Electromagnetic Scattering Analysis Based on Discrete Sources Method

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ABSTRACT. In this paper we examine theoretical and computational aspects of Discrete Sources Method applied to the solution of time-harmonic electromagnetic scattering problems

1 INTRODUCTION

Mathematical models in electromagnetic time-harmonic scattering theory are formulated as Boundary-Value Problems (BVPs). They consist of the Maxwell equations (Helmhotz equation), radiation conditions at the infinity and boundary conditions imposed on the discontinuity surfaces of permittivity and permeability of the medium. Their practical significance is determined by a wide field of applications in optics, radiophysics, computer tomography and etc. especially in connection with the development and implementation of new advanced technologies. From the theoretical point of view these problems are classical BVPs of mathematical physics. They represent exterior BVPs for a system of differential equations in partial derivatives, which in general case may be formulated as

$$\mathbf{L}\,(u) = 0, \quad M \in D_e = \Re^3 \setminus D_i\,, \qquad (1.1.a)$$

radiation conditions at infinity, (1.1.b)

boundary condition: $\mathbf{Q}\, u = -\,\mathbf{Q}\, u_0$ at ∂D. (1.1.c)

Here $\mathbf{L}$ is an operator of the external BVP imposed in outer region D_e; $\mathbf{Q}$ is a boundary condition operator; ∂D is a smooth closed surface; u_0 is an exciting field — either plane wave or a field of the local sources.

As a rule one considers both direct and inverse scattering problems. Treating the direct problems demands that physical characteristics of scattered field outside the obstacle should be determined. When investigating the inverse scattering problems such as the recognition problem or the synthesis one it is necessary to reconstruct the scatterers properties using a scattered field value. Note that most methods involved in the solution of inverse problems are based on a variational approach in which it is necessary to solve repeatedly direct problems [1]. This circumstance demands that the most effective tools for solving these problems be developed.

In this paper we are concerned with some aspects of constructing effective algorithms that may be employed for a wide variety of the scattering problems. Such problems are characterized by remoteness of exciting sources and observation points from an obstacle. The main feature of the problem consists in the fact that we are not interested in a detailed analysis of fields in the vicinity of the obstacle but we intend to obtain detailed information about scattered fields in a far zone from an obstacle. It enables one to avoid the time consumer algorithms similar to the precise solution of boundary integral equations but to use a Quasi-Solution (QS) concept for a BVP examination.

The essence of QS concept consists in the use of some semi-analytic construction u^δ corresponding to examined BVP. We shall construct QS in such a way that the following conditions hold

$$\mathbf{L}\,(u^\delta) = 0, \quad M \in D_e\,, \qquad (1.2.a)$$

radiation conditions at infinity, (1.2.b)

and approximate boundary condition:

$$\left\| \mathbf{Q}\, u^\delta + \mathbf{Q}\, u_0 \right\| \le \delta. \qquad (1.2.c)$$

Thus QS satisfies analytically a differential equation, radiation conditions and within the fixed accuracy the boundary condition in an appropriate norm at the obstacle surface. The latter circumstance should be provided by some numerical scheme that enables one determine all parameters of QS obeying (1.2.c). The advantage of the QS conception approach consists in the fact that we do not intend to solve any equations which solvability and equivalence to original BVP shall be proved.

It is based only on the assumption of solvability of the BVP for any external excitation. The fact that fitting (1.2.c) guarantees the closeness of QS to the exact solution of the BVP anywhere outside the obstacle plays the main role. In fact the existence of Green function (tensor) for the BVP (1.1) leads to the following estimation

$$\left\|u-u^{\delta}\right\|_{C(d)}=O\left(\left\|\mathbf{Q}u^{\delta}+\mathbf{Q}u_0\right\|_{L_2(d)}\right), \qquad (1.3)$$

where d is any compact in D_e. This relation means that to provide a closeness of the QS to the exact solution in a continuous metric outside the obstacle it is sufficient to approximate the boundary condition in $L_2(\partial D)$ norm [2]. Note that the approximation of boundary conditions in a mean square norm is not necessary for QS construction. One may use a weaken metric created by some kind of a smoothing operator.

QS conception may be realized in different forms. To investigate the scattering problems on a closed obstacle Discrete Sources Method (DSM) seems to be the most appropriate. DSM essence consists in the QS construction as a finite linear combination of elementary sources — dipoles and multipoles. Therefore the representation for the QS satisfies the Maxwell equations and radiation conditions at infinity. Discrete Sources (DS) amplitudes are to be determined from the boundary conditions on the local obstacle surface. Thus DSM is a semi-analytical method in the frame of which some conditions of the BVP are satisfied analytically but others — numerically. In fact the QS construction in this case is reduced to the approximation of the exciting field boundary value by a linear combination of the DS.

2 THEORETIC ASPECTS OF DSM

Let us begin to consider the basic theoretic aspects involved in the DSM realization. For simplicity we shall consider scattering of *TE* polarized plane wave by the infinite impedance cylinder with a smooth element $\partial D \in C^{(1,\alpha)}$. In this case

$$u=E_z, \quad \mathbf{L}=\nabla^2+k_e^2;$$

$$\mathbf{Q}=\partial/\partial n+\beta \quad (\operatorname{Im}\beta\geq 0, \quad \beta\in C^{(0,\alpha)}(\partial D)),$$

here $\partial/\partial n$ is a normal derivative at the contour ∂D.

In some publications DSM is regarded as a finite-dimension approximation of Auxiliary Current Method (ACM) [3-4]. Let us consider in detail the correlation between these two approaches. In the frame of ACM one may use for QS the following representation

$$u^{\delta}(M)=\int_{\gamma}\psi(M,p)\mu^{\delta}(p)\,dl_p, \qquad (2.1)$$

where ψ is a fundamental solution of Helmholtz equation $\psi(M,p)=-i/4\,H_0^{(2)}(k_eR_{Mp})$, contour $\gamma\in C^{(0,\alpha)}$ is situated completely inside D_i and $\mu^{\delta}\in L_t(\gamma)$ $(t\geq 1)$. Let us introduce an operator

$$\mathbf{G}\mu:=\int_{\gamma}\mathbf{Q}_p\psi(q,p)\,\mu(p)\,dl_p, \quad q\in\partial D \qquad (2.2)$$

Then to create QS of the BVP (1.1) it is sufficient to ensure that $R(\mathbf{G})$ (closure of range of values of the operator G) coincides with $L_2(\partial D)$. The latter fact means that for arbitrary $f\in L_2(\partial D)$ and for any $\delta>0$ there exists $\mu^{\delta}\in L_t(\gamma)$ so that

$$\left\|f-\mathbf{G}\mu^{\delta}\right\|_{L_2(\partial D)}\leq\delta. \qquad (2.3)$$

Let now a set of points $\{p_n\}$, $n=1, 2, \ldots$ be chosen on the contour γ and moreover the closure of $\{p_n\}=\gamma$. Then QS in the frame of the DSM takes the following form

$$u^N(M)=\sum_{n=1}^{N}\mu_n\,\psi(M,p_n). \qquad (2.4)$$

If for arbitrary $f\in L_2(\partial D)$ and any number $\delta>0$ there exist μ_n such that

$$\left\|f-\mathbf{Q}u^N\right\|_{L_2(\partial D)}\leq\delta \qquad (2.5)$$

holds, then by formula (1.3)

$$\lim_{N\to\infty}u^N(M)=u(M), \quad M\in d\subset D_e \qquad (2.6)$$

takes place. It is easy to prove that in case of a specific choice of the set $\{p_n\}$ so, that $\max\left|p_{n+1}-p_n\right|\to 0$ for $N\to\infty$ and from (2.3) the expression (2.5) may be obtained. Therefore to prove both the ACM and the DSM it is sufficient to ensure that closure of $R(\mathbf{G})=L_2(\partial D)$. In turn to prove the completeness of the $\mathbf{G}$ it is sufficient to establish that $\ker \mathbf{G}^* = \varnothing$. Here $\mathbf{G}^*$ is the conjugate to $\mathbf{G}$ operator having the following form

$$\mathbf{G}^* \nu := \int_{\partial D} \mathbf{Q}_q \psi(q,p)\, \nu^*(q)\, d\sigma_q, \qquad (2.7)$$

$$p \in \gamma, \quad q \in L_2(\partial D).$$

Let us determine the function

$$V(M) := \int_{\partial D} \mathbf{Q}_q \psi(q,M)\, \nu^*(q)\, d\sigma_q. \qquad (2.8)$$

Theorem 2.1. If the DS support γ is such that from $V(p) = 0$, $p \in \gamma$ one may receive $V(M) = 0$ in D_i then $\ker \mathbf{G}^* = \varnothing$ (cf. [5]).

Integral operator $\mathbf{G}$ acting from $L_t(\gamma)$ $(t \geq 1)$ to $L_2(\partial D)$ is a compact operator having an unclosed range of values. Therefore inverse to $\mathbf{G}$ operator $\mathbf{G}^{-1}$ is unbounded even in the range of $\mathbf{G}$. The latter fact leads to unboundedness of the currents sequence $\{j^\delta\}$ under $\delta \to 0$.

Theorem 2.2. Current sequence $\{j^N\}$ is limited in $L_t(\gamma)$ $(t > 1)$ norm then and only then when contour γ encloses all singularities of analytic continuation of a scattered field inside D_i (cf. [2]).

The above analysis shows that to provide the convergence of the QS u^N in a frame of the DSM to the exact solution u it is sufficient to select a DS support so that $V(p) = 0$, $p \in \gamma$ leads to $V(M) \equiv 0$ in D_i. Since V function (2.8) is an analytical function in D_i then selecting γ so that sufficient conditions of vanishing V in D_i will be fulfilled we will be able to construct a new complete functional system appropriate for the DSM.

Example 2.1. Let the support η be a part of analytic closed nonresonance contour γ (region bounded by γ is supposed to be nonresonance for the fixed k_e). Let $\{p_n\}$ have at least one limit point on γ then the corresponding complete system takes the form [2]

$$\psi_n(M) = \psi(M, p_n), \quad p_n \in \eta. \qquad (2.9)$$

Example 2.2. Let the support ζ be the segment of any $\eta \in C^{(1,\alpha)}$ curve inside the D_i and the closure of $\{p_n\} = \zeta$ then corresponding complete system takes the following form [5]

$$\left\{\psi(M,p),\ \partial\psi(M,p)/\partial n_p\right\}, \quad p = p_n \in \zeta \qquad (2.10)$$

System (2.10) is the most suitable for the analysis of the oblate obstacles or "thick" screens [2].

In the same manner as above it is easy to construct complete systems for an internal problem. For instance in the case when support is the same as in Example 2.1 the corresponding complete system is

$$\chi(M,p) = J_0(k_i R_{Mp}), \quad p = p_n, \quad p_n \in \gamma.$$

Note that in most other publication [3,4] DS having the singularities at a some contour in D_e are usually used for the representation of an internal field. So, completeness of a DS system in $L_2(\partial D)$ formally provides a convergence of QS to the exact solution outside D_e. For a selected scheme of the DSM realization it is necessary to coordinate the choice of support γ with the singularities of analytic continuation of the scattered field.

Let us consider a numerical scheme for determination of the amplitudes. We introduce the vector of amplitudes $\mathbf{p} = \{p_n\}$, $n = 1,\ldots,N$ and the value of $\mathbf{p}$ is to be determined as

$$\mathbf{p} := \arg\min \left\| \mathbf{Q} u^N + \mathbf{Q} u_0 \right\|_{L_2(\partial D)}.$$

Then vector $\mathbf{p}$ will be a solution of normal system [6]. It is known the determinant of that system is the Gramm's kind determinant $\mathbf{G}_N$. As the functional system $\{\mathbf{Q}_q \psi(q, M_n)\}$ is linear independent then we have $\det \mathbf{G}_N = g_N > 0$ for the all N.

Theorem 2.4. Let all points of $\{M_n\}$ be different and $\max |M_{n+1} - M_n| \to 0$ under $N \to \infty$, then the following estimate

$$g_N = O\left(\xi^N \prod_{n=1}^{N} h_n\right), \quad N \gg 1, \qquad (2.11)$$

holds (cf. [6]). Here ξ is constant depending only on mutual disposition of support γ and contour ∂D and $h_n = |M_{n+1} - M_n|$. It is easy to see that maximum of the estimate is achieved in case of an uniform step.

Corollary. For any $\delta > 0$ and arbitrary contours γ, ∂D there exists a such number $N_0(\gamma, \partial D)$ that $g_N < \delta$ holds for all $N \geq N_0$. Therefore the determination of DS amplitudes as a solution of the normal system is unstable under any disposition of γ inside D_i.

Now let us consider a scheme of amplitudes determination obtained from a point-matching approach. Let $\{q_j\}$, $j=1,\ldots,J$ be a set of matching points then the corresponding linear system for DS amplitudes determination is

$$\sum_{n=1}^{N} p_n \psi_n(q_j) = -u_0(q_j); \quad q_j \in \partial D, \qquad (2.12)$$

$j = 1, \ldots, J$. Then the product of singular numbers λ_n of the matrix $\mathbf{K}_{JN}$ (2.12) may be estimated as

$$\prod_{n=1}^{N} |\lambda_n| = h_0^{-N/2} g_N^{1/2}, \quad N, J >> 1. \qquad (2.13)$$

Matching points are assumed to be uniformly spaced on ∂D at a distance of h_0. It is evident that estimate (2.13) at $N = J$ is better than (2.11). This estimate enables one as previously to prove instability of the point-matching scheme for the determination of DS amplitudes. Note that instability occurs irrespective of the position of singularities of the analytic continuation. Since determinant (2.13) tends to zero there appear difficulties we have spoken above, in particular unboundedness of the DS amplitudes sequence.

Lately overdetermined linear systems have been preferably used for determination of DS amplitudes [6-7]. In this case the number of matching points is not equal to DS number $J > N$. Here the value of h_0 depends only on J. Thus, the estimate (2.13) is refined as J becomes larger. So, the overdetermined system of the matching-point method seems to realize a more stable numerical scheme. We shall use this approach for DS amplitudes determination. It is important to note that using an overdeterminated system it is not necessary to increase the number of matching points with respect to DS number. Conversely, using an overdeterminated system allows us to reduce the DS number [2].

Note that the DSM has some essential preferences:

- a simple structure of DS fields;
- the possibility of quickly passing from the solution of one BVP to the another, more complex;
- internal criterion for QS error estimation.

All these circumstances have allowed for a short period of time to adjust the DSM for solving a wide variety of BVPs in the scattering theory [2,8].

3 ANALYSIS OF AXI-SYMMETRICAL OBSTACLES

In this part we are going to consider problems of scattering by bodies of revolution. Electromagnetic scattering problems have some peculiarities for axi-symmetrical obstacles. One is able to formulate QS in such a manner that it takes into account both rotation symmetry of an obstacle and the polarization of excitation. In this case the selection of DS support plays a significant role. Before constructing QS for the Maxwell equations let us consider the main theoretical principles for Helmholtz equation. It seems to be reasonable because the complete sets of dipoles and multipoles are as a rule based upon a complete set of Helmholtz solutions [2]. In view of that approach completeness of the fields of dipoles and multipoles is provided by the appropriate choice of their orientation [2].

So we are going to consider the BVP in $\mathfrak{R}^3$ space. Let ∂D be a surface and $\partial D \in C^{(1,\alpha)}$, then there exists a unique solution of (1.1). In this case the fundamental solution of Helmholtz equation is

$$\psi(M,p) = \frac{k_e}{4\pi i} h_0^{(2)}\left(k_e R_{Mp}\right),$$

where $h_0^{(2)}$ is Hankel spherical function. When constructing QS of the BVP (1.1) there are different possibilities to select the support of DS. The simplest one seems to be the choice of DS support deposited on the auxiliary surface disposed inside the obstacle. Nevertheless this approach realizes a rather time consuming numerical schemes. It is correct both in the case for approximation of the fields at the obstacle surface ∂D and for the case when QS is represented as Fourier series with respect to azimuthal variable ϕ.

The approach developed in the previous part allows a set of arbitrary dimension as DS support, in particular a segment of symmetry axis situated inside of obstacle D_i to be chosen. Let region D_i be simply connected. Let us choose as a DS set $\overline{\{w_n\}_{n=1}^{\infty}} = \gamma_z$, where γ_z is a segment of axis Oz. Then the complete system corresponding to the geometry of this support is

$$V_m^e(M) = Y_m^e(q) \begin{matrix} \sin m\phi \\ \cos m\phi \end{matrix} \quad (3.1)$$

Here $q \in \Phi$, Φ is a semiplane ϕ = const, m = 0, 1, 2, ..., and functions Y_m^e have a form

$$Y_m^e(q,w) = h_m^{(2)}\left(k_e R_{qw}\right) P_m^m(\cos\theta_w) \times k_e^m / (2m-1)!!$$

where $P_m^m(.)$ is adjoint Legendre polynomial, $R_{qw}^2 = \rho^2 + (z-w)^2$, $q = (\rho, z)$, and (ρ,ϕ,z) are cylindrical coordinates. It is easy to ensure that functions $V_m^e(M)$ satisfy Helmholtz equation

$$\left(\nabla^2 + k_e^2\right) V_m^e(M) = 0 \quad \text{in} \quad D_e$$

and radiation condition, and for a fixed m they are lowest-order multipoles [2]. As

$$P_m^m(\cos\theta_w)/(2m-1)!! \; = \; (\rho/R_{qw})^m,$$

then (3.2) can be rewritten in the form

$$Y_m^e(q,w) = h_m^{(2)}(k_0 R_{qw}) \times (k_0 \, \rho / R_{qw})^m \quad (3.2)$$

For consideration of the similar internal BVP in D_i the complete system of the DS fields corresponding to the chosen support structure should have the form similar to (3.1)

$$V_m^i(M) = Y_m^i(q) \begin{matrix} \sin m\phi \\ \cos m\phi \end{matrix} \quad (3.3)$$

where functions $Y_m^i(q)$ have the following form

$$Y_m^i(q,w) = j_m(k_i R_{qw}) \times (k_i \, \rho / R_{qw})^m \quad (3.4)$$

Here $j_m(.)$ is the Bessel spherical function. In this case $Y_m^i(q)$ fields are regular functions having the singularities at infinity.

Theorem 3.1. Let closure $\{w_n\}_{n=1}^{\infty} = \gamma_z$, where γ_z is a segment of axis Oz. Then systems (3.1), (3.3) are complete in $L_2(\partial D)$ (cf. [6]).

Note that systems (3.1) and (3.3) are suitable for the analysis of scattering by axi-symmtrical obstacles for two reasons. First, these functions are orthogonal at the surface ∂D with respect to ϕ variable, and second, for any bounded m they may be represented as a finite linear combination of elementary functions.

As it was shown above the convergence of QS to the exact solution of BVP (1.1) is ensured not only by the completeness of the DS system but also by the behavior of DS amplitudes at $N \to \infty$ as well. The boundedness of the sequence of DS amplitudes may be achieved if the support is disposed in agreement with the location of singularities of the scattered field analytic continuation. Therefore the choice of the DS support γ_z at the symmetry axis essentially restricts the DSM applicability. In order to remove the mentioned restriction the procedure of analytic continuation of fields of the DS (3.1), (3.3) onto the complex plane W with respect to coordinate w [6] may be considered. Thus we will receive the following result.

Theorem 3.2. Let the DS set $\{w_n\}_{n=1}^{\infty} \subset D_w \subset W$ and there exists a limit point $w' \in D_w$, then the QS for an external (internal) BVP (1.1) can be constructed on the basis of functional system (3.1) ((3.3)) (cf. [6]).

Thus by selecting the DS support in the region D_w of the complex plane W we can coordinate the location of this support with the singularities of the analytical continuation of the scattered fields into region D_i. This approach proved to be especially efficient in investigating oblate obstacles [5]. We shall refer to the support used in Theorem 3.2 as γ_w.

4 EM SCATTERING BY PENETRABLE OBSTACLES

Let us consider scattering of an electromagnetic plane wave $\{\mathbf{E}^0, \mathbf{H}^0\}$ by the uniform magneto-dielectric body of revolution D_i. The mathematical formulation of BVP is

$$\operatorname{curl} \mathbf{H}_t = ik\varepsilon_t \mathbf{E}_t; \quad \operatorname{curl} \mathbf{E}_t = -ik\mu_t \mathbf{H}_t \quad \text{in } D_t, \quad t = i,e \quad (4.1)$$

$$\left.\begin{aligned} \mathbf{n}_p \times (\mathbf{E}_i(p) - \mathbf{E}_e(p)) &= \mathbf{n}_p \times \mathbf{E}^0(p), \\ \mathbf{n}_p \times (\mathbf{H}_i(p) - \mathbf{H}_e(p)) &= \mathbf{n}_p \times \mathbf{H}^0(p). \end{aligned}\right\} \quad p \in \partial D$$

Silver-Muller radiation conditions at the infinity holds as well [9]. Here $\mathbf{a}\times\mathbf{b}$ is a vector product, $\mathbf{n}_p$ is a unit normal to ∂D, $k = \omega / c$, $\varepsilon_e, \mu_e > 0$,

$\mathrm{Im}\,\varepsilon_i, \mu_i \ge 0$, $\partial D \in C^{(1,\alpha)}$, then there exists a unique solution of BVP (4.1). Let us introduce into consideration the systems of fields of the electric and magnetic multipoles

$$\begin{pmatrix}\mathbf{E}^e_{\upsilon t}\\ \mathbf{H}^e_{\upsilon t}\end{pmatrix} = \begin{pmatrix} i/k\varepsilon_t\mu_t & \mathrm{curl\,curl}\\ -1/\mu_t & \mathrm{curl}\end{pmatrix}\mathbf{A}^e_{\upsilon t};$$

$$\begin{pmatrix}\mathbf{E}^h_{\upsilon t}\\ \mathbf{H}^h_{\upsilon t}\end{pmatrix} = \begin{pmatrix} 1/\varepsilon_t & \mathrm{curl}\\ i/k\varepsilon_t\mu_t & \mathrm{curl\,curl}\end{pmatrix}\mathbf{A}^h_{\upsilon t}; \tag{4.2}$$

where Hertz vectors are $\mathbf{A}^{e,h}_{\upsilon t} = V^t_{mn}(q)\,\mathbf{e}_l$ and υ is multindex: $\upsilon = (n,m,l)$, $1 \le n \le N_t$, $0 \le m \le M$, $l = 1,2,3$; $\{\mathbf{e}_l\}$ is Cartesian basis. The following result holds

Theorem 4.1. Let ∂D be an arbitrary (not necessarily axi-symmetric) surface, and DS support be γ_z. Then QS of the BVP (4.1) can be constructed in the following form [2]

$$\begin{pmatrix}\mathbf{E}^S_t\\ \mathbf{H}^S_t\end{pmatrix} := \sum_{\upsilon}^{S} p^t_\upsilon \begin{pmatrix}\mathbf{E}^e_{\upsilon t}\\ \mathbf{H}^e_{\upsilon t}\end{pmatrix} \tag{4.3}$$

So, one may construct the QS representation based on the electric multipoles. The same result holds for the magnetic ones as well.

The representation (4.3) for axi-symmetric obstacles due to the choice of support γ_z has a form of the finite Fourier sum with respect to azimuthal variable ϕ. In future we shall consider the linearly polarized *TM/TE* plane wave as external excitation. So let us examine the convergence of the Fourier series for the plane wave at the obstacle surface. For the case of *TM* plane wave propagating at angle θ_0 to Oz axis the electromagnetic fields are

$$\begin{aligned}\mathbf{E}^0 &= (\mathbf{e}_1 \cos\theta_0 + \mathbf{e}_3 \sin\theta_0)\, e''\\ \mathbf{H}^0 &= -\mathbf{e}_2\, e''\end{aligned} \tag{4.4}$$

where $e'' = \exp\{ik_e(\rho\cos\theta_0\sin\theta_0 + z\cos\theta_0)\}$. Let us represent plane wave in the form of Fourier series

$$\exp\{ik_e\rho\sin\theta_0\cos\phi\} = \sum_{l=0}^{\infty}(2-\delta_{0l})\, i^l\, J_l(k_e\rho\sin\theta_0)\cos l\phi \tag{4.5}$$

where δ_{0l} is the Kroneker delta, $J_1(.)$ is the cyllindrical Bessel function. One easily sees that convergence of series (4.5) is determined by the value of $k_e a \sin\theta_0$, where a is the radius of the body D_i. Thus for the prolate bodies having small values of parameter a one may use only several number of Fourier harmonics for representation (4.3).

We shall determine the coefficients p^t_υ in a following manner

$$\begin{aligned} p^t_\upsilon = \arg\min \{ &\left\| \mathbf{n}_p \times \left(\mathbf{E}^s_i(p) - \mathbf{E}^s_e(p) - \mathbf{E}^0(p)\right)\right\|_{L^\tau_2(\partial D)}\\ &+ \left\| \mathbf{n}_p \times \left(\mathbf{H}^s_i(p) - \mathbf{H}^s_e(p) - \mathbf{H}^0(p)\right)\right\|_{L^\tau_2(\partial D)} \}\end{aligned} \tag{4.6}$$

where $L^\tau_2(\partial D)$ is the space of vectors situated in the tangential plane, and having components belonging to $L_2(\partial D)$. Then using formula (4.5) and taking into account that representation (4.3) is the sum of Fourier series on ∂D we shall determine the DS amplitudes from a sequence of optimization problems for Fourier harmonics. So we are able to determine the coefficients p^t_υ subsequently from *m = 0* to *m = M*. Below we shall determine the DS amplitudes as a solution of the point-matching overdetermined system.

If we use the representation (4.3) even for the case of an equal number both for matching points *J* and for the DS number *N* we will obtain an underdetermined linear system. This result follows from the fact that at each matching point we should satisfy four conditions (two tangential components for **E** field and two for **H**) but at any point w_n there are six sources: three (*l* = 1,2,3) for the external field representation and three for the internal one. This circumstance enables the number of DS to be reduced. It can be done if one takes into account a polarization of the external excitation. Let us consider the case of *TM* polarization.

Theorem 4.2. Let DS set be the same as in the Theorem 4.1, ∂D is the surface of revolution and $\{E^0, H^0\}$: (4.4). Then QS of the BVP (4.1) may be constructed in the form (cf. [6])

$$\begin{pmatrix}\mathbf{E}_t^S\\ \mathbf{H}_t^S\end{pmatrix} = \sum_s^S \left\{ p_s^t \begin{pmatrix}\mathbf{E}_{st}^e\\ \mathbf{H}_{st}^e\end{pmatrix} + q_s^t \begin{pmatrix}\mathbf{E}_{st}^h\\ \mathbf{H}_{st}^h\end{pmatrix}\right\} + \Sigma_{dip}^{TM},$$
$$\Sigma_{dip}^{TM} = \sum_{n=1}^{N} r_n^t \begin{pmatrix}\mathbf{E}_{nt}^e\\ \mathbf{H}_{nt}^e\end{pmatrix} \qquad (4.7)$$

(see also (4.2)) where Hertz vectors are

$$\mathbf{A}_{st}^e = Y_m^t(q,w_n)\left(\cos m\phi\, \mathbf{e}_1 - \sin m\phi\, \mathbf{e}_2\right),$$
$$\mathbf{A}_{st}^h = Y_m^t(q,w_n)\left(\cos m\phi\, \mathbf{e}_2 + \sin m\phi\, \mathbf{e}_1\right),$$
$$\mathbf{A}_{nt}^e = Y_0^t(q,w_n)\mathbf{e}_3.$$

In contrast to (4.3) the representation (4.7) is constructed by multipoles and dipoles oriented according to TM polarization of the plane wave (4.4). For the case of TE polarization the exciting field is

$$\mathbf{H}^0 = (\mathbf{e}_1 \cos\theta_0 + \mathbf{e}_3 \sin\theta_0)\, e''$$
$$\mathbf{E}^0 = \mathbf{e}_2\, e'' \qquad (4.8)$$

In this case the same result as in Theorem 4.2 occurs and QS representation has the form (4.7) but

$$\Sigma_{dip}^{TE} = \sum_{n=1}^{N} r_n^t \begin{pmatrix}\mathbf{E}_{nt}^h\\ \mathbf{H}_{nt}^h\end{pmatrix}, \qquad (4.9)$$
$$\mathbf{A}_{st}^e = Y_m^t(q,w_n)\left(\sin m\phi\, \mathbf{e}_1 + \cos m\phi\, \mathbf{e}_2\right),$$
$$\mathbf{A}_{st}^h = Y_m^t(q,w_n)\left(\cos m\phi\, \mathbf{e}_1 - \sin m\phi\, \mathbf{e}_2\right),$$
$$\mathbf{A}_{nt}^h = Y_0^t(q,w_n)\mathbf{e}_3,$$

One can see that instead of three electrical multipoles at each point w_n used in representation (4.3), in formulas (4.7), (4.9) only two (electrical multipole situated at the E-plane and magnetic at the H-plane) are used. The latter circumstance allows us when the number of sources is equal to the number of matching points to obtain the square matrix for the determination of DS amplitudes.

Let us proceed to the numerical scheme for DS amplitude determination based on representations (4.7), (4.9). We introduce into consideration the amplitude vectors

$$\mathbf{p}_m = \{p_m^e; p_m^i\},$$
$$p_m^e = \{p_{nm}^e, q_{nm}^e\}_{n=1}^{N_e}, \quad p_m^i = \{p_{nm}^i, q_{nm}^i\}_{n=1}^{N_i}$$
$$\mathbf{r} = \{r^e; r^i\}, \quad r^e = \{r_n^e\}_{n=1}^{N_e}, \quad r^i = \{r_n^i\}_{n=1}^{N_i}$$

The values of vectors $\mathbf{p}_m$ and $\mathbf{r}$ are calculated as a pseudosolution of the overdetermined linear systems obtained from a point-matching approach for the tangential components of Fourier harmonics of electromagnetic fields. Let $\{\eta_j\}_{j=1}^{J}$ be the set of matching points. Then we will obtain the point-matching system for the determination of DS amplitudes in the form

$$\mathbf{B}_m \mathbf{p}_m = 0.5\{\mathbf{Q}_m(1+\delta_{0m}) + \mathbf{R}_{m+1} + \mathbf{T}_{m+2}\} \qquad (4.10)$$

$m = \overline{0, M}$. Here $\mathbf{B}_m$ is $4J \times 2(N_i + N_e)$ matrix, whose rows contain the tangential components of multipoles fields at the matching points $\{\eta_j\}_{j=1}^{J}$, $j = 1,\dots,J$, $\mathbf{p}_m$ are $2(N_i + N_e)$ vectors, $\mathbf{Q}_m$, $\mathbf{R}_m$, $\mathbf{T}_m$ are $4J$ vectors. For TM case these vectors have the forms

$$\mathbf{Q}_m = \{\xi^j \cos\theta_0; -\cos\theta_0; \xi^j; 1\}\, S_m^j,$$
$$\mathbf{T}_m = \{\xi^j \cos\theta_0; -\cos\theta_0; \xi^j; 1\}\, S_m^j,$$
$$\mathbf{R}_m = \{R_m^j; 0; 0; 0\}, \quad R_m^j = 2\zeta^j \sin\theta_0\, S_m^j,$$
$$S_m^j = i^m (2 - \delta_{0m})\, J_m(k_e \rho^j \sin\theta_0) \exp\{ik_e z^j \cos\theta_0\},$$

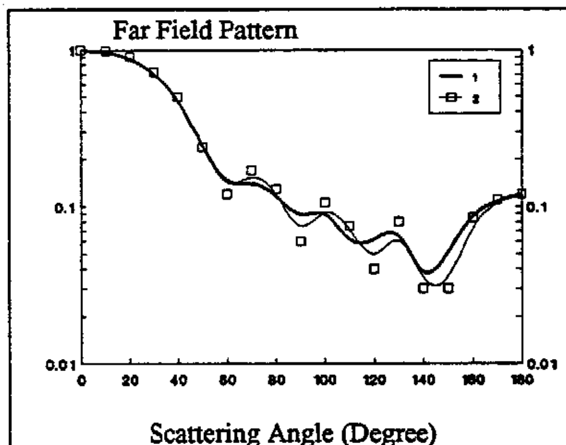

Figure 1. Comparison of scattering diagrams F_ϕ with the method of integral equations.

where $(\xi^j, \zeta^j) = \tau^j$ is a vector tangential to the surface element $\Im$ at the η_j point. To find the $\mathbf{r}$ vector the following linear system should be used

$$\mathbf{B}_{-1}\mathbf{r} = 0.5\,\mathbf{c}, \qquad (4.11)$$

here $\mathbf{B}_{-1}$ is $2J \times (N_i + N_e)$ matrix and $\mathbf{c}$ is $2J$ vector having the form

$$\mathbf{c} = \{\xi^j \cos\theta_0\, S_1^j + R_0^j; S_1^j\}.$$

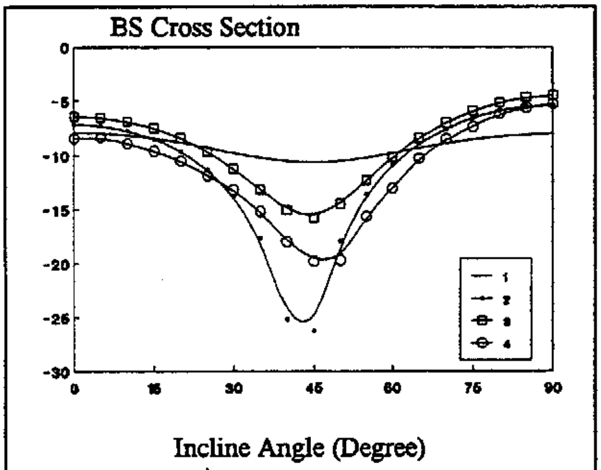

Figure 2. Back scattering cross-section as function of the incline angle. The curves 1,2,3,4 correspond to $\alpha = 0.2; 0.4; 0.6; 0.8$.

Representations (4.7), (4.9) allow us not only to construct the QS taking into account the exciting field polarization but also to calculate simultaneously the DS amplitudes for *TM* and *TE* polarizations as a solution of the linear system (4.10) with the same matrix but different right hand parts.

The basis for computing of the energy scattering characteristics in a far zone appears to be a scattering diagram **F** [9]

$$\mathbf{E}_e = \frac{\exp\{-ik_e r\}}{r}\mathbf{F} + o\left(\frac{1}{r}\right), \quad r \to \infty,$$

where $\mathbf{F} = \{0; F_\theta; F_\varphi\}$. After DS amplitudes have been determined it is easy to calculate the value of the scattering diagram. Its components for *TM* polarization are

$$F_\theta(\theta,\phi) = i\sum_{m=0}^{M}\{(-ik_e)^{m+1}\sin^m\theta\cos(m+1)\phi \times \sum_{n=1}^{N_e}(p_{nm}^e\cos\theta + q_{nm}^e)G_n\} - k_e\sin\theta\sum_{n=1}^{N_e} r_{nm}^e G_n$$

$$F_\phi(\theta,\phi) = -i\sum_{m=0}^{M}\{(-ik_e)^{m+1}\sin^m\theta\sin(m+1)\phi \times \sum_{n=1}^{N_e}(p_{nm}^e + q_{nm}^e\cos\theta)\,G_n\} \qquad (4.12)$$

here $G_n = \exp\{-ik_e z_n^e\cos\theta\}$, θ is a scattering angle.

The comparison of scattering diagrams F_ϕ obtained by two different methods is demonstrated in Figure 1. The obstacle is the prolate dielectric rod $\varepsilon = 2.5$, frequency $k_e a = 132$ ($2a$ is the rod diameter), rod length/diameter ratio is equal to 5; an exciting plane wave propagates along the rod symmetry axis. The curve 1 is obtained as DSM result, and markers (curve 2) show the result of an integral equation method [2]. We used for our calculations PC 486DX-50. The integral equation method spent 4.5 min. and DSM spent 10 sec (48 dipoles were used).

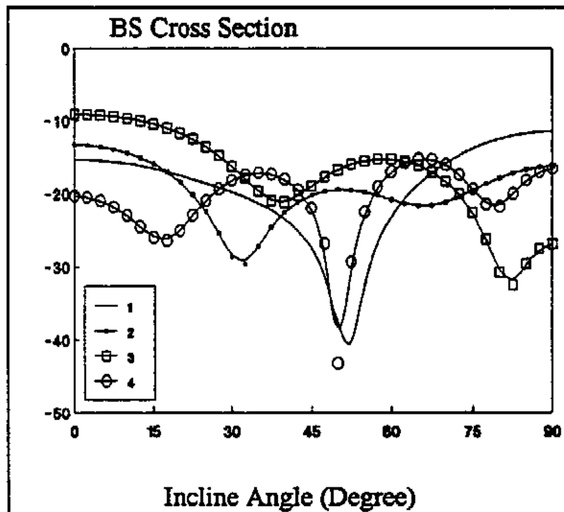

Figure 3. Back scattering cross-section as function of the incline angle. The curves 1, 2, 3, 4 correspond to the value $\alpha = 1$ and distance between obstacles as $\delta/R_0 = 0.05; 0.5; 1; 2$.

We will illustrate the described technique for the obstacle having the following surface element

$$r(t) = R_0\{1-\alpha+7/5\alpha^2 - 9/35\alpha^3\}^{-\frac{1}{3}}(1+\alpha\cos 2t)$$

here $R_0 = const$ and parameter α determines the obstacle shape. So the obstacle can vary in shape from spherical ($\alpha = 0$) to the prolate one ($\alpha \le 0.2$), then to the dumb-bell ($0.2 < \alpha < 1$) and doubled ($\alpha = 1$) shape. Such models are of interest in the scattering problems when one investigates the propagation of radio wave through the active melting layer of atmosphere [10].

We let the wave length to be $\lambda = 3.2$ cm, $R_0 = 1$ cm, refractive index of particle $m = 1.78 + 0.0024i$. The dependencies $\Psi = 10\log(\sigma/\lambda^2)$ are given in Figures 2,3 as a function of angle θ_0. The curves 1,2,3,4 in Figure 2 correspond to the values of $\alpha =$

0.2; 0.4; 0.6; 0.8; in Figure 3 the same curves correspond to $\alpha = 1$, and the distances between obstacles are $\delta/R_0 = 0.05$; 0.5; 1; 2. The expended time for each curve was 1.5 min.

5 PENETRABLE PARTICLE ON SUBSTRATE

The control of a contamination of a silicon wafer surface is very important in semiconductor industry. To detect contamination or to recognize a particle from other defects one needs to solve a problem of light scattering by structures on a wafer surface. The inherent difficulty in the modeling of such scattering problem consists in that an obstacle now is not a local scatterer. So one should take into account the presence of a wafer surface.

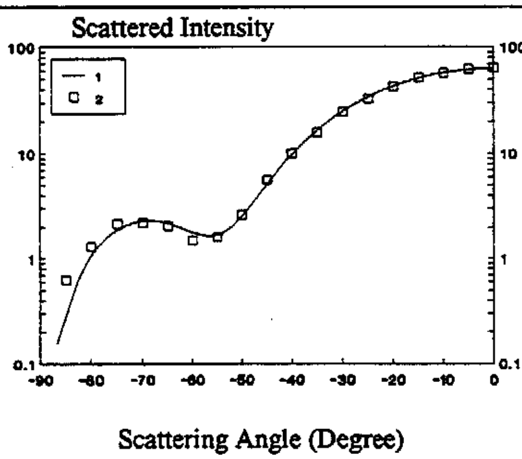

Figure 4. Scattered intensity (curve 1) compared with Wojcik [11] results (curve 2).

Let an axi-symmetric penetrable particle be placed at the plane Ξ so that the axis of symmetry is normal to Ξ. Let us choose the Cartesian coordinate system so that the origin of coordinates is located at the contact point between particle and the plane Ξ. The external excitation is assumed to be a *P/S* polarized plane wave propagating at angle θ_0 with respect to *Oz* axis. Then the mathematical statement of the light scattering problem has a form

$$\operatorname{curl} \mathbf{H}_t = ik\varepsilon_t \mathbf{E}_t; \quad \operatorname{curl} \mathbf{E}_t = -ik\mu_t \mathbf{H}_t \quad \text{in } D_t,$$

$$\begin{aligned} \mathbf{n}_p \times (\mathbf{E}_i(p) - \mathbf{E}_e(p)) &= \mathbf{n}_p \times \mathbf{E}^0(p), \\ \mathbf{n}_p \times (\mathbf{H}_i(p) - \mathbf{H}_e(p)) &= \mathbf{n}_p \times \mathbf{H}^0(p), \end{aligned} \quad p \in \partial D$$

$$\begin{aligned} \mathbf{e}_z \times (\mathbf{E}_0(p) - \mathbf{E}_e(p)) &= 0, \\ \mathbf{e}_z \times (\mathbf{H}_0(p) - \mathbf{H}_e(p)) &= 0, \end{aligned} \quad p \in \Xi \tag{5.1}$$

with the radiation/attenuation conditions at the infinity ($z \neq 0$). Here $t = i,0,1$; $\operatorname{Im}\varepsilon_i, \mu_i \geq 0$. The index i corresponds to the particle, 0 to the ambient space, and 1 to the substrate half-space. Let $\partial D \in C^{(1,\alpha)}$ then there exists a unique solution of the BVP (5.1).

For the BVP (5.1) the Green tensor is

$$\tilde{G}(M,P) = \begin{Vmatrix} g & 0 & 0 \\ 0 & g & 0 \\ -\partial f/\partial x & -\partial f/\partial y & \sigma \end{Vmatrix}, \tag{5.2}$$

where the tensor elements are the following functions

$$g^{e,h}(M,P) = \exp\{-ik_0 R\}/R - \exp\{-ik_0 R'\}/R'$$
$$+ \int_0^\infty J_0(\lambda r)\, \kappa^{e,h} \exp\{-\eta_0(z+z_P)\}\, \lambda d\lambda,$$

$$f(M,P) = \int_0^\infty J_0(\lambda r)\, \varsigma \exp\{-\eta_0(z+z_P)\}\, \lambda d\lambda,$$

$$\sigma^{e,h}(M,P) = g^{h,e}(M,P) \tag{5.3}$$

here $r = \{(x - x_P)^2 + (y - y_P)^2\}^{1/2}$ is the distance on the plane and $R = \{r^2 + (z - z_P)^2\}^{1/2}$ is distance in $\Re^3$ space; R' means $R' = \{r^2 + (z + z_P)^2\}^{1/2}$. The spectral functions $\kappa^{e,h}$ and ζ are given by

$$\kappa^e = 2\mu_1/(\mu_1\eta_0 + \mu_0\eta_1),$$
$$\kappa^h = 2\varepsilon_1/(\varepsilon_1\eta_0 + \varepsilon_0\eta_1),$$
$$\zeta = 2(\mu_1\eta_0 + \mu_0\eta_1)/\{(\mu_1\eta_0 + \mu_0\eta_1)(\mu_1\eta_0 + \mu_0\eta_1)\},$$
$$\eta_t = (\lambda^2 - k_t^2)^{1/2}, \tag{5.4}$$

For the electrical dipole directed along $\mathbf{e}_x$ in the presence of half-space the Hertz vector has the form

$$\mathbf{A}^e = \mathbf{e}_x g^e + \mathbf{e}_z(\mathbf{e}_x, \nabla) f, \tag{5.5}$$

and for the case of magnetic dipole along $\mathbf{e}_y$

$$\mathbf{A}^h = \mathbf{e}_y g^h + \mathbf{e}_z(\mathbf{e}_y, \nabla) f. \tag{5.6}$$

Expanding the components of the Green tensor in the Fourier series with respect to azimuthal variable ϕ we receive

$$G_{ij}(Q,P) = \sum_m \Gamma_{ij}^m(q,p) \begin{matrix} \sin m\phi \\ \cos m\phi \end{matrix},$$

where Γ_{ij}^m are the Fourier harmonics of functions G_{ij} and points $p,q \in \Phi$. Let us introduce into consideration the following functions

$$G_{ij}^m(q;z_p) = \lim_{\rho_p \to 0} \{\Gamma_{ij}^m(\rho,z;\rho_p,z_p)/\rho_p^m\}, \quad (5.7)$$

then the functions forming the tensor (5.7) accept the following form

$$g_m^{e,h} = Y_m(q,w_n) - Y_m(q,-w_n) + \int J_m(\lambda\rho)\,\kappa^{e,h} \exp\{-\eta_0(z+w_n)\}\,\lambda^{1+m} d\lambda,$$

$$f_m = \int J_m(\lambda\rho)\,\zeta \exp\{-\eta_0(z+w_n)\}\,\lambda^{1+m} d\lambda \quad (5.8)$$

Let us denote

$$\mathbf{A}_{mx}^{e,h} = \mathbf{e}_x\, g_m^{e,h} + \mathbf{e}_z(\mathbf{e}_x,\nabla) f_m,$$
$$\mathbf{A}_{my}^{e,h} = \mathbf{e}_y\, g_m^{e,h} + \mathbf{e}_z(\mathbf{e}_y,\nabla) f_m,$$

and form the following combinations

$$\begin{aligned} \mathbf{A}_m^e &:= \mathbf{A}_{mx}^e \cos m\phi - \mathbf{A}_{my}^e \sin m\phi \\ \mathbf{A}_m^h &:= \mathbf{A}_{mx}^h \sin m\phi + \mathbf{A}_{my}^h \cos m\phi \end{aligned} \quad (5.9)$$

Then the QS of BVP (5.1) accepts the form

$$\begin{pmatrix} \mathbf{E}_s \\ \mathbf{H}_s \end{pmatrix} = \sum_s \{p_s \begin{pmatrix} i/k\varepsilon\mu & \mathrm{curl\,curl} \\ -1/\mu & \mathrm{curl} \end{pmatrix} \mathbf{A}_s^e + q_s \begin{pmatrix} 1/\varepsilon & \mathrm{curl} \\ i/k\varepsilon\mu & \mathrm{curl\,curl} \end{pmatrix} \mathbf{A}_s^h\} + \sum_n r_n \begin{pmatrix} i/k\varepsilon\mu & \mathrm{curl\,curl} \\ -1/\mu & \mathrm{curl} \end{pmatrix} \mathbf{A}_{n0}^e \quad (5.10)$$

Here $s = \{m, n\}$ is multindex, $0 \le m \le M$, $1 \le n \le N_t$. Assuming that the external excitation is P-polarized plane wave we can represent the sum of incident and reflected fields as

$$\mathbf{E}^0 = \mathbf{e}_1 \cos\theta_0(\chi_1 - R_P\,\chi_2) + \mathbf{e}_3 \sin\theta_0(\chi_1 + R_P\,\chi_2),$$
$$\mathbf{H}^0 = -\mathbf{e}_2 \cos\theta_0(\chi_1 + R_P\,\chi_2), \quad (5.12)$$

where R_P, is the Fresnel coefficient

$$R_P = \frac{\varepsilon_1\cos\theta_0 - (\varepsilon_1 - \sin^2\theta_0)^{1/2}}{\varepsilon_1\cos\theta_0 + (\varepsilon_1 - \sin^2\theta_0)^{1/2}},$$

$$\chi_1 = \exp\{-ik_0(x\sin\theta_0 - z\cos\theta_0)\},$$

$$\chi_2 = \exp\{-ik_0(x\sin\theta_0 + z\cos\theta_0)\}.$$

To obtain the scattering diagram one may use an asymptotic technique for approximation of the Sommerfeld integrals. So we have for the P case

$$F_\theta = ik_0 \sum_m \cos(m+1)\phi(ik_0\sin\theta)^m \times \sum_n \{p_{nm}\cos\theta\,[G_n' + (\bar\kappa^e - \sin^2\theta\,\zeta)G_n] + q_{nm}(G_n' + \bar\kappa^h G_n)\} - ik_0\cos\theta \sum_n r_n(G_n' + \bar\kappa^h G_n),$$

$$F_\phi = -ik_0 \sum_m \sin(m+1)\phi(ik_0\sin\theta)^m \sum_n \{p_{nm}\cos\theta \times (G_n' + \bar\kappa^e)G_n] + q_{nm}[G_n' + (\bar\kappa^e - \sin^2\theta\,\zeta)G_n]\},$$

where $\psi_0 = \exp\{-ik_0 r\}/r$, $G_n = \exp\{-ik_0 w_n\cos\theta\}$, and $G'_n = \exp\{ik_0\,w_n\cos\theta\}$,

$$\bar\kappa^e = \sqrt{\varepsilon_1 - \sin^2\theta} - \cos\theta\Big/\sqrt{\varepsilon_1 - \sin^2\theta} + \cos\theta,$$
$$\bar\kappa^h = \sqrt{\varepsilon_1 - \sin^2\theta} - \varepsilon_1\cos\theta\Big/\sqrt{\varepsilon_1 - \sin^2\theta} + \varepsilon_1\cos\theta,$$
$$\zeta = 2(\varepsilon_1 - \varepsilon_0) \times \left((\sqrt{\varepsilon_1 - \sin^2\theta} + \cos\theta)(\sqrt{\varepsilon_1 - \sin^2\theta} + \varepsilon_1\cos\theta)\right)^{-1}$$

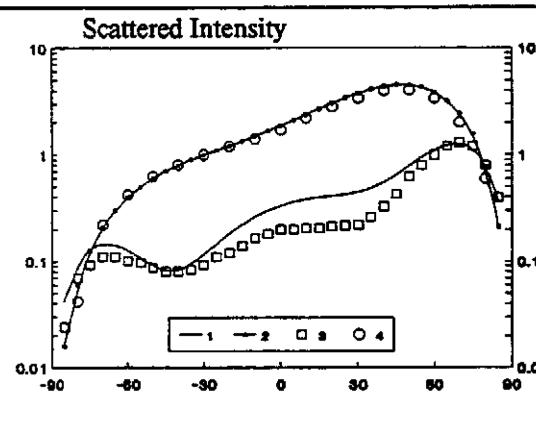

Figure 5. Scattered intensity compared with [12] results. Curves 1,2 correspond to DSM for P,S polarization; curves 3,4 correspond to [12] for P,S cases.

We intend to illustrate the DSM for the model of a spherical particle on a silicon wafer surface. Figure 4 shows the intensities of scattered E field by polysterin latex (PSL) particle as compared with the Wojcik [11] results for the following parameters: $m_{PSL} = 1.59$, $m_{Si} = 3.8$, $\lambda = 632.8$ nm, $D = 0.54$ mkm, S case, normal incidence (where D is a particle diameter, λ is a wavelength). Time consumed is 1.5 min. The same characteristics for the $D = 0.3$ mkm, $\theta_0 = -65^0$ are shown in Figure 5 as compared with the [12] results for S and P polarizations. Here $m_{Si} = 3.88 - 0.02i$, results of [12] are shown by markers.

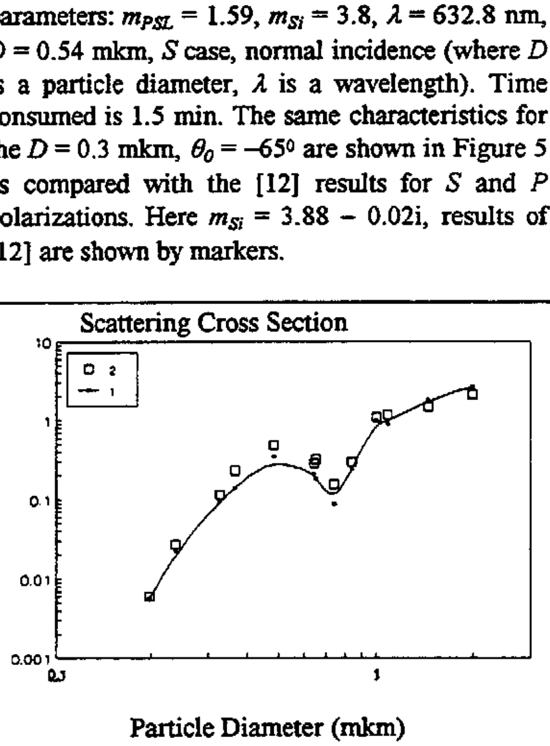

Figure 6. Scattering cross section R_{sca} as function of the particle diameter. Curve 1 corresponds to PSL particle (calculated values), curve 2 means the experimental data [11].

The calculation of the scattering cross-section R_{sca} is shown in Figure 6. This characteristic means an amount of the total scattered light and is very useful for some engineering applications. Thus the response of the wafer surface scanner can be described as

$$R_{sca} = \int_0^{2\pi} \int_{\theta_1}^{\theta_2} \{I_S \sin^2\phi + I_P \cos^2\phi\} \sin\theta \, d\theta \, d\phi.$$

In Figure 6 R_{sca} is shown as a function of diameter of the *PSL* sphere (curve 1, $m_{PSL} = 1.59$). Markers (curve 2) shows the data obtained from the experiment with the wafer surface scanner [11].

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