FPO-Based Shooting and Bouncing Ray Method for Wide-Band RCS Prediction

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Abstract — The fast physical optics (FPO) method for calculating the monostatic radar cross section (RCS) of an object over a range of frequencies is extended to the shooting and bouncing rays (SBR) method where the multi-bounce phenomena of the launched rays is considered. The FPO method is an improved version of the physical optics (PO) method, which is efficient when calculating the monostatic RCS over a wide range of frequencies or/and angles. The SBR method, based on the combination of geometrical optics (GO) and PO methods, can reach a higher accuracy compared with the PO method. However, due to different length of ray tube paths, it is difficult to implement the phase compensation and phase retrieval in the SBR as that in the FPO method. In this paper, a coordinate transformation is introduced in the integral equation, which transforms the original ray tubes model into a new one. The FPO technique can then be taken on the revised model without difficulty. The validity and efficiency of the proposed method are validated though a couple of numerical experiments.

Index Terms — Coordinate transformation, electromagnetic scattering, fast physical optics (FPO), shooting and bouncing rays (SBR), and wide-band RCS.

I. INTRODUCTION

The radar cross section (RCS) is an important characterization of the electromagnetic scattering properties of large objects and it is common practice to use this data in the fields of war monitoring, radar imaging simulation, and target identification. Techniques for computing RCS can be categorized into high-frequency techniques and low-frequency techniques [1]. The low-frequency techniques include the method of moments (MoM) [2], the finite element method (FEM) [3] and other MoM-type methods [4-10]. The high-frequency techniques include the geometrical optics (GO) and physical optics (PO) [11] methods, and more complicated methods, such as ILDCs [12], GO/UTD-PO/PTD [13], and SBR [14]. The high-frequency techniques, usually based on the ray-tracing and edge diffraction, are fast and accuracy acceptable when used to electrically large objects [15]. For example, an aircraft at L wave band will up to 130 wavelengths, which will result a large number of unknowns if 0.1 wavelength mesh size is used in the MoM method. This large number of unknowns puts forward high demanding in computer performance. However, the high-frequency techniques express the scattering field in an analytical approach and avoid solving the matrix equations as that in the low-frequency techniques, which reduces the memory and CPU time cost significantly.

A lot of researchers have paid attention to the calculation of wide-band RCS of an object. The asymptotic waveform evaluation (AWE) [16-18] technique and model-based parameter estimation (MBPE) [19] have been developed to decrease the computation burden associated with repeated point-by-point calculations [18]. However, the techniques mentioned above are based on low-frequency techniques, and the objects analyzed are not electrically large. Recently, a fast physical optics (FPO) [20-25] has been developed to fast evaluate wide-band or/and wide-angle monostatic RCS. The algorithm is based on the observation that the scattering pattern of a finite scatterer is an
essentially band limited function of the aspect angles and frequencies [20, 26]. In the FPO method, the rapid oscillation of the integrand is cancelled by a phase compensation process, the RCS response over a range of frequencies or angles can then be obtained by interpolating the phase-compensated field at much sparser sampling grids. The backscattering echo of a 2-dimensional rectangular cylinder [20] and double-bounce scattering phenomena involving two surfaces [21] have been analyzed by the FPO. However, the method is rather complicated and limited to double-bounce scattering phenomena.

The shooting and bouncing rays (SBR) [14], proposed by H. Ling etc., is a robust and an accurate method in analyzing electrically large objects. In the SBR approach rays representing the incident fields in a GO manner are used to determine the resulting equivalent surface currents and finally the resulting field contributions at the given observation points are derived by PO integration [27]. Besides the first-order scattered fields, the SBR provides more accurate results by including the scattered fields arising from multiple bounces [29]. In [29], a GPU-based SBR that implemented on the graphics processing unit (GPU) is proposed to reduce the computation time. The ray tracing is modified to evaluate the exit point and field quickly. The electromagnetic computing is integrated into the process of central ray tracing, including the evaluation of the reflected and scattered field. The main contribution of this paper is utilizing the GPU to accelerate both ray tracing and electromagnetic computing of the SBR, which is a parallelization of the SBR method under a certain frequency and incident angle. The wide-band and wide-angle monostatic RCS can then be obtained by repeating this process at different frequencies and angles. Whereas the method proposed in [20] is focused on getting wide-band or/and wide-angle monostatic RCS with interpolation technique. According to this method, only a few monostatic RCS need to be calculated directly, the RCS at required frequencies and angles can then be obtained by interpolating these calculated ones.

In this paper, the FPO method for calculating the monostatic RCS of an object over a range of frequencies is extended to the SBR method. Unlike the method in [29], the method proposed in this paper tries to get wide-band RCS by interpolating values calculated at sampling frequencies. The RCS at these sampling frequencies can be obtained by either a CPU-based or a GPU-based SBR method as that in [29]. However, due to different length of ray tube paths, it is difficult to implement the phase compensation and phase retrieval in the SBR as that in the FPO method. To conquer this problem, a coordinate transformation is introduced in the integral equation, which transforms the original ray tubes model into a new one. The FPO technique can then be taken on the revised model. The remainder of this paper is organized as follows. In section II, theory and formulations are discussed. Numerical results are presented and discussed in section III. Section IV concludes this paper. The time factor $e^{j\omega t}$ is assumed and suppressed throughout this paper.

II. THEORY AND FORMULATION

Consider a perfectly electrically conductor (PEC) shown in Fig. 1, a plane wave is incident on this object ($\vec{E}_{in}$ and $\vec{H}_{in}$ denote the electric and magnetic field of the incident wave, respectively). $R_0$ is half of the diagonal line of the cube that contains the object. To facilitate analysis, the object is meshed with small patches (for example, triangular patches), the number of meshed triangles of which is dependent on the frequency and the size of the object. In section A below, the theory of the FPO is briefly discussed. The SBR and its improvement are discussed in section B.

**Fig. 1. Scattering model of an object.**

**A. FPO**

As shown in Fig. 1, according to the PO method, the back-scattered field of the object at a fixed frequency can be written as,
\[ \tilde{E}_s(f) = 2j\eta \frac{e^{-jkR}}{4\pi R} \sum_{i=1}^{N_t} \int s \times (\hat{n} \times \hat{H}_m) \, e^{2j{k} s \hat{n} ds}, \]  
\[ = 2j\eta \frac{e^{-jkR}}{4\pi R} \sum_{i=1}^{N_t} \int p \, e^{2j{k} s \hat{n} ds}, \]  

where \( \hat{n} \) is a unit outward normal vector at the point \( r \), \( s \) is the unit vector in the direction of observation, \( k \) is the wavenumber, \( \eta \) is the intrinsic wave impedance, \( N_t \) is the number of triangle patches in the illuminative area, \( R \) is the distance between the observation point and the object, and \( j = \sqrt{-1} \). In the high frequency region, \( k \) is a large number, and function \( p \) is a slowly varying function regarding frequencies. In this paper, the integral technique proposed by Gordon [30] is adopted to calculate the integral in equation (1).

\[ \frac{\hat{n}}{\eta} = \frac{\hat{r}}{R} \]  

When used to wide-band scattering problems (for example, the required frequency band is \([f_{\text{min}}, f_{\text{max}}]\) with frequency interval \( \Delta f \)), one has to repeat the calculation of equation (1) at a sequence of frequency samples, which is time-consuming when used to electrically large object. According to the Nyquist sampling theorem, the sampling interval of the frequency should satisfy \( \Delta f < c/(4R_o) \), where \( c \) is the velocity of light. Consequently, the number of frequency samples is,

\[ N_f = \Omega_f \frac{4R_o (f_{\text{max}} - f_{\text{min}})}{c}, \]  

where \( \Omega_f \) is the oversampling ratio satisfying \( \Omega_f > 1 \). It is observed that the number of frequency samples is proportional to the size of the object \( R_o \). One possible way to reduce the \( N_f \) is by dividing the object into several non-overlapped groups as shown in Fig. 2. The \( N_f^i \) of the \( i \)-th group is obtained by substituting \( R_o \) with \( R_o^i \) in equation (2). Since \( R_o^i < R_o \), the sampling points \( N_f^i < N_f \). The back-scattered field of each group at the required frequencies can then be interpolated by these sparser sampling grids. However, the scattered field obtained by direct interpolation is not accurate. To improve the interpolation accuracy, phase compensation is applied to the back-scattered field prior to interpolation, and a phase retrieval algorithm is used after the interpolation stage. The phase compensated field of the \( n \)-th group can be written as,

\[ \tilde{E}_s(f) = 2j\eta \frac{e^{-jkR}}{4\pi R} \sum_{i=1}^{N_t} \int p \, e^{2j{k} s \hat{n} ds}, \]  

where \( N_t^i \) is the number of triangle patches in the illuminative area of the \( n \)-th group, \( \hat{r}_n \) is the center coordinate of the \( n \)-th group. The rapid oscillation of the integrand is cancelled by the phase compensation process, which makes the interpolation process more accurate. After interpolation, the back-scattered field of each group at the required frequency point can be retrieved by phase restoration, i.e.,

\[ \tilde{E}_s(f) = \tilde{E}_s(f) e^{2j{k} s \hat{n} \hat{r}_n}. \]  

The total back-scattered field \( \tilde{E}_s(f) \) can then be obtained by aggregation of the back-scattered field of all groups and the monostatic RCS can then be written as,

\[ \text{RCS}(f) = \lim_{R \to \infty} 4\pi R^2 |\frac{\tilde{E}_s(f)}{\tilde{E}_s(f)}|^2 | \tilde{E}_s(f) |^2. \]  

\[ \text{RCS}(f) = \lim_{R \to \infty} 4\pi R^2 |\frac{\tilde{E}_s(f)}{\tilde{E}_s(f)}|^2 | \tilde{E}_s(f) |^2. \]  

B. SBR and its improvement

The SBR method involves two steps: ray tube tracing and electromagnetic computing at the exit point. First, the incident plane wave is modeled as a dense grid of ray tubes at a virtual aperture, which are shot toward the object. When used to wide-band RCS prediction, the number of ray tubes is set according to the highest frequency to be analyzed. Each ray tube is recursively traced to obtain the exit position and its scattered field. Figure 3 shows a possible path of a ray tracing. It
should be noted that the ray tracing process is time-consuming when the object is meshed with large number of triangles and when the number of ray tubes is large. In [28], the angular Z-buffer (AZB), the volumetric space partitioning (SVP) and the depth-limited search method are combined to accelerate the ray tracing process, whereas a stackless kd-tree traversal algorithm is adopted to evaluate the exit position and field quickly in [29]. In this paper, the octree [31-32] technique is used to recursively subdividing the box into eight children to decrease the number of intersection tests. Second, the PO integral is preformed to obtain the scattered field of this ray tube based on the pre-calculated exit positions and field [29].

The equation $r = r - r_i$ implies that the final integral can be taken on a new model that moves the exit points in the reverse direction of $\hat{s}$ . Comparison between equations (6) and (9) shows that the coordinate transformation extracts the exponential term in the square brackets and combines with the exponential term outside the square brackets. As shown in Fig. 3, the original integral of equation (6) at the exit point (Point C) can be replaced by the integral of equation (9) at point D. In other words, the new model (or integral domain) has taken into account the effect of path of each ray tube, whereas the original integral is taken at the exit point. However, the coordinate transformation does not affect the fields and the medium through the transformation because equation (9) is consistent with equation (6) in essence and there is only a change in variables. To make this clearer, we consider a dihedral corner reflector as shown in Fig. 4 (a), the incident and observation angles are $\theta_i = 45^0$, $\phi_i = 0^0$, $\theta_i' = 45^0$, $\phi_i = 0^0$. After ray tracing and coordinate transformation, the integral of equation (9) is taking on a new model as shown in Fig. 4 (b). Then the new model can be regrouped without difficulty and take the FPO technique as in section II A to fast get the wide-band RCS prediction.
Points A, B, and C are intersection points during the ray tracing. Point D is obtained by moving point C in the reverse direction of \( \frac{s}{g_4} \) such that the distance between C and D (remarked as \( r_i \)) satisfies \( r_i = r_{i1} + r_{i2} + r_{i3} \).

Firstly, we consider a trihedral corner reflector with a side length of 1\( \text{m} \), the geometry of the trihedral corner reflector is shown in Fig. 5. The monostatic RCS on the \( \theta = 60^0 \) plane is calculated. The virtual aperture is also meshed with 0.01\( \lambda \) as that in [29]. The results of the HH-polarization at 3 GHz are shown in Fig. 6, which show a good agreement between the method proposed in this paper and the method in [29]. The total computation time of the proposed method in this paper is about 281 seconds, whereas the GPU-based method in [29] only needs 8.73 seconds. The reason is that the method introduced in this paper mainly deals with wide-band RCS prediction. The result may become unacceptable when used to wide-angle problems because of the drastic change in the path of each ray tube at different angles. So the proposed method in Fig. 6 degenerates to the conventional SBR indeed. However, as mentioned in the introduction, the GPU-based SBR method in [29] can be combined together with the method proposed in this paper to accelerate the computation at sampling frequencies when dealing with wide-band RCS problems.

Secondly, we validate accuracy of the proposed method. As shown in Fig. 7, we consider a trihedral corner reflector with a side length of 10 meters. The number of ray tubes is set to be 68,730. The incident angles are \( \theta_i = 45^0 \), \( \varphi_i = 45^0 \), respectively. \( l_1 = l_2 = 0.4m \). The frequency range of interest is 0.5 GHz – 1.5 GHz, with frequency interval \( \Delta f = 20 \text{MHz} \) (resulting 51 frequencies points). By using equation (2) with \( \Omega_f = 2 \) and

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**Fig. 4.** Schematic drawing of scattering model of a dihedral corner reflector; (a) the dihedral corner reflector and (b) the new model after ray tracing and coordinate transformation.

**Fig. 5.** The geometry of a trihedral corner reflector.
\( R_0 = \sqrt{\frac{3l_2}{2}} \), only 10 frequencies points are needed to be calculated. Figure 8 is the monostatic RCS for VV polarization versus the frequency. The result simulated by the commercial software CST [33] is also shown in the figure, which shows a good agreement among these methods.

Thirdly, we consider an F15 aircraft as shown in Fig. 1, the size of the F15 aircraft is \( 4.78m \times 3.35m \times 1.06m \). The incident angles are \( \theta_i = 120^\circ \), \( \varphi_i = 0^\circ \), respectively. The frequency range of interest is 1 GHz – 15 GHz, with frequency interval \( \Delta f = 20 \) MHz (resulting 701 frequencies points and 627,592 ray tubes). \( l_1 = l_2 = 0.1m \). Figure 9 is the monostatic RCS for VV polarization versus the frequency with different oversampling rates of the proposed method, which shows a good agreement with the direct SBR method. Table I lists the CPU time and relative error of the proposed method, where the CPU time cost by the ray tracing is not considered (Because the CPU time for the ray tracing are the same for all of the 4 cases). The relative error is given by,

\[
\text{relative error} = \frac{\sum_{i=1}^{\text{frequency points}} |RCS_{\omega \Omega i} - RCS_{\text{SBR}}|}{\sum_{i=1}^{\text{frequency points}} |RCS_{\text{SBR}}|},
\]

where the RCS calculated by the SBR method is used as the correct solution. It can be seen from the table that the proposed method can reduce the CPU time significantly with reasonable oversampling rates, and the relative error decreases with the oversampling rate increases.

Table I: CPU time cost and relative error for different oversampling rates.

<table>
<thead>
<tr>
<th>( \Omega_i )</th>
<th>No. of frequency points</th>
<th>CPU time (Sec.)</th>
<th>relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBR</td>
<td>( 701 )</td>
<td>( 433 )</td>
<td>--</td>
</tr>
<tr>
<td>Proposed method</td>
<td>( \Omega_i = 2 )</td>
<td>( 34 )</td>
<td>( 18 )</td>
</tr>
<tr>
<td></td>
<td>( \Omega_i = 3 )</td>
<td>( 50 )</td>
<td>( 26 )</td>
</tr>
<tr>
<td></td>
<td>( \Omega_i = 4 )</td>
<td>( 68 )</td>
<td>( 35 )</td>
</tr>
</tbody>
</table>

Finally, we consider a tank model as shown in Fig. 10. The size of the tank is \( 8.11m \times 2.59m \times 1.82m \). 690,454 ray tubes is used to get an accurate result over the C, X, and
Ku wave band (i.e., 4 GHz − 18 GHz). The frequency interval of interest is \( \Delta f = 10 \) MHz. The incident angles are \( \theta = 60^0 \), \( \varphi = 0^0 \). \( l_1 = l_2 = 0.1 m \) is adopted as that in the third example. Figure 11 shows the monostatic RCS versus the frequency with different oversampling rates of the proposed method, which shows a good agreement with the conventional SBR method. Table II lists the CPU time and error values of the proposed method, from which we can see that the proposed method can reduce the CPU time with a factor of 26 ~ 44 without losing accuracy compared with the conventional SBR. In terms of relative error, the error values in Table I are higher than those in Table II. The reason is that the monostatic RCS in Fig. 9 is much smaller than that in Fig. 11. The error values can be reduced by a double precision or higher order interpolation technique.

<table>
<thead>
<tr>
<th>( \Omega )</th>
<th>No. of frequency points</th>
<th>CPU time (Sec.)</th>
<th>relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBR</td>
<td>---</td>
<td>1401</td>
<td>711</td>
</tr>
<tr>
<td>Proposed method</td>
<td>( \Omega = 2 )</td>
<td>33</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>( \Omega = 3 )</td>
<td>49</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>( \Omega = 4 )</td>
<td>65</td>
<td>27</td>
</tr>
</tbody>
</table>

Fig. 9. Monostatic RCS versus frequency computed by the proposed method with different oversampling rates and the SBR method.

Fig. 10. The geometry of a tank model.

Fig. 11. Monostatic RCS versus frequency computed by the proposed method with different oversampling rates and the SBR method.

**IV. CONCLUSION**

In this paper, the FPO method for calculating monostatic RCS of an object over a range of frequencies is extended to the SBR method where the multi-bounce phenomena of the launched rays are considered. A coordinate transformation is used to cancel the influence of different length of ray tubes on phase compensation and phase retrieval, which transforms the original integral model into a new one. The FPO technique can then be taken on the revised model without difficulty. The proposed method and its efficiency are validated by numerical experiments, which show that the proposed method can reduce the CPU time significantly. The proposed method is especially suited for generation of synthetic data for radar imaging simulation.

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