Optimisation of the FGT for Electroheat Multiphysics Modelling

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Abstract: Multiphysics modelling of coupled systems is indispensable for accurate, reliable and objective characterization of electroheat phenomena. This paper examines the usefulness of the Fourier series and Green's function methods for deriving the analytical solutions of such models. An optimized Fourier-Green technique (FGT) is vital for accurate model description and analysis. A numerical solution for a EM-heat flow model was realized for an aluminium metal sample using F90 program code and F95 compiler.

A homogeneous time-dependent E-field (μV/m) source, consisting of 500 time steps, was used to study the temperature distribution of the metal sample. The results satisfy the set initial and boundary conditions. Besides, the visualisations show that the temperature profile increases towards the centre, averaging 20^{o}C within the bulk of the sample. The optimized FG function promises to revolutionise biomedical applications of EM-based heating, communications devices, RF and microwave heating and industrial processes amongst others.

Keywords: EM-heat flow, Fourier-Green Optimisation, Multiphysics modelling

1. Introduction

Heating results when non-ionising forms of electromagnetic radiation are absorbed by a material medium. The resulting radio-frequency (1 MHz – 300 MHz) or microwave (300 MHz – 3000 MHz) system can be described as a coupled multiphysics problem [1, 2].

The EM-heat transfer modelling equation is given by: [1, 2]

$$\rho C_p(\partial T/\partial t) = \nabla k \nabla T + P_{abs} \tag{1}$$

where T is the temperature (^{o}C) within the medium, t is the process time (s), ρ is the density (kgm^{-3}), C_p is the heat capacity (Jkg^{-1o}C^{-1}), ∇ is the Laplacian operator $(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)$, k is the thermal conductivity (Wm^{-1o}C^{-1}), P_{abs} is the power, per unit volume generated by the E-field distribution and given by the following equation:[1, 2]

$$P_{abs} = 2\pi f \varepsilon_o \varepsilon'' \left| E \right|^2 \tag{2}$$

where f is the frequency (Hz) of the radio-wave generator, ε_o is the permittivity of free space (Fm^{-1}), ε" is the relative dielectric loss factor of the medium and $|E|^2$ is the root-mean-square value (or modulus) of the E-field (Vm^{-1}).

2. Fourier-Green Optimisation

Fourier series (FS) and Green's function (GF) methods have been extensively used for solving the heat equation [3, 4]. In principle, both approaches are similar.

Equation 3 is a one spatial dimension version of the heat equation given by equation 1 [3, 4]. The partial differential equation models the temperature profile T(x, t) in a bar.

$$\rho C_p\, \partial T/\partial t - k\partial^2 T/\partial x^2 = g(x, t),\ 0<x<L,\ t>t_o. \tag{3}$$

The initial-boundary conditions of equation 3 then become $T(x, t_o) = \psi(x)$, $0<x<l$; $T(0, t) = 0$, $t>to$; $T(L, t) = 0$, $t>t_o$.

The solution of equation 3 by the FS methods yields thus: [3]

$$T(x, t) = \sum_{n=1}^{\infty} a_n(t)\sin(n\pi x/L) \tag{4}$$

where $a_n(t) = (2/L) \int_0^L T(x, t)\sin(n\pi x/L)dx$ and satisfies the boundary conditions in equation 3.

The choice of $a_n(t)$ must be such that equation 3 and the initial condition are met. This is accomplished by obtaining the FS of the right-hand side of equation 3 and introducing the initial conditions. The final solution of the initial value problem becomes:

$$a_n(t) = [(2/L \int_0^L F(x)\sin(n\pi x/L))\exp(-\alpha n^2\pi^2(t - t_o)/L^2)$$
$$+ (1/\rho C_p) \int_{to}^{t} \exp(-\alpha n^2\pi^2(t - s)/L^2)\ (2/L)ds \int_0^L g(x, s)\sin(n\pi x/L)dx] \tag{5}$$

The GF solution to equation 3 is given by: [4]

$$T(x, t) =(2/L) \sum_{m=1}^{\infty} \exp(-\alpha\beta_m^2 t)\ \sin\beta_m x \int_{x'=0}^{L} \sin\beta_m x' F(x')dx'$$
$$+ (2\alpha/kL)\sum_{m=1}^{\infty}\exp(-\alpha\beta_m^2 t)\ \sin\beta_m x \int_{\tau'=0}^{t} \exp(-\alpha\beta_m^2\tau)\ d\tau \int_{x'=0}^{L}\sin\beta_m x' g(x', \tau)dx' \tag{6}$$

where $\beta_m = m\pi/L$, m = 1, 2, 3, …

Equation 5 makes equations 4 and 6 similar with α (thermal diffusivity) = $k/\rho C_p$.

The accuracy of both methods is influenced by the choice of the terms parameter n (FS) or m (GF). Increasing n or m to infinity would eliminate the ringing effect; this occasions computational overhead in terms of time and resources. Furthermore, both analytical techniques consider rigid temperature variations without recourse to dynamic thermal phenomena as in RF and/or MW welding with moving heat sources [5]. Therefore, weighting parameters are required to cater for situations where combined rigid and dynamic time- and position-dependent temperature changes occur. This optimized function would greatly reduce the ringing effect exhibited by the medium during visualization for code verification. Similar analyses can be carried out for a 3-D case [3, 4].

3. Results

Figure 1 shows the excitation signal used for analyzing the electroheat problem of an aluminium metal sample. The E-field source consisted of 500 time steps with a corresponding time duration of 322.55ps. The excitation exhibits numerical dispersion due to the presence of higher number of spectral components for which its phase velocity differs significantly from the speed of light in a vacuum; this can be eliminated using appropriate spectral decomposition (in the spatial domain) and reconstruction (in the time domain) algorithms. The medium was of dimension 0.1m x 0.1m x 0.1m. The initial temperature of the sample was 20^{o}C. Fortran 90 program algorithm and code were used to numerically solve the EM-heat flow problem. 100 terms of the series were used to generate the curves.

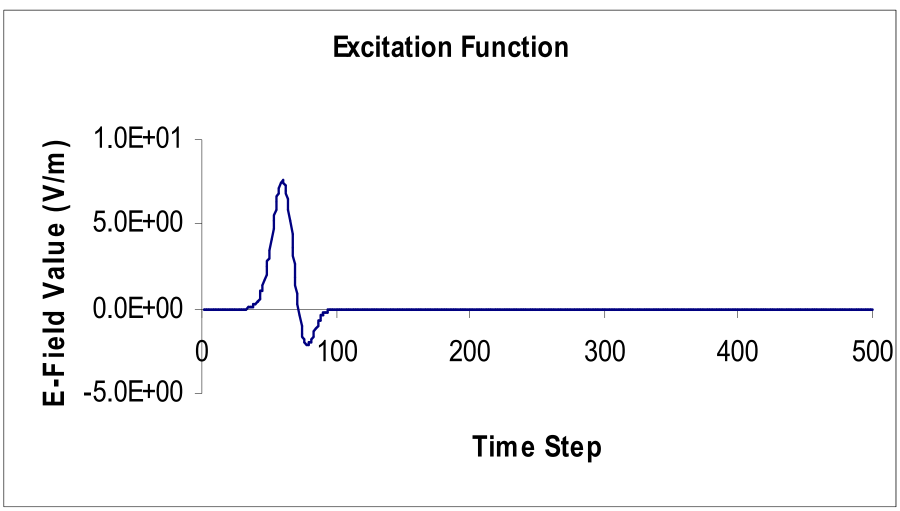

Figure 1. Homogeneous E-Field Source

The E-field temperature distribution over the time using the Green's function method is depicted in figure 2. The profile averages at about 0.2 p^{o}C within the bulk of the metal. Ringing effect occurs with a maximum spike of about 0.226 p^{o}C at 0.001m and 0.099m.

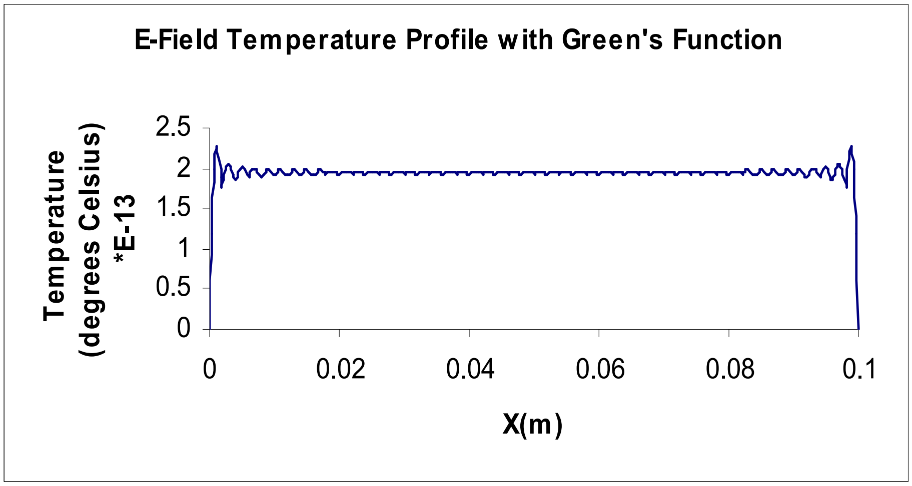

Figure 2. E-Field-dependent Temperature Profile using GF

The complete temperature profile using the Green's function method is shown in figure 3. The profile averages at about 20^{o}C within the bulk of the metal. Ringing effect occurs with a maximum spike of about 23.57^{o}C at 0.001m and 0.099m.

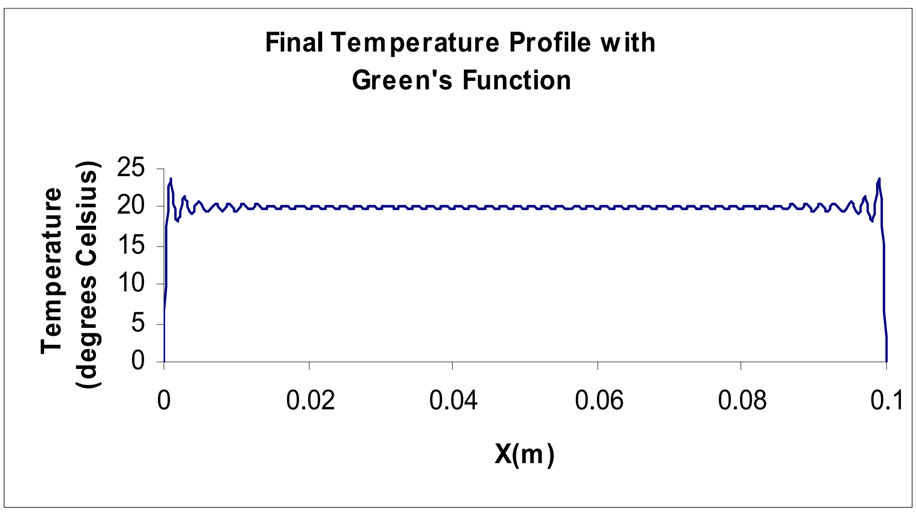

Figure 3. Complete Temperature Profile using GF

Figures 2 and 3 have the "ringing" phenomenon present due to the finite number of terms (n and m) chosen for the iteration.

4. Conclusion

The Fourier series and Green's function techniques reveal the response of the analytical solution to the initial and boundary value problem (IBVP) of heat transfer in an E-field. The GF results of the coupled multiphysics modelling satisfied the IBVP.

The final temperature distribution averages 20^{o}C within the investigated sample due to the low E-field contribution; in telecommunications, the low thermal surges would mean much for low noise characterisation of devices. Moreover, the visualization reveals that temperature profile increases towards the centre of the medium sample. Besides, the ringing effect, in which high thermal spikes are recorded, reduces as the number of terms of the series increases with a computational overload. With 100 terms of the series, the ringing effect yielded 13% and 17.85% maximum spikes above the average constant values of the E-field-dependent and final temperature profiles respectively. The optimised FG function promises a better thermal response with a greatly reduced or no computational overload. This novelty will objectively enhance the rapid design, development and production of reliable electroheat devices and equipment; this promises a great tool for modelling the noise temperature deep space communication systems.

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