ON THE USE OF BIVARIATE SPLINE INTERPOLATION OF SLOT DATA IN THE DESIGN OF SLOTTED WAVEGUIDE ARRAYS

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Abstract : *Essential to the design process of slotted waveguide arrays is an accurate knowledge of the self-properties of the individual slots. These properties can be computed with sufficient accuracy using moment method codes, which are too computationally intensive to be incorporated into design codes for large arrays. This paper describes an interpolation scheme which provides both sufficiently accurate, and rapid, computation of slot data from a pre-determined database to be viable for use in a slotted waveguide array design code.*

1. INTRODUCTION

It is correct to state that the structured design of high-performance slotted waveguide arrays began with the publication by Elliott [1] in 1978. All the design procedures available in the literature are, to the best of the authors' knowledge, based on this original approach [2,3], with further discussions [3] aimed principally at examining the correlation of the design method with experiment or the details of its implementation.

Essential to the design process is an ability to determine the self-properties of the individual slots. These properties can be computed with sufficient accuracy using moment method techniques. In a recent paper [3], which very carefully compares theory and experiment, the inclusion of the moment method computation itself in the design process has been suggested. However, because of the extensive computational effort involved, this has only been used in connection with the design of linear arrays of slots with no more than 21 elements. An even braver approach has been presented by Gulick and Elliott [4], where the moment method is used to obtain a complete solution for the integral equation formulation of the array antenna considered as a whole. The advantage of this scheme is that it does not require individual characterisation of the radiating elements, with mutual coupling, both internal and external, being handled automatically. However, this approach has thus far only been demonstrated on an array of 8 slots and is at present not a practical procedure for the design (as opposed to analysis only) of large planar arrays.

Although the moment method formulations [5,6,7] are quite efficient, they are still too computationally intensive to be incorporated into design codes for large arrays. Although it is possible to use approximate formulas for slot properties, it is possible to avoid any compromise and use numerically generated slot data along with a spline interpolation scheme for use in the design phase. This has already been an approach recommended in microstrip circuit design [8] and the modeling of antenna near a half-space [11].

When designing slotted waveguide arrays, the waveguide dimensions (e.g. full-height, half-height, quarter-height) and wall thickness would have been determined before the start of the design. Practical fabrication considerations usually require that the slot width be the same pre-determined value for each of the slots. Also, although the array performance may be analysed at a number of frequencies, any design run is done at some single frequency. Thus, if the results of the moment method code, namely complex s_{11} and s_{21} (or complex self-immittances of the equivalent network model) are obtained over the range of slot offsets x_o and lengths 2ℓ (these are shown in Fig.1), such sets of data can be considered as bivariate functions of the variables x_o and 2ℓ. Ready-to-use computer routines are available [9] for performing a bivariate spline interpolation, and these have been used by the present authors.

2. BIVARIATE CUBIC SPLINE INTERPOLATION OF SLOT SCATTERING PARAMETERS

The procedure is to establish (via the moment method code) the slot data at some single frequency at selected values of the two independent variables x_o and 2ℓ. The range of this data will usually be known from previous experience or examination of the literature. This data may be the complex s_{11} and s_{21}, from which equivalent network parameters are easily obtained, or such equivalent network parameters themselves. The routine *SPLIE2* [9,P.100] is then run with this data as input in order to compute certain derivative data needed for the final spline interpolation. Thereafter the original slot data, plus that generated by *SPLIE2*, is used with routine *SPLIN2* [9,p.101] to compute interpolated values of the slot data for any given x_o and 2ℓ combination as requested by a design code. This process is as rapid as if direct closed-form formula such as the variational expression in [10] were utilised (which is not sufficiently accurate of course), even when many data nodes are used. The latter remark applies to the other quantities as well. Note that the above-mentioned routines also require the availability of two others (*SPLINE* and *SPLINT*), also available in [9,pp.86-89].

In order to illustrate this point, Table 1 shows the scattering parameters (magnitude and phase) for various slot lengths and offsets (at 9.0 GHz in half-height X-band waveguide) computed with the moment method code and with the spline interpolation procedure from a database. The width of the slots were taken as 1.6 mm and the wall thickness of the waveguide as 0.9 mm. Database values were computed with the moment method code at 23 node points in slot length (13 mm to 24 mm in steps of .5 mm) and 12 node points in slot offset (.8 mm and 1 mm to 6 mm in steps of .5 mm). The database was computed to an accuracy of 6 significant digits, so as not to lose accuracy during the interpolation process through round-off error. All the lengths and offsets shown in Table 1 are in between these node points and examination of the data reveals that the difference between the exact moment method computed data differ only very slightly from the scattering-parameter data computed from the database. Fig.2 has been included to give a global view of the variation of the magnitude of s_{11} with the slot parameters x_o and 2ℓ. We have shown a portion of the x_o=3mm dataset in Fig.3, obtained using a decreasing number of nodal points to give some indication of the number needed. The accuracy achievable can easily be increased simply by using more nodes in the database until it is well within that required for design purposes (and certainly up to that accuracy with which slot properties can be measured).

3. CONCLUDING REMARKS

The interpolation schemes whose use has been discussed in this paper provide both sufficiently accurate, and rapid, computation of slot data for their use in a slotted waveguide array design code to be viable. Given a set of measured or computed (using accurate moment method techniques) data, node points can be selected in such a way as to obtain sufficient accuracy with as few nodes as possible (and thus interpolate as rapidly as possible).

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		Data computed with the moment method code				Data computed from database using spline interpolation			
2ℓ	x_o	$\|S_{11}\|$	$\angle S_{11}$	$\|S_{21}\|$	$\angle S_{21}$	$\|S_{11}\|$	$\angle S_{11}$	$\|S_{21}\|$	$\angle S_{21}$
13.2	2.1	.027	76.05	.994	-1.62	.027	76.00	.994	-1.63
14.4	3.4	.104	63.76	.959	-5.93	.104	63.76	.959	-5.93
15.6	3.7	.188	43.46	.874	-9.21	.188	43.46	.874	-9.21
16.8	4.1	.284	15.47	.732	-7.37	.284	15.67	.733	-7.43
17.2	4.8	.337	15.41	.684	-9.85	.337	15.51	.685	-9.87
18.7	5.2	.380	-8.46	.622	0.74	.380	-8.44	.622	0.73

Table 1: Comparison between scattering parameter data generated by a moment method code, and data computed from a data base using a spline interpolation routine. (Lengths and offsets given in mm, and phase in degrees).

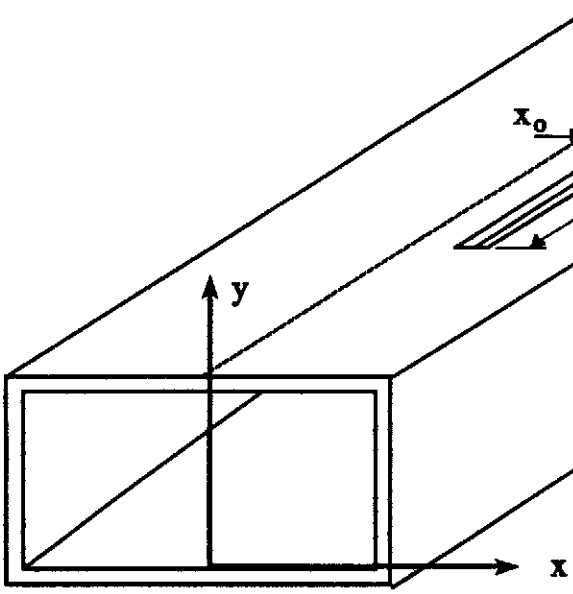

Figure 1: Geometry of a longitudinal slot in rectangular waveguide, showing the notation used in the paper.

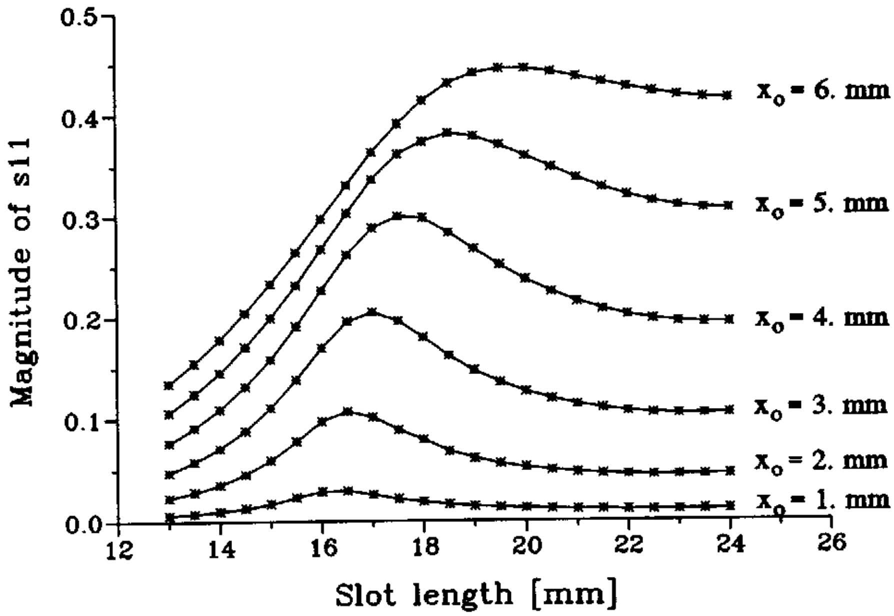

Figure 2: Magnitude of s_{11}, for varying x_o and 2ℓ, computed using the moment method.

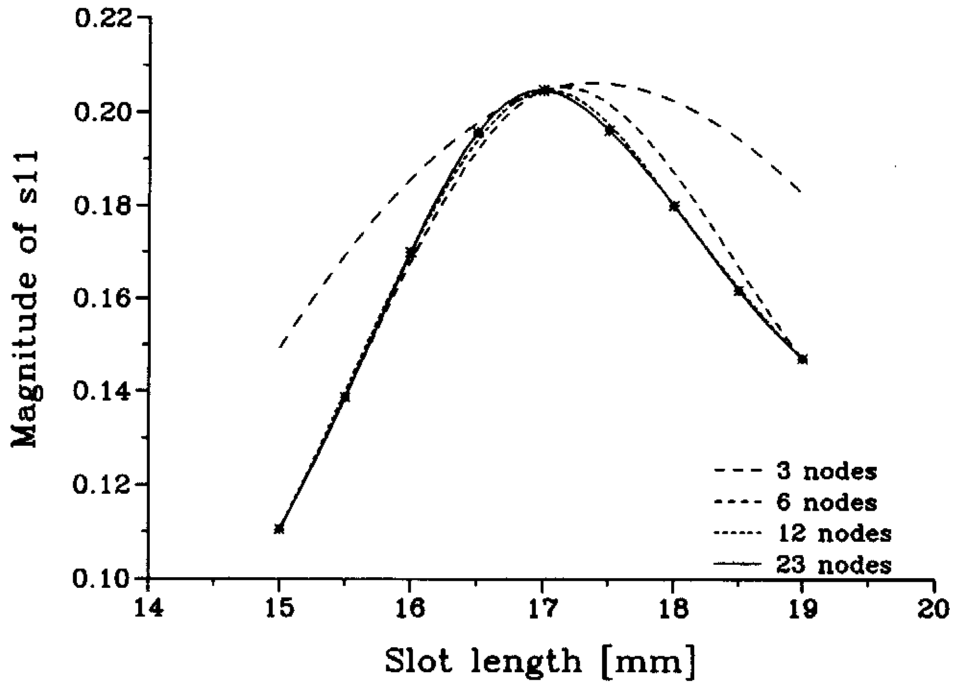

Figure 3: Magnitude of s_{11}, for x_o=3mm and varying 2ℓ, computed using the spline interpolation procedure with different numbers of nodal points. Some moment method results are shown (*) for reference.