A High Resolution Algorithm for Null Broadening Beamforming Based on Subspace Projection and Virtual Antenna Array

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Abstract **─** In this paper, we proposed a high resolution algorithm for null broadening beamforming. The algorithm is based on the property of subspace orthogonal principle between signal and noise, and on virtual antenna array. By utilizing Capon spectrum estimator, we construct the correlation matrix to obtain projection transformation matrix, the width of null increases when original covariance matrix is processed by projection transformation matrix. In order to improve the depth of null and increase array resolution, virtual antenna technique is introduced. Also diagonal loading technique is used to form robust beam pattern. With the theoretical analysis and computer simulation, it's demonstrated that the superiority of proposed algorithm corresponding other null broadening beamforming methods.

Index Terms **─** Array resolution, diagonal loading, null broadening beamforming, remove redundancy, subspace projection, virtual antenna array.

I. INTRODUCTION

Adaptive beamforming can suppress the interferences and noises by producing null at the direction of interferences, so it has been widely used in radar, sonar, mobile communications and many other fields [1-3]. One of the challenges of designing such a beamformer arises when the interference direction may be inaccurately known by Direction-of-Arrival (DOA) estimation. It's desired that interferences are suppressed within an angular region for enhancing fault tolerance. Thus many approaches of null broadening beamforming have been proposed [4-8].

Mailloux [4] and Zatman [5] have proposed pattern troughs techniques, respectively. Two methods are the same essentially and are unified by introducing the concept of covariance matrix tapers (CMT) [6]. However, the depth level of null degrades when a wide trough is obtained because of interference power dispersion. Amar [7] has proposed a new approach that called linear constraint sector suppressed (LCSS), by which the depth of null is improved, the performance of LCSS is degradation with high SNR, unfortunately. Recently, a novel algorithm named projection and diagonal loading null broadening beamforming (PDNBB) was proposed [8], this method has excellent performance compared with previous algorithms, but depth of null is not enough in some certain cases and array resolution degrades when the null is broadened. In this paper, we construct the projection transformation matrix by PDNBB algorithm, and a high resolution algorithm for null broadening beamforming is proposed, in which covariance matrix is processed by projection transformation matrix to enhance the orthognality of subspace and real antenna array is transformed into virtual antenna array by virtual antenna technique [9,10] to improve relative power distribution. Virtual antenna technique is an advanced array signal processing technique, array resolution could be improved by new virtual array elements. Compared with other algorithms, deeper null and higher array resolution can be got by proposed method when we broaden the null, and it's insensitive to snapshots.

II. THE SIGNAL MODEL

We consider a uniform linear array (ULA) with N omnidirectional antennas with spacing half a wave length uniformly. Assume that there are $M+1$ narrowband far-field signals from the directions θ_p, p=0,1,2,…, M, where θ_0 represents the direction of desired signal and θ_q (q=1,2,…, M) are the direction of interference signal. The receive signal at the time index k could be expressed as follows:

$$\mathbf{X}(k) = \mathbf{A}\mathbf{S}(k) + \mathbf{N}(k), \tag{1}$$

where $\mathbf{A}$ denotes array manifold matrix, $\mathbf{S}(k)$ is signal complex envelop vector, $\mathbf{N}(k)$ represents a vector modeled as zero-mean white Gaussian noise and complex vector $\mathbf{X}(k) = \left[x_1(k), x_2(k), ..., x_N(k)\right]^T$ is an observation data vector at the sample snapshot kth. We assume that desired signal, interference signals and noise are statistically independent of each other. So output signal of beamformer at the receiving terminal can be denoted as:

$$y(k) = \mathbf{W}^H \mathbf{X}(k), \tag{2}$$

where $\mathbf{W}$ is a complex weight vector with dimension

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$N \times 1$ and $(\cdot)^H$ denotes Hermitian transpose.

According to the criterion of maximizing the output signal-to-interference-plus-noise ratio (SINR), the minimum variance distortionless response (MVDR) beamformer can be formulated as the following linearly constrained quadratic optimization problem [11,12]:

$$\begin{cases} \min\limits_{\mathbf{W}} & \mathbf{W}^H \mathbf{R}_{i+n} \mathbf{W} \\ \text{subject to} & \mathbf{W}^H \mathbf{a}(\theta_0) = 1 \end{cases}, \tag{3}$$

where $\mathbf{a}(\theta_0)$ represents steering vector of desired signal and $\mathbf{R}_{i+n}$ is the interference-plus-noise covariance matrix. The solution of optimal weight vector can be solved by lagrangian multiplier method and expressed as follows:

$$\mathbf{W}_{OPT} = \frac{\mathbf{R}_{i+n}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{R}_{i+n}^{-1}\mathbf{a}(\theta_0)}, \tag{4}$$

In practice, we can't obtain the covariance matrix $\mathbf{R}_{i+n}$ because of the existence of desired signal in the receive signal, so interference-plus-noise matrix is commonly replaced by the sample covariance matrix (SCM) with K snapshots, SCM can be described as:

$$\hat{\mathbf{R}} = \frac{1}{\mathrm{K}} \sum_{i=1}^{K} \mathbf{X}(i)\mathbf{X}^H(i), \tag{5}$$

III. THE PDNBB APPROACH [8]

We construct correlation matrix $\mathbf{R}_\omega$ for the steering vector as follows:

$$\mathbf{R}_\omega = \int_\omega \mathbf{a}(\theta)\, \mathbf{a}^H(\theta) d\theta, \tag{6}$$

where ω is the angular sector that interference signals may appear. Because there is a high computational complexity in integral operation, we replace the integral with summation operation by selecting a series of discrete points within the desired null angular sector. Then $\mathbf{R}_\omega$ is decomposed with eigenvalue λ_i and eigenvector $\mathbf{v}_i$ as:

$$\mathbf{R}_\omega = \sum_{i=1}^{N} \lambda_i \mathbf{v}_i \mathbf{v}_i^H \quad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N, \tag{7}$$

We assume there are Q lager eigenvalues which occupy the most energy of receiving signal and they are satisfied with inequality as follows:

$$\frac{\sum_{i=Q+1}^{N} \lambda_i}{\sum_{i=1}^{N} \lambda_i} \leq \varepsilon, \tag{8}$$

where parameter ε decides the depth of null. So projection transformation matrix can be obtained by $\mathbf{T}$:

$$\mathbf{T} = \mathrm{span}(\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_Q), \tag{9}$$

where $span(\cdot)$ represents the generative vector space of selected based vector.

The original covariance matrix is replaced as follows:

$$\overset{\triangleq}{\mathbf{R}} = \mathbf{T}\hat{\mathbf{R}}\mathbf{T}^H + \gamma \mathbf{I}, \tag{10}$$

where γ is the diagonal loading factor and it can be selected in the range of $10^{-6} \leq \gamma \leq 10^{-4}$ by practical experience, and $\mathbf{I}$ is identity matrix.

IV. THE PROPOSED METHOD

A. The proposed method

In this paper, the PDNBB algorithm is taken to obtain new covariance matrix which strengths the orthogonality between signal subspace and noise subspace, but array resolution degrades when wider null is obtained. In order to get deeper null in some certain circumstances, we introduce the concept of Kronecker product [13], by which real array is transformed into virtual array and the number of antenna array increases, therefore, both covariance matrix and steering vectors are transformed as follows:

$$\overset{\cong}{\mathbf{R}} = \overset{\triangleq}{\mathbf{R}} \otimes \overset{\triangleq}{\mathbf{R}}^*, \tag{11}$$

$$\overset{\cong}{\mathbf{a}}(\theta) = \mathbf{a}(\theta) \otimes \mathbf{a}^*(\theta), \tag{12}$$

where $\otimes$ operator denotes Kronecker product and $(\cdot)^*$ is conjugate operator.

B. Theoretical analysis

According to subspace decomposition theory, we write covariance matrix obtained by PDNBB algorithm as follows:

$$\begin{aligned} \overset{\triangleq}{\mathbf{R}} &= \mathbf{T}\mathbf{U}\Sigma\mathbf{U}^H\mathbf{T}^H \\ &= \mathbf{T}\left[\mathbf{U}_s\ \mathbf{U}_n\right]\begin{bmatrix} \mathbf{\Sigma}_s & \\ & \mathbf{\Sigma}_n \end{bmatrix}\left[\mathbf{U}_s\ \mathbf{U}_n\right]^H \\ &= \mathbf{A}_s \Sigma_s \mathbf{A}_s^H + \mathbf{B}_n \Sigma_n \mathbf{B}_n^H \end{aligned}, \tag{13}$$

where $\mathbf{U}_s$ represents signal subspace and it can be written as $\mathbf{U}_s = [\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_{M+1}]$, $\mathbf{U}_n$ represents noise subspace and it can be written as $\mathbf{U}_n = [\mathbf{v}_{M+2}, \mathbf{v}_{M+3}, ..., \mathbf{v}_N]$, $\mathbf{\Sigma}_s$ and $\mathbf{\Sigma}_n$ are diagonal matrixes which could be expressed as $\mathbf{\Sigma}_s = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_{M+1})$ and $\mathbf{\Sigma}_n = \mathrm{diag}(\lambda_{M+2}, \lambda_{M+3}, ..., \lambda_N)$ corresponding eigenvalues of signal and noise subspace, respectively. In addition, let $\mathbf{A}_s = \mathbf{T}\mathbf{U}_s$, $\mathbf{B}_n = \mathbf{T}\mathbf{U}_n$, the latter tends to zero, ideally. Then covariance matrix of proposed method is expressed as:

$$\begin{aligned} \overset{\cong}{\mathbf{R}} &= (\mathbf{A}_s \Sigma_s \mathbf{A}_s^H) \otimes (\mathbf{A}_s \Sigma_s \mathbf{A}_s^H)^* \\ &= \mathbf{V}(\Sigma_s \otimes \Sigma_s^*)\mathbf{V}^H \end{aligned}, \tag{14}$$

where let $\mathbf{V} = \mathbf{A}_s \otimes \mathbf{A}_s^*$.

It can be seen from (14) that only signal subspace is retained, ideally. It is noteworthy that there are redundant items in covariance matrix and steering vector processed by Kronecker product, so it needs to be handled to remove redundancy. The power of desired signal and interference components are enhanced furtherly so that the deeper null can be presented.

C. The method of removing redundancy

The dimension of covariance matrix increases after virtual transformation, which means there are more array antenna elements so that higher array resolution could be obtained, but many redundant items exist in covariance matrix processed by Kronecker product. According to the correspondence between steering vector and covariance matrix as follows:

$$\mathbf{a}(\theta) = [1, e^{\frac{-j2\pi d\sin\theta}{\lambda}}, \cdots, e^{\frac{-j2\pi(N\text{-}1)d\sin\theta}{\lambda}}]^T , \tag{15}$$

$$\begin{aligned}\overset{\cong}{\mathbf{a}}(\theta) &= \mathbf{a}(\theta)\otimes\mathbf{a}^*(\theta) \\ &= [1, e^{\frac{j2\pi d\sin\theta}{\lambda}}, e^{\frac{j2\pi 2d\sin\theta}{\lambda}}, \cdots, e^{\frac{j2\pi d(N\text{-}1)\sin\theta}{\lambda}}, \\ &\quad e^{\frac{\text{-}j2\pi d\sin\theta}{\lambda}}, 1, \cdots, e^{\frac{\text{-}j2\pi(N\text{-}2)d\sin\theta}{\lambda}}, \cdots, \\ &\quad e^{\frac{\text{-}j2\pi(N\text{-}1)d\sin\theta}{\lambda}}, \cdots, 1]^T_{N^2\times 1}\end{aligned} \tag{16}$$

$$\overset{\cong}{\mathbf{R}} = \overset{\triangleq}{\mathbf{R}}\otimes\overset{\triangleq}{\mathbf{R}}^* , \tag{17}$$

where d represents the spacing between elements and θ is signal incident direction. We can remove the redundant items from the covariance matrix and steering vector processed by Kronecker product. Figure 1 denotes covariance matrix after removing the redundancy.

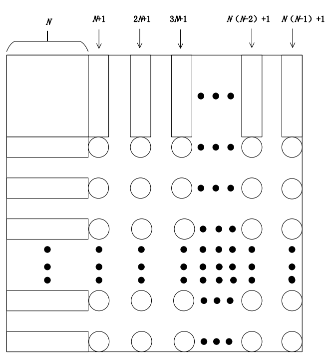


Fig. 1. Covariance matrix after removing the redundancy.

In the Fig. 1, large square box represents a matrix with dimension $N^2\times N^2$, small square box denotes a matrix which dimension is $N\times N$, the rectangle boxes denote vector with dimension $N\times 1$, and circles are elements extracted from original covariance matrix. The original covariance matrix with dimension $N^2\times N^2$ becomes a new covariance matrix with dimension $(2N-1)\times(2N-1)$ after using the method of removing redundancy.

In the proposed method, the main computational complexity lies in construction of $\mathbf{R}_\omega$ and matrix inversion operation, the former is $O(PSN^2)$ where P denotes the number of null broadening and S is the number of samples taken in the summation with $S\gg N$, the latter is $O((2N-1)^3)$, so the overall computational complexity is $O(\max(PSN^2,(2N-1)^3))$. The PDNBB and LCSS have same computational complexity with $O(PSN^2,N^3)$. CMT has a lower complexity of $O(N^3)$. Although the proposed method has higher complexity than other algorithms, deeper null and higher array resolution can be got in practical application.

D. The summary of proposed method

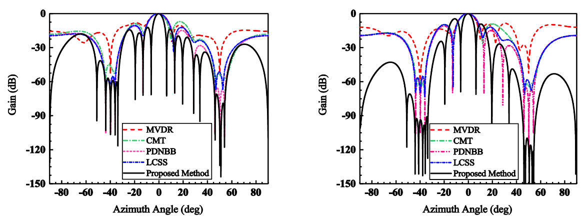
The proposed algorithm can be implemented by several steps and summarized as follows:

Step 1) Construct steering vector correlation matrix $\mathbf{R}_\omega$ as Equation (6);
Step 2) Eigen decomposition of $\mathbf{R}_\omega$ as Equation (7);
Step 3) Construct projection transformation matrix $\mathbf{T}$ as Equation (8);
Step 4) Projection and diagonal loading as Equation (10);
Step 5) Kronecker product transforms as Equation (11) and Equation (12);
Step 6) Removing the redundant items as Fig. 1;
Step 7) Calculate optimal weight value as Equation (4).

V. SIMULATION RESULTS

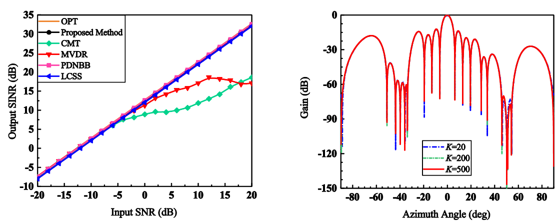
We consider a uniform linear array (ULA) with 10 omnidirectional antennas spaced half a wave length uniformly. The direction-of-arrival of desired signal is 0°. The DOAs of the two interferences are -40° and 50°, respectively. The interference to noise ratio (INR) is 30 dB, SNR is 0 dB unless it's specified. The number of snapshot is 200. The width of null is 10°, so projection angular sector selects as $[-45^\circ,-35^\circ]$ and $[45^\circ,55^\circ]$. The parameter ε decides the depth level of null, we select ε as 6×10^{-6} or 6×10^{-10}. The beam patterns of proposed method and other algorithms are compared in Fig. 2.

From the Fig. 2 (a) and Fig. 2 (b), it can be seen that there are deeper null and higher array resolution when null is broadened in proposed algorithm. In addition, there are different depth of null when parameter ε selected different values, the reason is that orthogonality between signal subspace and noise subspace increases when we select smaller ε. Figure 3 (a) demonstrates the output SINR performance versus SNR where the number of antenna array is 19, except from proposed method, it's consistent with what have been said before, the reason is that the proposed method has changed the number of array elements from N to $2N$-1 after Kronecker product transformation, where N takes 10. Figure 3 (b) shows beam patterns of proposed method when the number of snapshot takes different values, and K=20, K=200 and K=500 are used in the simulations. For each scenario, there are Monte-Carlo simulations performed with the number of 200 and diagonal loading factor γ is 10^{-4}. We can see that the proposed method have deeper null, high resolution and good robustness which verifies the feasibility of the algorithm.



(a) (b)

Fig. 2. (a) Normalized beampatterns of different algorithms when parameter ε is equal to 6×10^{-6}, and (b) normalized beampatterns of different algorithms when parameter ε is equal to 6×10^{-10}.



(a) (b)

Fig. 3. (a) Output SINR versus the input SNR when parameter ε is equal to 6×10^{-10}, and (b) beam patterns of the proposed method when the number of snapshot takes different values and ε is equal to 6×10^{-6}.

VI. CONCLUSION

A high resolution algorithm for null broadening beamforming based on subspace projection and virtual antenna array is presented in this paper. The proposed method expands the direction of interference incidence through the projection transformation technique, deepens the depth of null, and improves the orthogonality between signal subspace and the noise subspace. At the same time, through the virtual antenna technology, the proposed method furtherly deepens the depth of null while achieving higher array resolution. Theoretical analysis and simulation results show that the proposed method has a good performance on null broadening, array resolution and robustness, it can still work steadily with a small number of snapshots, which enhances its practicality and saves the hardware storage resources.

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