, SINR is defined as the output signal power divided by the output interference-and-noise power and is given by:

$$\mathrm{SINR}(\boldsymbol{w}) = \frac{\sigma_0^2\left|\boldsymbol{w}^{\mathrm{H}}\boldsymbol{a}_0\right|^2}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{j+n}\boldsymbol{w}}. \tag{18}$$

The signal power σ_0^2 can be estimated according to the following formulation:

$$\hat{\sigma}_0^2 = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{w} = \frac{1}{\boldsymbol{a}_0^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{a}_0}. \quad (19)$$

From equation (18) and equation (19), SINRs of CMT-mp-QP MVDR beamformer and mp-QP MVDR beamformer can be achieved, respectively.

Definition 4 For an adaptive antenna array system, the array gain is defined as the output Signal-to-Interference-and-Noise Ratio (SINR) divided by the input SINR and is given by:

$$G = \frac{\mathrm{SINR}_{\mathrm{out}}}{\mathrm{SINR}_{\mathrm{in}}} = \frac{\sigma_0^2 \left|\boldsymbol{w}^{\mathrm{H}} a(\theta_0)\right|^2 \Big/ (\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{j+n}\boldsymbol{w})}{\sigma_0^2 \Big/ \sum_{j=1}^{J}\sigma_j^2 + \sigma_n^2}. \quad (20)$$

To calculate the output due to the desired signal, a distortionless constraint is imposed on $\boldsymbol{w}$ that $\boldsymbol{w}^{\mathrm{H}} a(\theta_0) = 1$. Consider the special case of spatial white noise and identical noise spectra at each sensor, the noise cross-spectral density matrix $\boldsymbol{\rho}_n$ reduces to an identity matrix. Thus the array gain for white noise is given by:

$$\begin{aligned} G &= \frac{\sum_{j=1}^{J}\sigma_j^2 + \sigma_n^2}{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_{j+n}\boldsymbol{w}} \\ &= \frac{\sum_{j=1}^{J}\sigma_j^2 + \sigma_n^2}{\sum_{j=1}^{J}\sigma_j^2 \left|\boldsymbol{w}^{\mathrm{H}} a(\theta_j)\right|^2 + \sigma_n^2 \boldsymbol{w}^{\mathrm{H}}\boldsymbol{w}} \\ &= \frac{\sum_{j=1}^{J}\sigma_j^2 / \sigma_n^2 + 1}{\sum_{j=1}^{J}\sigma_j^2 / \sigma_n^2 \left|\boldsymbol{w}^{\mathrm{H}} a(\theta_j)\right|^2 + \left\|\boldsymbol{w}\right\|^2}, \end{aligned} \quad (21)$$

where $\|\cdot\|$ stands for the Euclidean norm.

From equation (21), the array gain of CMT-mp-QP MVDR beamformer and mp-QP MVDR beamformer can be calculated, respectively, according to the weight vectors given by equation (16) and equation (2).

V. SIMULATION RESULTS

In this section, we conduct some simulations to validate the proposed approach. Assume that the Uniform Linear Array (ULA) consists of seventeen sensors (M=17) equispaced by half-wavelength. Assume that the desired signal and two interference signals are plane waves impinging on the ULA from the directions 0° and $30^\circ, -30^\circ$, respectively. In these simulations, the Signal-to-Noise Ratio (SNR) is set to 0 dB, 35 dB and 35 dB for the desired signal and the two interference signals, respectively. The notch width $W = 9^\circ$ and $q = 5$. The sidelobe beampattern areas $[-90^\circ, -10^\circ] \cup [10^\circ, 90^\circ]$ are chosen and a uniform grid is used to obtain the angles. Sensor noises are modeled as spatially and temporally white Gaussian processes. It is assumed that $\xi^2 = 10^{-5}$ and $\varepsilon^2 = 10^{-2}$, i.e., we require the beampatterns null level below -50 dB and sidelobe level to be below -20 dB.

In the first simulation, 1024 snapshots are used to compute the direction patterns of mp-QP MVDR and CMT-mp-QP MVDR, which are plotted in Fig. 1. As shown in this figure, all the beampatterns place deep nulls at the DOAs of the interference signals and maintain a distortionless response for the signal-of-interest. However, mp-QP MVDR is not able to broaden the interference nulls and it has very high sidelobe level. The proposed algorithm can overcome these above shortcomings, with tapered covariance matrix that is and it can not only broaden the null width but also achieve lower sidelobe level.

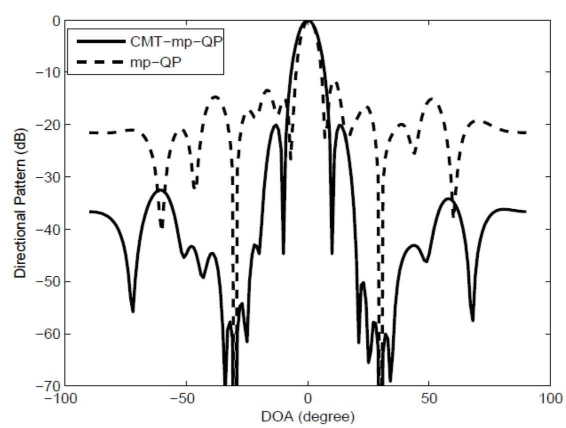

Fig. 1. Directional pattern curves of the proposed CMT-mp-QP method and pure mp-QP method.

In the second simulation, Fig. 2 gives the array gain curves (which are computed by equation (21)) of the aforementioned two beamformers, based on 200 independent trials under the hypothesis that SNRs range from -20 dB to 20 dB and the number of snapshots is equal to 1024. When $\mathrm{SNR} \le 0\mathrm{dB}$, mp-QP MVDR has a little better array gain than the proposed method. With the increase of SNR, it can be seen that CMT-mp-QP MVDR shows better array gain.

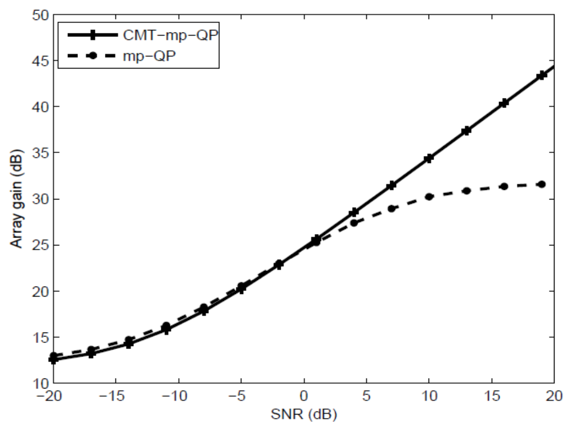

Fig. 2. Array gain versus SNR for various algorithms.

The third simulation considers that the number of snapshots is varied. Figure 3 shows the average output SINR curves (which are computed by equation (18) and equation (19)) of the aforementioned methods, based on 200 independent trials with the SNR equal to 10 dB. From the figure, it can be seen that the proposed method has better convergence even few samples are available. As the number of snapshots increases, the performances of both methods tend to stabilize, while CMT-mp-QP MVDR has higher output SINR than mp-QP MVDR.

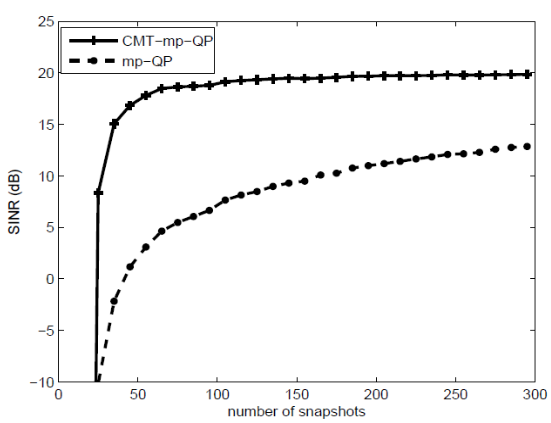

Fig. 3. Array gain versus number of snapshots.

VI. CONCLUSIONS

This paper presents an effective robust adaptive beamforming method with null broadening and sidelobe control. By modifying the measured covariance matrix, null broadening can be regarded as adding the coherent signals to each signal sources. Multiple quadratic inequality constraints outside the mainlobe beampattern area are used to guarantee the beampattern sidelobe level are strictly below some prescribed threshold. Then, the robust adaptive beamforming problem is formulated as a multi-parametric quadratic programming problem, such that the optimal weight vector can be estimated by real-valued computation. The performance of the presented method is verified by simulation.

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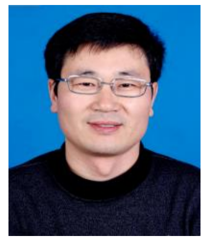

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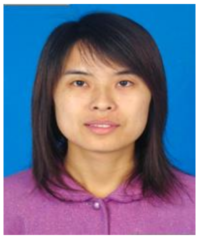

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