# Error Bound due to the Random Position Errors in Adaptive Processing Using a Nonuniformly Spaced Array

Seunghyeon Hwang
Department of Electrical Engineering and Computer Science
121 Link Hall, Syracuse University
Syracuse, New York 13244-1240 USA
sehwang@syr.edu

**Abstract:** In this paper, the gain power error bound due to random position errors in the array elements is presented. From the gain power error bound, the relationship between the output signal to interference and noise ratio (OSINR) and the coefficient of variation of the powerdensity function is investigated by adaptive processing using a single snapshot of the measured voltage at the feed points of a semi-circular antenna (SCA) elements in an array.

**Keywords:** Adaptive signal processing, a semicircular array antenna, error bound.

## 1. Introduction

By using a transformation matrix to change the voltages induced in a nonuniformly spaced array antenna, such as a SCA, to the uniformly spaced linear virtual array (ULVA), we are able to remove the various electromagnetic effects like the presence of mutual coupling and near-field effects. A least square method is used to generate a transformation matrix which preprocesses the measured voltages from a single snapshot of data. This is then used in a direct data domain (DDD) algorithm which has been proposed in [1]-[2].

Because the transformation matrix is a preprocessing technique, it is possible for the location of the antenna elements in a real array system to fluctuate due to an installation error or environmental effect. This will degrade the performance of the system. Using the gain pattern error model [3], we introduce the gain power error bound according to the variation of the position of the SCA elements and investigate the relationship between the OSINR and the coefficient of variation of the powerdensity function in the position of the elements. The OSINR is obtained using the DDD algorithm**.** In this paper we review a transformation matrix based on the least square method. We present the error bound according to the variation of the position of the phased array antenna. We apply the DDD algorithm to data which is obtained from a perturbed SCA (PSCA) antenna through the example.

## 2. A Transformation Matrix Based on the Least Square Method

A preprocessing technique is used to compensate for the lack of non-uniformity in a real array contaminated by the mutual coupling effects. It is based on transforming the non-uniformly spaced array into a (ULVA) operating in the absence of mutual coupling and other undesired electromagnetic effects. Such a transformation is achieved through the use of a compensation matrix. Hence, our goal is to select the best-fit transformation, $[\Im]$, between the real array manifold, $[\mathrm{A}(\phi)]$, and the array manifold corresponding to an ULVA, $[\mathrm{A}_{\mathrm{v}}(\phi)]$ such

 

$$[\Im]\,[A(\phi)] \;=\; [A_v(\phi)] \tag{1}$$

for all azimuth angles $\phi$ within a predefined sector. Since such a transformation matrix is defined within a predefined sector, the various undesired electromagnetic effects such as non-uniformity in spacing and mutual coupling between the elements and near field obstacles for an array are made independent of the angular dependence of all the signals including the SOI and all the interferers. This procedure is similar in concept to the procedure described in [4-7] with the difference that all the electromagnetic effects including presence of near field obstacles are characterized in an accurate fashion satisfying the real physics through Maxwell's equations. An important point to note is that here we are carrying out the processing using a single snapshot of the data.

We compute the transformation matrix $[\Im_q]$ for each one of the sector q such that $[\Im_q][A(\Phi_q)] = [A_v(\Phi_q)]$ using the least squares method. The this solution of is given by

$$\left[\Im_q\right] = \left[A_v(\Phi_q)\right]\left[A(\Phi_q)\right]^H \left\{\left[A(\Phi_q)\right]\left[A(\Phi_q)\right]^H\right\}^{-1} \tag{2}$$

where the super script H represents the conjugate transpose of a complex matrix.

This transformation matrix needs to be computed only once *a priori* for each sector and this computation can be done off-line. Hence, once $[\Im_q]$ is known we can compensate for the various undesired electromagnetic effects such as mutual coupling between the antenna elements, including the effects of near field scatterers, as well as non-uniformity in the spacing of the elements in the real array simultaneously. Since the transformation matrix $[\Im_q]$ is defined within the predefined angular sector we can eliminate both the non-planar effects and the mutual coupling effects independently within each sector.

Then using (2), we can obtain the processed input voltages in which the effects of non-uniformity in the spacings and the mutual coupling effects including the presence of near-field scatterers have been eliminated from the actual voltages. The corrected voltages $\left[\overline{x}_c(m)\right]$ are obtained by

$$\left[\overline{x}_c(m)\right] = [\Im_q]\left[\overline{x}(m)\right] \tag{3}$$

Now, once (3) is obtained, we can apply the direct data domain algorithms to these preprocessed set of voltages $[\overline{x}_c(m)]$ without any significant loss of accuracy.

### 3. A Transformation Gain pattern Error Bound based on the position random fluctuations.

In this section we investigate the relationship between the output SNR and Error bound. We use the gain pattern error model as the error bound model. According to the [3], The coefficient of variation is given by:

$$\varepsilon_y = \frac{\sigma_y}{<Y>} \tag{4}$$

where <Y> is the sample estimate of a population Y with mean $\mu_y$ and a sample standard deviation $\sigma_y$. Here we only consider variation of x and y position of antenna elements. From this, we can write the powerdensity function as:

$$S(\theta,\phi) = \sum_{i=1}^{N}\sum_{k=1}^{N} F_i F_k^* \tag{5}$$

 

where $F_i = \exp(j\,2\pi/\lambda\,(x_i \sin\theta\cos\phi + y_i \sin\theta\sin\phi))$ .

The powerdensity function consists of variables showing random fluctuations: The x, y positions. The mean and variance of this powerdensity function is given by:

$$\mu_S = S(\mu_{x1}, \mu_{x2}, ..., \mu_{xn}, \mu_{y1}, \mu_{y2}, ..., \mu_{yn}) \tag{6}$$

$$\sigma_S^2 = \sum_{k=1}^{N}\left\{\frac{dS}{dx_k}\right\}^2 \sigma_x^2 + \sum_{k=1}^{N}\left\{\frac{dS}{dx_y}\right\}^2 \sigma_y^2 \;\Big|\{\mu_{x1}, \mu_{x2}, ..., \mu_{xn}, \mu_{y1}, \mu_{y2}, ..., \mu_{yn}\} \tag{7}$$

The next step is to determine the derivative of the powerdensity function with respect to the x and y position per radiating element k;

$$\frac{\partial S}{\partial x_k} = \frac{\partial}{\partial x_k}\left\{\sum_i F_i F_k^* + \sum_i F_k F_i^*\right\} = 2k\sin\theta\cos\phi \sum_{i=1}^{N}\sin\left[(x_k - x_i)\sin\theta\cos\phi + (y_k - y_i)\sin\theta\cos\phi\right] \tag{8}$$

$$\frac{\partial S}{\partial y_k} = 2k\sin\theta\sin\phi \sum_{i=1}^{N}\sin\left[(x_k - x_i)\sin\theta\cos\phi + (y_k - y_i)\sin\theta\cos\phi\right] \tag{9}$$

Therefore we get

$$\sigma_s^2 = \sum_{k=1}^{N}\left\{\frac{\partial S}{\partial x_k}\right\}^2 \sigma_x^2 + \sum_{k=1}^{N}\left\{\frac{\partial S}{\partial y_k}\right\}^2 \sigma_y^2 \tag{10}$$

and using equation (7) we can get the error bound based on the position random fluctuations. The coefficient of variation then follows as

$$\varepsilon_S = \sigma_S / \mu_S \tag{11}$$

Next we first focus on error bound from the fluctuated elements position and OSINR by the D3LS algorithm [1].

## 4. Simulation Results

We consider a semi-circular array consisting of 24 elements as shown in Figure 1. The radius of the semi-circular array is 3.82 λ. The elements of the semi-circular array are composed of electrically thin dipoles spaced at equal angles as shown in Figure 1. Each element of the array is identically point loaded by $50\Omega$ at the center. The dipoles are z-directed, of length L = $\lambda/2$ and radius r = $\lambda/200$. Then the semicircular array is interpolated into a ULVA consisting of 17 isotropic omni directional point radiators which are uniformly spaced across the diameter of the SCA and are radiating in free space. The elements of the ULVA are equally spaced and their inter element spacing is given by $0.4775\lambda$. The measured/computed voltages are computed using the electromagnetic analysis code WIPL-D [8]. The interpolation region is

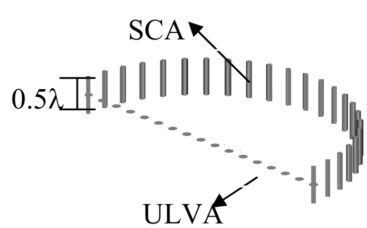


Fig. 1. Geometry of a SCA and a ULVA

 

[Φ]=[30°, 150°] and the incremental size $\phi$ in the interpolation region is chosen to be 1°. After we compensate for the various electromagnetic effects like the presence of mutual coupling between the elements of the array using (2), we transform the voltages measured at the feed points of the SCA to an equivalent set of voltages that will be induced in a ULVA operating in free space. Therefore we get the transformation matrix which is obtained the uniformly located SCA(USCA) to a ULVA.

Then we consider that the element positions of the USCA are randomly changed and we use the data from the PSCA. To define the error bound according to the randomly position, firstly we look up the half power beamwidth (HPBW). Of special interest is the error made at the HPBW (18.6°). We choose the worst case between the HPBW from Fig.2. and 3. which show the normalized variation-coefficient as a function of $\phi$,

$$\varepsilon_S / \sigma_x = 10\ \mathrm{m}^{-1} \qquad \varepsilon_S / \sigma_y = 5.88\ \ \mathrm{m}^{-1}. \tag{12}$$

The overall error bounds of the power density function can now be determined by substituting the worst-case values in (10)

$$\varepsilon_S^2 = 10^2 \sigma_x^2 + 5.88^2 \sigma_y^2 . \tag{13}$$

We assume that variation of x and y positions are same. According to $\sigma_x$ and $\sigma_y$, we change the x and y position of the elements of the SCA. Then we apply the data from the PSCA to the transformation matrix which is obtained from the USCA to the ULVA. Then we get the graph by D3LS algorithm that shows the relationship between the position variation of elements and error bound.

Let us assume that we receive the signal and jammers into the PSCA as like as table 1. The received signal-to-noise ratio (SNR) is set to 26 dB at each of the antenna elements. The noise is additive and is modeled as a Gaussian random variable.

When a uniformly SCA elements are located at the exact position, we get OSINR 37.8 dB by the forward-backward method and 32.3 dB by the forward and the backward method and we get the beampattern of Fig. 4. Then we apply the data which are obtained from the randomly fluctuating position of x and y which is obtained from PSCA by using the transformation matrix, which is obtained by the preprocessing and is obtained from the USCA to the ULVA.

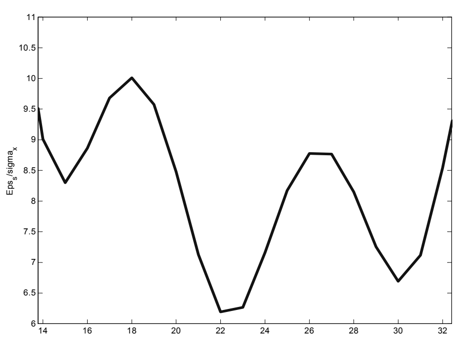


Fig.2. The normalized variation-coefficient $\varepsilon_S / \sigma_x$ as a function of $\phi$

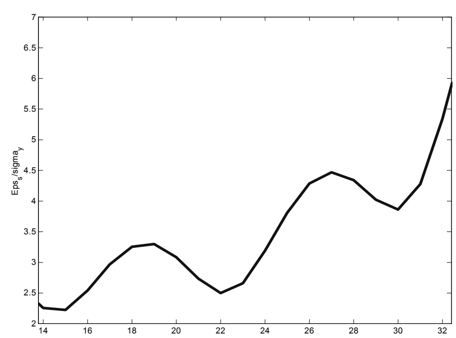


Fig.3. The normalized variation-coefficient $\varepsilon_S / \sigma_y$ as a function of $\phi$

 

**Table 1.** Complex amplitudes of the signal and jammers along with their DOA

| | Magnitude | Phase | DOA | | Magnitude | Phase | DOA |
|---|---|---|---|---|---|---|---|
| **Signal** | 1.0 V/m | 0° | 100° | **Jammer 2** | 100.0V/m | 0° | 97° |
| **Jammer 1** | 100.0V/m | 0° | 125° | **Jammer 3** | 10.0V/m | 0° | 70° |

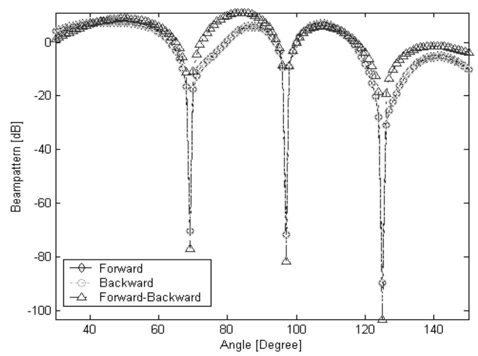


Fig. 4. Beam pattern when we get the data from the USCA.

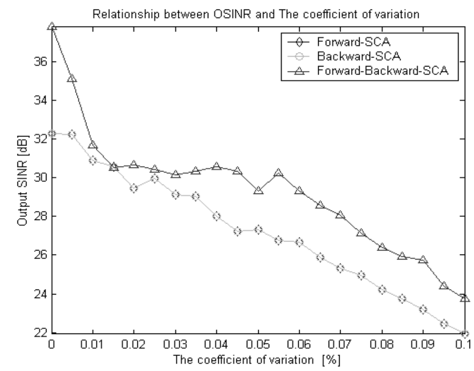


Fig. 5. The relationship between the OSINR and the coefficient of variation.

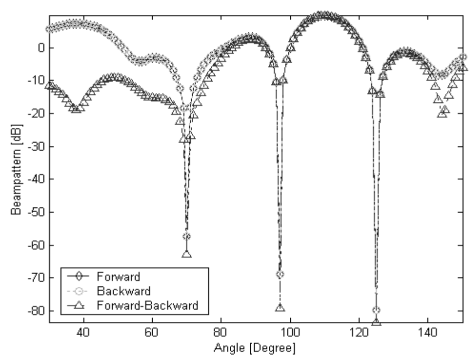


Fig. 6. Beampattern when we get the data from the PSCA in case of 0.035 % error.

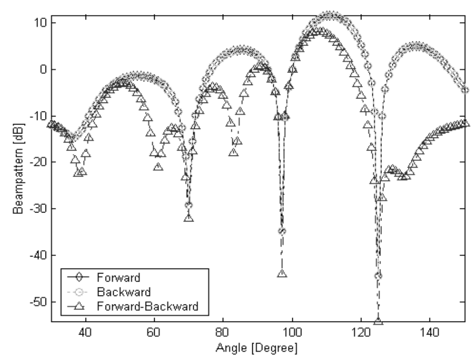


Fig. 7. Beampattern when we get the data from the PSCA in case of 1.17 % error.

Fig 5 shows the results of the relationship of OSINR [dB] and the coefficient of variation [%]. Fig.6 shows the Beampattern when we get the data from the PSCA in case of 0.035 % error**.**

As we see the OSINR decreases as Error bound increases. When we choose the error bound to be 0.035% at 3dB point below the 0% error by the Forward method, then $\sigma_{x,y} = 3.0171\times10^{-5}$ [m]**,** $3.0171\times10^{-5}\lambda$, is obtained physically. If the elements are moved randomly with $\sigma_{x,y}$ = 1 mm, 0.001 λ, we get $\varepsilon_S = 0.0117 = 1.17$ % as the error bound. Then using the above example we get approximately 4 dB OSINR. Fig.7 shows the Beampattern when we get the data from the PSCA in case of 1.17 % error.

 

## 5. Conclusion

This paper presents the gain pattern error bound according to the random position fluctuations. We investigate the relationship between the gain pattern error bound based on the random position fluctuation and OSINR using the DDD algorithm through the example. The transformation matrix has the strong advantage to extract the SOI in the presence of coherent jammers, and thermal noise using a single snapshot of data obtained from an antenna array. An electromagnetic preprocessing technique is first applied which transforms a nonuniformly spaced array operating in the presence of mutual coupling into a virtual array of omnidirectional isotropic point elements that is amenable to the application of a direct data domain least square algorithm. By a certain effect, the positions of the elements are randomly moved. We have to consider how much we can give the error bound according to the gain pattern error model. Given an error bound by the position changed, we can still apply the preprocessing which obtains the transformation matrix.

## References

[1] T. K. Sarkar, S. Park, J. Koh and R. A. Schneible, "A Deterministic Least Squares Approach to Adaptive Antennas", *Digital Signal Processing - A Review Journal*. Vol. 6, 185-194, 1996.

[2] T. K. Sarkar, Michael C. Wicks, Magdalena Salazar-Palma, and Robert J. Bonneau, "Smart Antennas", John Wiley & Sons, New Jersey, 2003.

[3] P. Snoeij and A. R. Vellekoop, "A Statistical Model for the Error Bounds of An Active Phased Array Antenna for SAR Applications" *IEEE Transactions Geoscience and Remote Sensing*. Vol. 30, 736 –742, 1992

[4] B. Friedlander, "The Root-MUSIC Algorithm for Direction Finding with Interpolated Arrays", *Signal Processing.* Vol. 30, 15-19, 1993.

[5] T. S Lee and T. T. Lin, "Adaptive Beamforming with Interpolation Arrays for Multiple Coherent Interferes"**,** *Signal Processing.* Vol. 7, 177-194, 1997.

[6] B. Friedlander and A. J. Weiss, "Direction Finding Using Spatial Smoothing with Interpolated Arrays"**,** *IEEE Transactions Aerospace and Electronics Systems.* Vol.28, No.2, Apr. 1992.

[7] B. Friedlander and A. J. Weiss, "Direction Finding in the Presence of Mutual Coupling"**,** *IEEE Transactions Antennas and Propagation*, Vol. 39, No.3, Mar 1991.

[8] B. M. Kolundzija, J. S. Ognjanovic, and T. K. Sarkar, "WIPL-D: Electromagnetic Modeling of Composite Metallic and Dielectric Structures", Artech House, Norwood, MA, 2000

 