Particle Swarm Optimization Applied to EM Problems

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Abstract: This paper summarizes results for preferred values of the Particle Swarm Optimizer (PSO) parameters when it is applied to EM problems. The PSO is applied to two different EM optimization problems. The first problem is the optimization of the position of a rectangular waveguide feed, which has two optimization variables. The second problem is the optimization of the excitations of a broadside antenna array, which has twenty optimization variables. The results show that the preferred parameters of PSO are somewhat different for optimization problems with different number of dimensions of the optimization space.

Keywords: Broadside Antenna Array, EM Optimization, Particle Swarms Optimization, Rectangular Waveguide Feed, WIPL-D code

1. Introduction

The Particle Swarm Optimization (PSO) is a relatively new approach to EM optimization and design [1-3]. It is based on the analogy of movement of bird flocks or fish schools, on one side, and the optimization, on the other side. The PSO algorithm belongs to the class of the heuristic optimization algorithms. Since it was introduced ten years ago, its possibilities and limits, when applied to EM problems, are not fully explored. The aim of this paper is to summarize the results for the found preferred parameters of the PSO when it is applied to the optimization of EM systems.

In Section 2 we briefly describe the implementation of PSO used in the following numerical examples. In Section 3, we present results for optimization of the position of the rectangular waveguide feed, for different values of the PSO parameters. In Section 4, we present results for the optimization of the excitations of the broadside antenna array using PSO.

2. Description of the Used Implementation of PSO

The PSO algorithm minimizes the cost-function of an EM problem by simulating movement and interaction of particles in a swarm. The position of a particle (or an agent) corresponds to one possible solution of the EM problem, i.e., it corresponds to one point in the optimization space. Since we assume that there is no *a priori* knowledge of the optimization problem, all solutions in the optimization space are well suited for the beginning of the optimization. Therefore, PSO starts with randomly chosen positions of particles. Each particle keeps track of its personal best position found in the optimization space, $\mathbf{p}_{\text{best}}$, which is the position-vector in the optimization space. The swarm keeps track of the global best position, $\mathbf{g}_{\text{best}}$, found with all particles together. Hence, every particle knows the best position found by itself and the best position found by the whole swarm. (There are other possible formulations of the PSO, when particles do not know the global best position, but rather know the best position of a subset of the swarm that it belongs to. However, these formulations are out of the scope of this paper.) The velocity vector for the calculation of the particle position in the next iteration is calculated as

$$\mathbf{v}_n = w \cdot \mathbf{v}_{n\text{-}1} + c_1 \cdot \mathrm{rand}() \cdot \left(\mathbf{p}_{\mathrm{best}} - \mathbf{x}_{n-1}\right) + c_2 \cdot \mathrm{rand}() \cdot \left(\mathbf{g}_{\mathrm{best}} - \mathbf{x}_{n-1}\right), \tag{1}$$

where $\mathbf{v}_{n-1}$ is the particle velocity in the previous iteration, w is so-called inertia coefficient, $\mathrm{rand}()$ is the function that generates uniformly distributed random numbers in the interval from 0.0 to 1.0, c_1 is so-called the cognitive coefficient (it controls the pull to the personal best position), and c_2 is so-called the social rate coefficient (it controls the pull to the global best position).

The next position of the particle in the optimization space is calculated as

$$\mathbf{x}_n = \mathbf{x}_{n\text{-}1} + \mathbf{v}_n \Delta t, \tag{2}$$

where Δt is most often considered to be of a unit value.

It is found that if there are no limits for the velocity of the particles, they might fly-out of the meaningful optimization space [1-3]. Therefore, maximal velocity $V_{\max}$ is introduced as another parameter of the PSO algorithm. $V_{\max}$ represents the maximal percentage of the dynamic range of the optimization variable for which the velocity can change in successive movements of the particle. In our implementation of the optimization, all dynamic ranges of the optimization variables are scaled to the interval $[-1.0, +1.0]$, and we will use one unique value of $V_{\max}$ for all optimization variables.

Default parameters of PSO found in the literature [3] are: the number of particles $p = 15$, the inertia $w = 0.729$, the maximal velocity $V_{\max} = 0.2$, and the cognitive coefficient and the social rate $(c_1, c_2) = (1.494, 1.494)$.

3. The First Optimization Problem: Optimal Position of Feeding Probe in Rectangular Waveguide

We consider a rectangular waveguide open at one end and closed at the other end. The width of the waveguide is 86 mm, the height is 43 mm, and the length is 200 mm. One half of the structure is shown in Fig. 1. The operating frequency is 4 GHz.

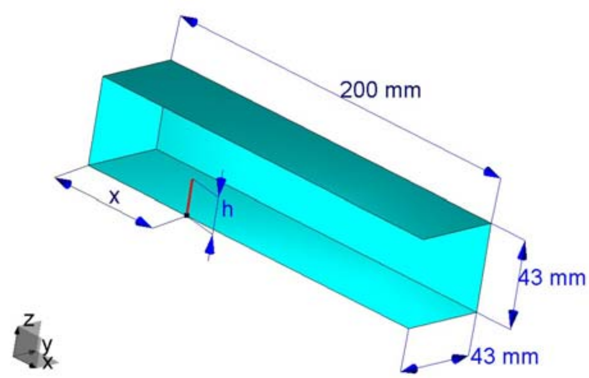


Fig. 1. One half of the rectangular waveguide used in the first optimization problem.

The feeding probe is positioned on the central axis of the rectangular waveguide. The probe distance from the closed end of the waveguide, x, and its height, h, are optimized for the minimal possible reflection coefficient (s_{11}). The optimization parameters are varied in the ranges $1 \le x \le 101$ mm and $10 \le h \le 30$ mm. Therefore, the optimization space has two dimensions.

The cost-function used in this optimization is calculated as

$$f_{\text{cost}} = 100\,[\text{dB}] - s_{11}[\text{dB}]\,, \tag{1}$$

where s_{11} is the reflection coefficient, in dB, at the feeding probe.

PSO algorithm parameters are varied one-at-a-time with all the others equal to the default values, as given in Section 2, except the number of particles in the swarm. It is found that the swarm of five particles performs better than setups with higher number of particles and for that reason it is used as a default value in this optimization problem.

The PSO coefficients are taken from the following sets

$p \in \{5, 10, 15, 20, 25, 30, 35\}$,

$w \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$,

$V_{\max} \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$, and

$(c1, c2) \in \{(0.0, 3.0), (0.5, 2.5), (1.0, 2.0), (1.5, 1.5), (2.0, 1.0), (2.5, 0.5), (3.0, 0.0)\}$.

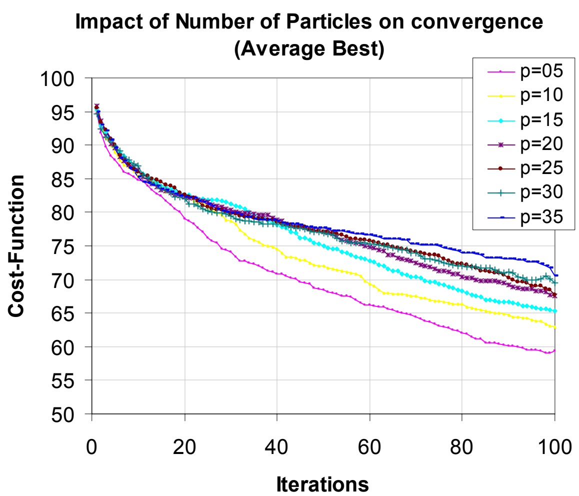


Fig. 2. Impact of number of particles.

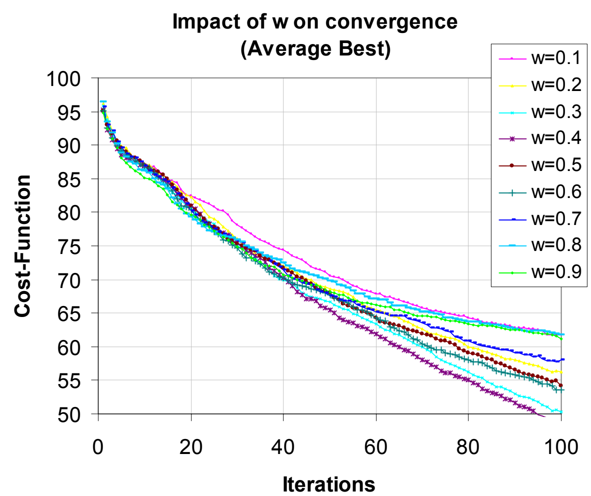


Fig. 3. Impact of inertia coefficient *w*.

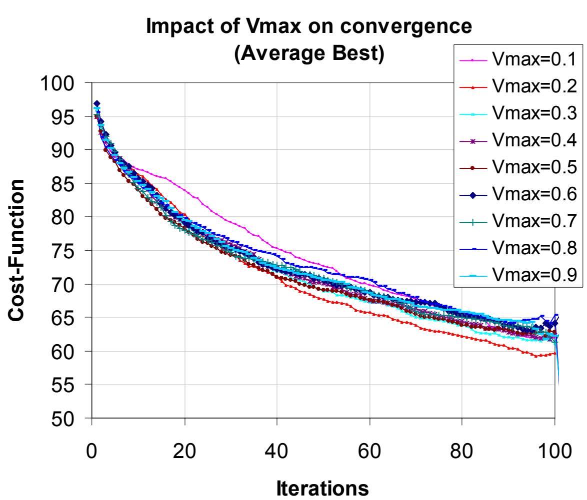


Fig. 4. Impact of maximal velocity $V_{\max}$.

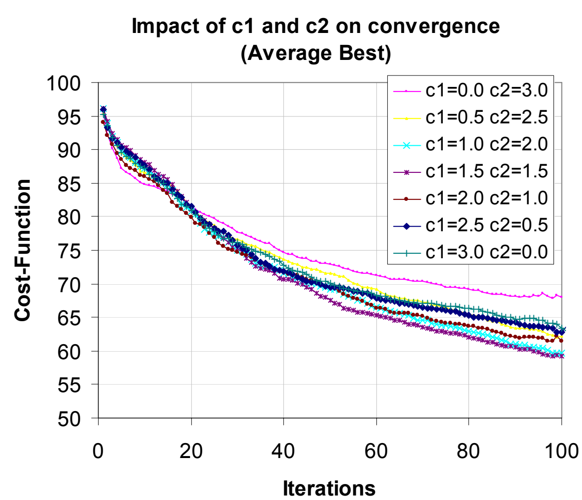


Fig. 5. Impact of cognitive and social rate coefficients.

One optimization lasts for 100 iterations (EM solver calls). We use WIPL-D Pro. v5.1 [4] as the EM solver.

For each setup of the PSO coefficients, the optimization is repeated for 100 times to get a good estimation of the average outcome of the optimization. The average minimal cost-function found after the number of iterations is shown is Figs. 2-5.

From the presented results we conclude that preferred values for the PSO algorithm in this problem are: the number of particles $p = 5$, the inertia coefficient $w = 0.4$, the maximal velocity $V_{\max} = 0.2$, and cognitive and social rate coefficients $(c_1, c_2) = (1.5, 1.5)$.

4. The Second Optimization Problem: Optimal Excitations of Broadside Antenna Array

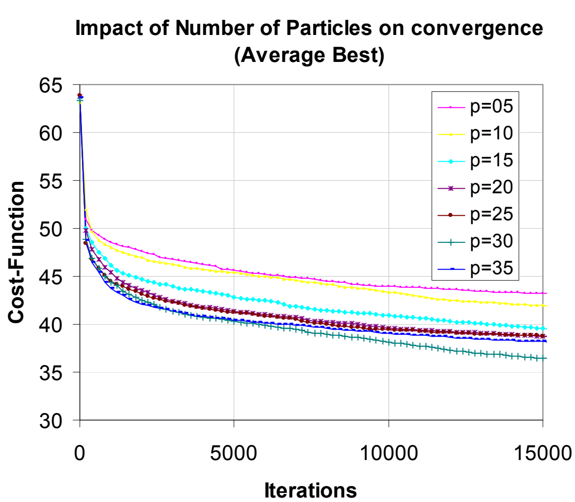
The second optimization problem is a broadside antenna array of 42-point sources located along the *z*-axis and at uniform distance of one half of the wavelength at the operating frequency. Optimization of excitation magnitudes is done for the lowest possible side lobe levels. The solution is required to have symmetry of the excitation amplitudes, and therefore only one half of the excitations are varied. To avoid infinite number of solutions due to scaling, the 21^{st} coefficient is predefined. Hence, we have a 20-dimensional optimization problem.

The criterion for the optimization is that the side lobe levels should be lower then –80 dB everywhere for $\theta < 65°$, where θ is the angle between the array axis and the radiation direction. The cost-function is calculated as:

$$f_{\text{cost}} = \sqrt{\frac{1}{n}\sum_{i=0}^{n-1}\left\{\max\left[0,\quad 80 - F_{\max} - F(\theta_i)\right]\right\}^2}\,, \tag{2}$$

where $F_{\max}$ is in the direction $\theta = 90°$, $\theta_i = i[°]$, $n = 65$, and $F(\theta_i)$ is given in dB. The theoretical result exists in the form of the binomial distribution of the amplitudes. The ratio of the highest and the lowest amplitude is of the order of 10^{11}, which is inconvenient from the standpoint of the numerical optimization. Hence, we represent each coefficient as $s_k = \ln(a_k)$, $k = 1,2,\ldots,20$, so that the maximal ratio of the coefficients is less than 30. Each optimized parameter, s_k, has the lower bound equal to zero and the upper bound equal to the largest (21^{st}) coefficient.

PSO algorithm parameters are varied in the same way as in the previous example, with the difference that the default value for the number of particles in the swarm is 15. The same sets of the parameter values are used.



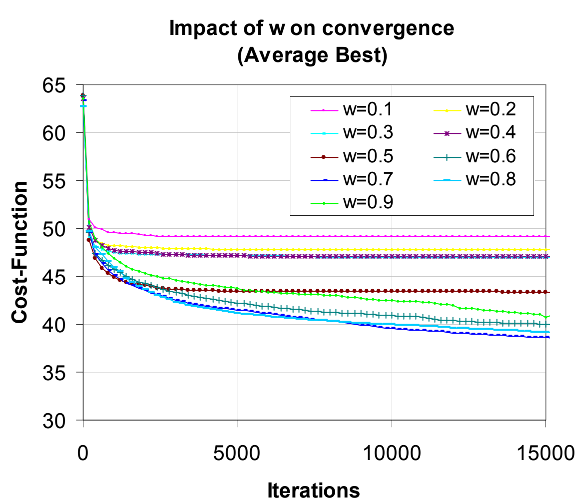


Fig. 6. Impact of number of particles.

Fig. 7. Impact of inertia coefficient w.

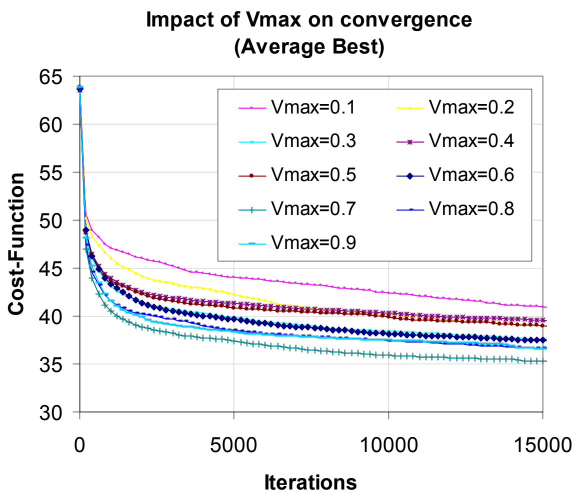


Fig. 8. Impact of maximal velocity $V_{\max}$.

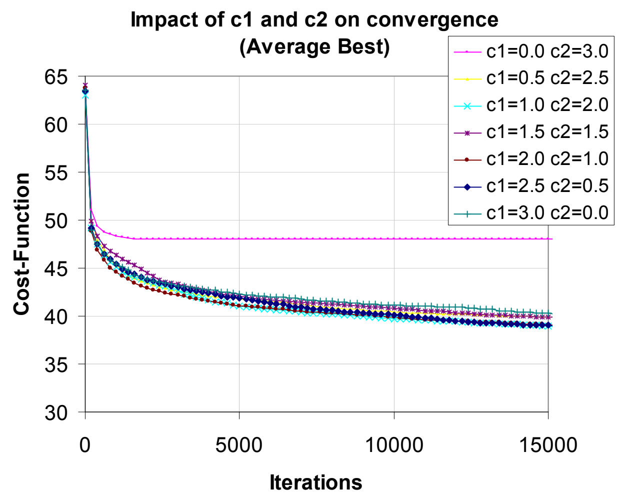


Fig. 9. Impact of cognitive and social rate coefficients.

One optimization lasts for 15,000 iterations (EM solver calls). Each optimization is repeated for 100 times to get the good estimation of the average outcome of the optimization. The average minimal cost-function found after the number of iterations is shown in Figs. 6-9.

From the presented results, it can be seen that the preferred values for the PSO algorithm in this problem are: the number of particles $p = 30$, the inertia coefficient $w = 0.7$, the maximal velocity $V_{\max} = 0.7$, and cognitive and social rate coefficients $(c_1, c_2) = (1.0, 2.0)$. Note that all values for (c_1, c_2) coefficients except $(c_1, c_2) = (0.0, 3.0)$ show very similar behavior.

5. Conclusions

On the basis of results obtained through numerical experiments, presented in the paper, we can conclude that preferred PSO parameters change with the increase of the number of optimization variables. The number of particles in the swarm seems to be proportional to the number of dimensions of the optimization problem. However, more experiments should be preformed to verify that. For the other parameters of the PSO algorithm, the default values performed well, but careful tuning can yield a slightly more efficient optimization. The future research should go in the direction of comparing the efficiency of PSO to the efficiency of other optimization algorithms that are used to optimize EM systems.

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