MOMENT METHOD ANALYSIS OF RADIATION AND SCATTERING BY THIN WIRES IN AN INFINITE DISSIPATIVE MEDIUM

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ABSTRACT

In this report, a modified version of AMP (Antenna Modelling Program), a forerunner of NEC (Numerical Electromagnetics Code), is validated for applications involving thin wire structures immersed in an infinite dissipative medium. A Pascal derivative of MiniNEC2, the Mini Numerical Electromagnetics Code, is also used to provide numerical comparison as needed when other data are not available. Computations from both programs are compared with analytical and pre-existing experimental results. Drive point impedances, currents and near fields for a range of thin wire radiators and scatterers are presented. Useful results are obtained for the highest conductivity case tested ($\delta = \sigma/\omega\varepsilon_o\varepsilon_r = 8.8$) and for less conductive media, agreement of computed parameters with available comparative data of approximately 10% or better is achieved.

KEYWORDS: thin-wire antennas, moment methods, dissipative media, metallic implants.

1. INTRODUCTION

Over the past two decades, the thin wire moment method [Harrington, 1968] has been applied to a wide variety of applications via programs such as AMP [Selden and Burke, 1974], NEC [Burke and Poggio, 1981] and MiniNEC [Julian, Logan and Rockway, 1982]. With few exceptions, these applications have involved conductors immersed in free space, or in the case of lossy ground modelling, a 'lossless half-space'. Whilst not too many published papers exist, in the public domain at least, some references to the subject of cylindrical wire modelling in a conducting medium can be found [King, Sandler and Wu, 1969], [King and Sandler, 1973], [Richmond, 1975], [Anders, 1987]. To the best of the authors' knowledge, no published numerical details obtained using moment methods exist for the canonical cases of a cylindrical dipole radiator and scatterer as shown in figure 1.*

King and Harrison [1969] studied the general problem of linear antennas in a dissipative medium from a theoretical viewpoint. They derived analytical expressions for the currents and near fields for wires of various lengths in a range of conducting media. In addition, measurements were made of currents on, and fields near quarter wave monopoles in a series of saline solutions by King and Iizuka [1963]. Recently King and Smith [1981] presented an overview of research in this

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* *Richmond used a piecewise sinuosoidal current interpolation basis and the resulting code, similar to the modified version of AMP presented here, is capable of analyzing both near and far zone radiation and scattering by thin wires in an arbitrary lossy, homogeneous medium. We are advised that Richmond's code and documentation is available from the IEEE.*

field including the important case of a buried insulated wire [King, 1977].

In this paper, AMPC, a modified version of the thin wire moment method program AMP is discussed; these modifications allow analysis of structures immersed in an infinite conducting medium. In particular, the complex propagation constant, the appropriate current interpolation bases, and Green's functions are discussed. Next, AMPC is validated using analytical solutions and experimental results obtained previously by King and Harrison [1969] and King and Iizuka [1963] for dipole and monopole radiation. Drive point impedances, wire currents and near fields are presented. A Pascal derivative [Davis et al., 1987] of MiniNEC2 [Julian, Logan and Rockway,1982] was also modified, termed MiniNEC2C, and used to provide numerical comparison for the case of scattering by a dipole.

The authors' need for such a thin wire analytical capability stems from the potential heating effects induced by metallic objects that may be imbedded within the human body, usually as a result of surgical treatment. A number of medical procedures such as cardiac surgery and knee and hip replacements involve metallic implants that are left inside the patient, who may be required to work in an environment which contains significant levels of radiofrequency radiation. A knowledge of the immediate near field distribution surrounding the implant is required prior to estimating the heat flow relationships and temperature increases existing at a particular physiological location. A preliminary assessment of the health aspects has been discussed elsewhere [Joyner et al., 1988].

In addition to the analysis of potential bioelectromagnetic hazards, several other important generic applications exist such as arctic, desert, submarine and subsurface communication [King,1975] and hyperthermia treatment of cancer [Moidel et al., 1976].

2. AMPC - MODIFIED VERSION OF AMP

Using the free space version of AMP, three major modifications were made in order to allow the user to analyze wire structures immersed in a conducting medium. These were the propagation constant and associated parameters, the current interpolation base used to approximate the currents within a segment, and the Green's functions required by the electric field integral equation formulation. The resulting code was called AMPC.

The propagation constant, k, was now complex since the permittivity, ε, and conductivity, σ, of the surrounding medium were problem variables.

$$k^2 = \omega^2 \mu\varepsilon - j\omega\sigma\mu, \tag{1}$$

All dependent parameters were calculated such as the complex impedance, η, and the wavelength in the medium, λ_i. A number of variables and arrays were required to be converted to complex equivalents. A normalization of complex media may be used to simplify analysis. Equivalently, k may be written as

$$k = \omega \sqrt{\mu\varepsilon^*} \tag{2}$$

where the equivalent complex permittivity, ε^*, is given by

$$\varepsilon^* = \varepsilon - j\sigma/\omega \quad . \tag{3}$$

Thus the propagation constant can be related to its free space counterpart

$$k = k_o \sqrt{\varepsilon_r^*} = k_o \Lambda \tag{4}$$

where $\varepsilon_r^* = \varepsilon_r - j\sigma/\omega\varepsilon_o = \varepsilon_r(1 - j\delta)$ is the corresponding relative complex permittivity and normalization constants, Λ and δ, are defined as

$$\Lambda = \sqrt{\varepsilon_r - j\sigma/\omega\varepsilon_o} \tag{5a}$$

$$\delta = \sigma/\omega\varepsilon_o\varepsilon_r \tag{5b}$$

When wire currents resulting from radiation in dissipative media are analyzed it is found that the currents tend to cluster around the region of the source; those flowing on remote parts of the wire are reduced in magnitude from the corresponding currents in free-space for a given normalized structure [King and Harrison, 1969]. Examination of a cylindrical dipole at resonance shows that as the degree of conductivity of the medium increases, the normalized currents near the source first decrease in magnitude and eventually increase for $\delta > 1$; the current slope discontinuity across the gap grows as more charge density exists in the gap region. In the limit when the surrounding medium is a perfect conductor ($\sigma = \infty$), no currents flow along the wires since the voltage source is effectively shorted out.

Within segments, current interpolation used by AMP is based upon the sinusoidal nature of the currents in free space. AMP uses a three-term approximating function, namely sin, cos and constant. To enable a more accurate representation of the spiked current flows as a result of lossy media, AMPC employed a complex sin, cos and constant approximation. The method of obtaining intersegment continuity remained unchanged from that used by AMP [Selden and Burke, 1974]. The basis function for any particular segment is evaluated at the centres of adjacent segments. In this fashion, the function for each segment may be constrained to reduce the numbers of unknowns from three to one.

The electric field equation used in the thin-wire formulation employs Green's dyadic for an infinite homogeneous medium.

$$\bar{\bar{G}}\,(\bar{r},\bar{r}_0) = (k^2\,\bar{\bar{I}} + \bar{\nabla}\,\bar{\nabla}\,)\;g\,(\bar{r},\bar{r}_0) \tag{6}$$

where $\bar{\bar{I}}$ is the unit dyadic and g the scalar Green's function

$$g\,(\bar{r},\bar{r}_0) \;=\; \exp\,(-jk\,|\bar{r}-\bar{r}_0\,|)\,/\,|\bar{r}-\bar{r}_0\,| \tag{7}$$

Two conjugate Greens functions, G_1 and G_0, were used within AMPC for calculating either wire currents or external fields. Referring to Fig. 2, the form G_0 was used to model outwardly travelling or scattered waves in the unbounded exterior medium when the electric fields were required to be computed, while the conjugate form G_1 was employed for numerical reasons by the moment method within the bounded interior region.

In terms of the generalized weighted residual technique, the numerical errors associated with the inner product of the three-term current approximating function times the weighting function was minimized in this time-harmonic application by using a conjugate form of the Green's function, G_1, for calculating the wire currents. This requirement was determined empirically using a number of trial tests for which results were available. In all such preliminary tests, the use of a conjugate form, G_1, gave much better agreement between computed and known results.

3. VALIDATION

Much use was made of the previous analytical and experimental work of King and Harrison [1969] and King and Iizuka [1963] in testing AMPC for convergence and accuracy. As mentioned, a second thin wire program, MiniNEC2 [Davis,1987], was modified to provide numerical comparison where other alternatives were not available. These modifications were similar to those outlined in Section 2 and the result termed MiniNEC2C. The current interpolation base remained a simple pulse; intersegment continuity was achieved as in MiniNEC2 by a system of staggered pulses. Unfortunately, MiniNEC2 does not calculate the near fields and so no numerical comparison was available for near-field validation*

The first stage of testing involved a series of radiating half-wave dipole problems, each normalized with respect to frequency and permittivity. A comparison of the driving point impedances (resistance and reactance) determined by both AMPC and MiniNEC2C and analytical results [King and Harrison, 1969] are shown in Fig. 3 as a function of the normalization constant δ. The wire radius in each case was such that the thickness parameter, $\Omega = 2 \ln \left(\frac{2h}{a} \right) = 10$, where a is the wire radius and 2h is the total length of each dipole. It is seen that both

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* A near field capability is included in MiniNEC3 which the authors have *recently obtained in the form of NEEDS 1.0 (ACES Software Library).*

moment method programs compare reasonably well with the analytically computed results. Due to simplifications made to facilitate computations, the theoretical results only remain accurate for the range $\delta \leq 0.3$.

A comparison of the current distribution along these same dipoles was performed. Although not shown, the general agreement was of the same order shown for the drive point impedances of Fig. 3. It is however appropriate to note that as δ gets larger, AMPC and the analytically computed currents near the voltage source, especially the imaginary components, are increasingly less in agreement. A major factor causing this behaviour may have been the spiked nature of the currents near the feed region as the conductivity becomes large. On the other hand MiniNEC2C showed no such problem since intersegment continuity is achieved more effectively at segment junctions, equivalent to employing a system of staggered charge and current pulse bases. In AMPC, the method of enforcing continuity at adjacent segment centres appears to cause instability and convergence problems not found in MiniNEC2C.

Convergence tests were performed by examining the wire currents as the number of segments per half-wave dipole was varied. Again, numerical and analytical results were compared using both AMPC and MiniNEC2C. These tests indicated that AMPC was sensitive to the number of segments per half-wave dipole. As δ increases, the number of segments for accurate modelling decreases. Thus a 5 segment model was employed for δ = 0.149, 0.298 and 0.373. MiniNEC2C was found to be fairly stable for a wide range of segmentation (5-10% variation using 4-16 segments).

Next, a series of radiating monopoles were modelled and the drive point currents compared with experimental results obtained by King and Iizuka [1963]. For these monopoles, a saline solution of varying conductivity had been used, giving a wider range of δ, $0 \leq \delta < 8.8$, than was available from the analytical computations. Table 1 shows the various properties and constants for the five solutions used. Of importance for the application to wire implants, this range includes normalized values appropriate for biological modelling. Additionally, a range of monopole wire lengths was used from $\beta Z = \pi/2$ up to $5\pi/4$ wavelengths. From experience gained in the previous convergence testing, the segmentation used with AMPC was reduced as the conductivity increased.

The monopole current distributions for two of the five solutions, A and D, are shown in Fig. 4. The numerical results were found to be significantly different when compared with the experimental data; on the other hand, a fairly close agreement between AMPC and MiniNEC2C was noted. In fact, due mainly to the size of the box containing the solution, the experimental results had been adversely affected as recognized by King and Harrison [1969, pp.244-248], especially for the low conductivity cases.

Whilst some implants such as heart pacemakers act as radiators, most are passive and act as scatterers. Hence a parallel round of tests for both the normalized dipoles and the experimental monopoles was conducted with these structures acting as scatterers.

Since analytical and experimental scattering results were unavailable, a comparison of AMPC and MiniNEC2C was used with some degree of confidence gained from previous test results. In the scattering case, the currents are not generally clustered in pockets of the wires as is the case for radiators. Hence the difficulties of approximating the currents are reduced. Fig. 5 shows the magnitude and phase at the centres of half-wave scatterers as the number of segments is varied. These dipoles were immersed in the same media as the radiating monopoles (Table 1). It is seen that the results agree reasonably well for all cases except the high conductivity case E. Since the charge density distribution on the wires is continuous for the quarter wave case, the stability and convergence of AMPC was improved and the segmentation restrictions were reduced. In case E, there still appeared to be an instability which was more pronounced than in cases A to D. In all cases except E, employing 7 segments led to a difference between numerical results for the magnitude of the drive point current of less than 12%; for case E the difference was 27%. MiniNEC2C was found to be very stable over the range of segmentation used which was up to 23 segments per half wave; the change in both magnitude and phase was less than 5% for the entire range of conductivities.

The final stage of validation was to test the near fields over a wide range of locations and radial distances up to about a wavelength. Analytic expressions [King and Harrison, 1969] for a series of normalized radiating dipoles provided results to compare with AMPC. A range of conductivities $0 \leq \delta \leq 1.0$ was employed. The electric fields for a range of cylindrical distances at three axial heights were sought. These heights were chosen to examine (a) the feed point, (b) segment interfaces and (c) the wire tips.

Fig. 6 shows the results for $\delta = 0.373$ for a range of cylindrical distances as the tip is approached. As can be seen, these results appeared to confirm the program for use in calculating the fields near the tips of metallic implants. The results shown approach to within about 1.6 radii in distance from the point of the tip. They confirm that consistent numerical results can be obtained by AMPC where the regions tested do not lie on the wire surface. Since AMPC (a) models the currents at the wire ends to be zero and (b) interpolates current continuity at segment centres rather than at the ends, the method of interpolation used by AMPC leads to some inherent problems when trying to interpret the near-field results at the surface and free ends of wire models. As a result then, the fields adjacent to intersegment junctions showed a more pronounced lack of agreement with the analytic results, particularly in the vertical component. This increased as the dipole was approached. Similarly, the fields adjacent to the feed point were adversely affected with additional uncertainty introduced through the spiked shape of the currents near the feed. Comparison of the fields at the wire tips was conducted (within 1 radius distance). Due to the use of the thin wire kernel approximation whereby each self-term of the moment method impedance matrix is approximated by a ring of current surrounding the segment centre at the radius of the wire, the numerically obtained fields were substantially below the analytical field levels which are singular at the tips.

5. CONCLUSIONS

In this report, a modified version of AMP has been validated for applications involving thin wire structures in an infinite conducting medium. The validation procedure used analytical, experimental and numerical tests. Radiation and scattering characteristics in the form of both wire currents and electric near fields were compared for a wide range of lossy dielectrics. As well, drive point impedances for a range of radiators was examined.

Difficulties found in modelling wire radiation currents for high conductivities were due to errors in approximating the shape of the actual currents and to the method used in AMPC for numerically enforcing current and charge continuity between segments.

The range of application of AMPC in its present form is approximately $\delta \leq 3.0$. It should be noted that fewer segments per wavelength are required for accurate radiation modelling than is the case in free space, and this requirement increases along with conductivity. Useful results were obtained using only 3 segments per half-wave dipole for the highest conductivity case tested ($\delta = 8.8$).

More research needs to be performed in certain areas such as the behaviour of a range of current bases and wire radii, intersegment continuity, the voltage gap phenomenon and the immediate near-fields especially for different shaped ends such as flat, round and pointed tips. In this last case, a conical shaped tip might allow analysis of jagged, point-shaped conductors.

Finally, at the time of writing, AMPC does not include a lossy ground plane capability and various other options have not been tested. As further applications require, the authors envisage either developing the program to utilize these presently unused routines or using a more recent thin-wire program such as NEC2 as a basis. It would be most interesting to compare results if more sophisticated modelling programs such as NEC, NEC2, NEC3 and NEEDS could be successfully modified to include lossy media.

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Solution	ε_r	δ	λ_i (m)	a (m)
A	78	0.036	0.2977	0.001002
B	78	0.35	0.2934	0.0009885
C	77	1.06	0.2704	0.0009109
D	74	2.64	0.2211	0.0007449
E	69	8.8	0.1426	0.0004804

Table 1 Various Experimental Saline Solutions and their Properties

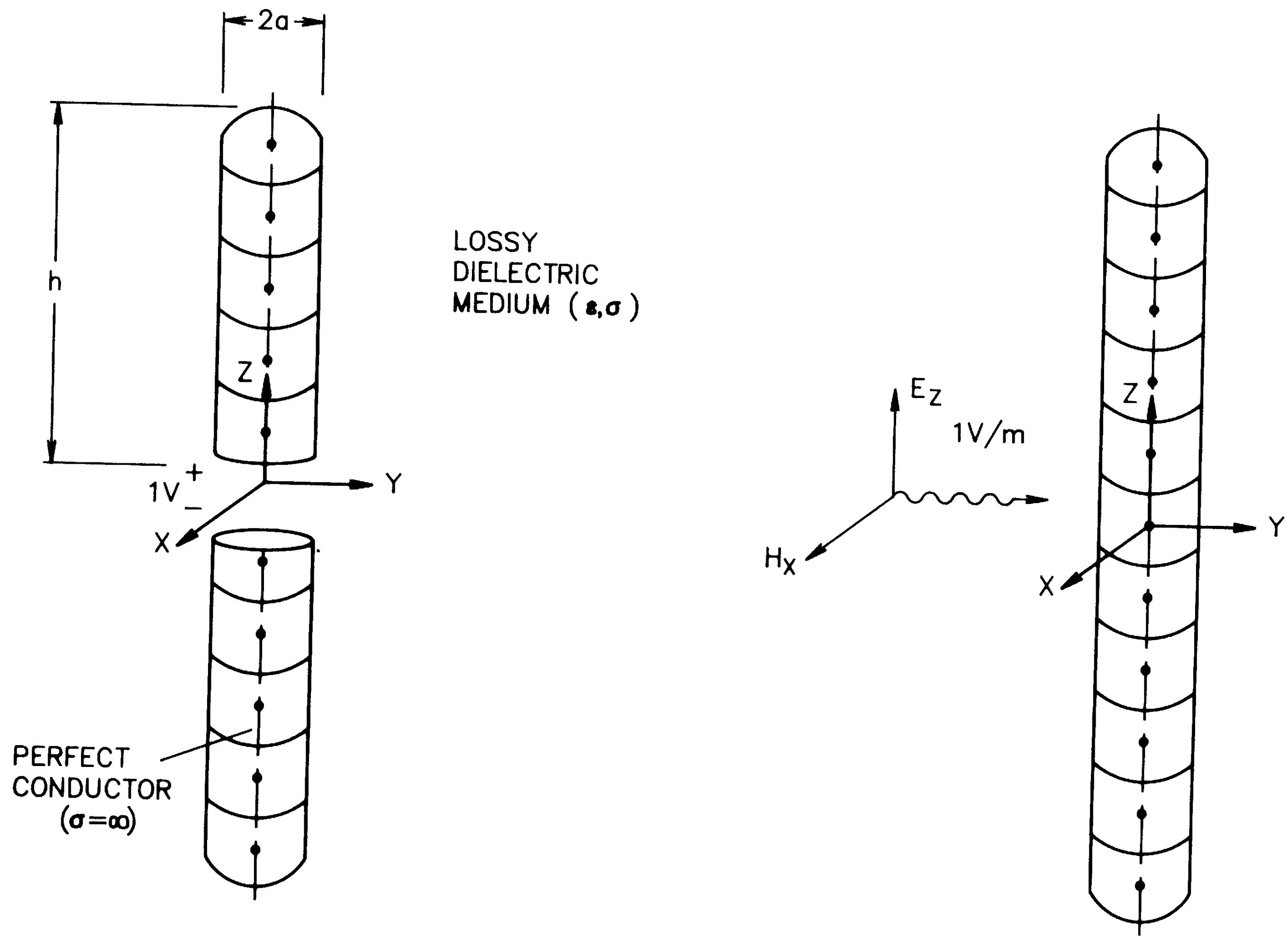

FIG.1 THIN WIRE STRUCTURES – RADIATOR AND SCATTERER.

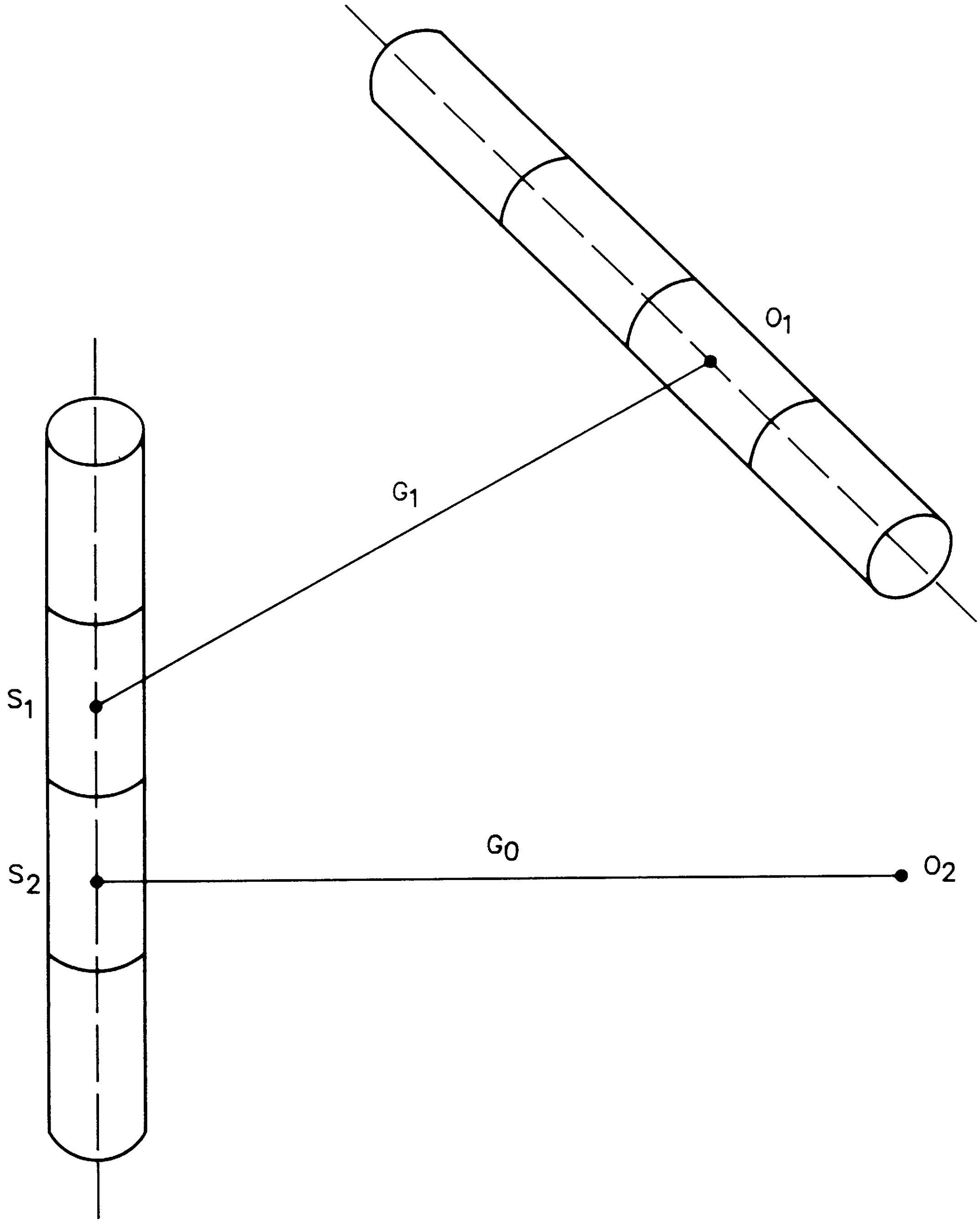

FIG.2 GREEN'S FUNCTION AND CONJUGATE FOR DISSIPATIVE MEDIA.

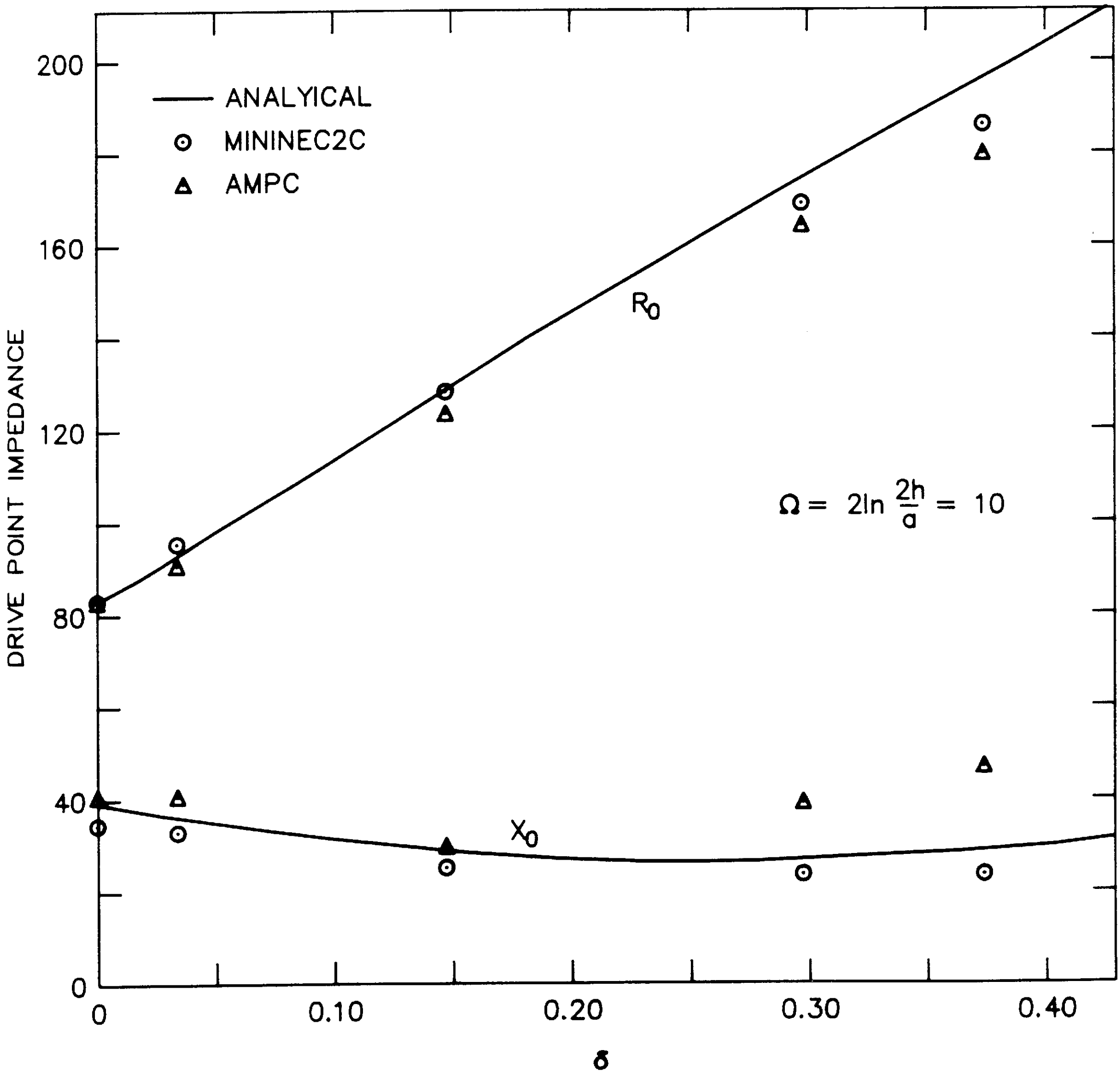

FIG.3 DRIVE POINT IMPEADANCE FOR RADIATING HALF—WAVE DIPOLES IN NORMALIZED CONDUCTING MEDIA

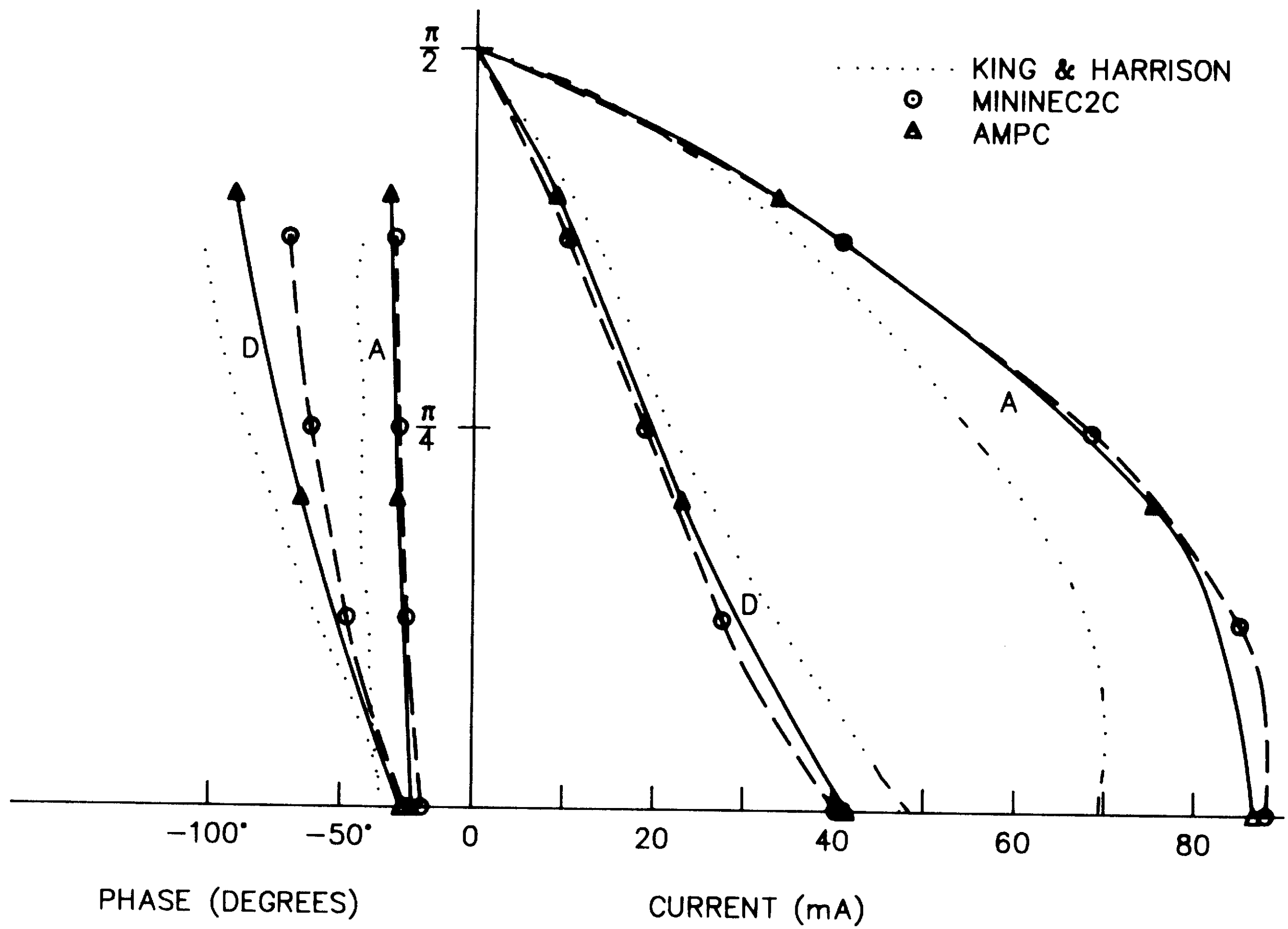

FIG.4 AMPLITUDE AND PHASE OF CURRENT DISTRIBUTION ALONG QUARTER-WAVE MONOPOLE RADIATORS IN DIFFERENT MEDIA AT 114 MHz.

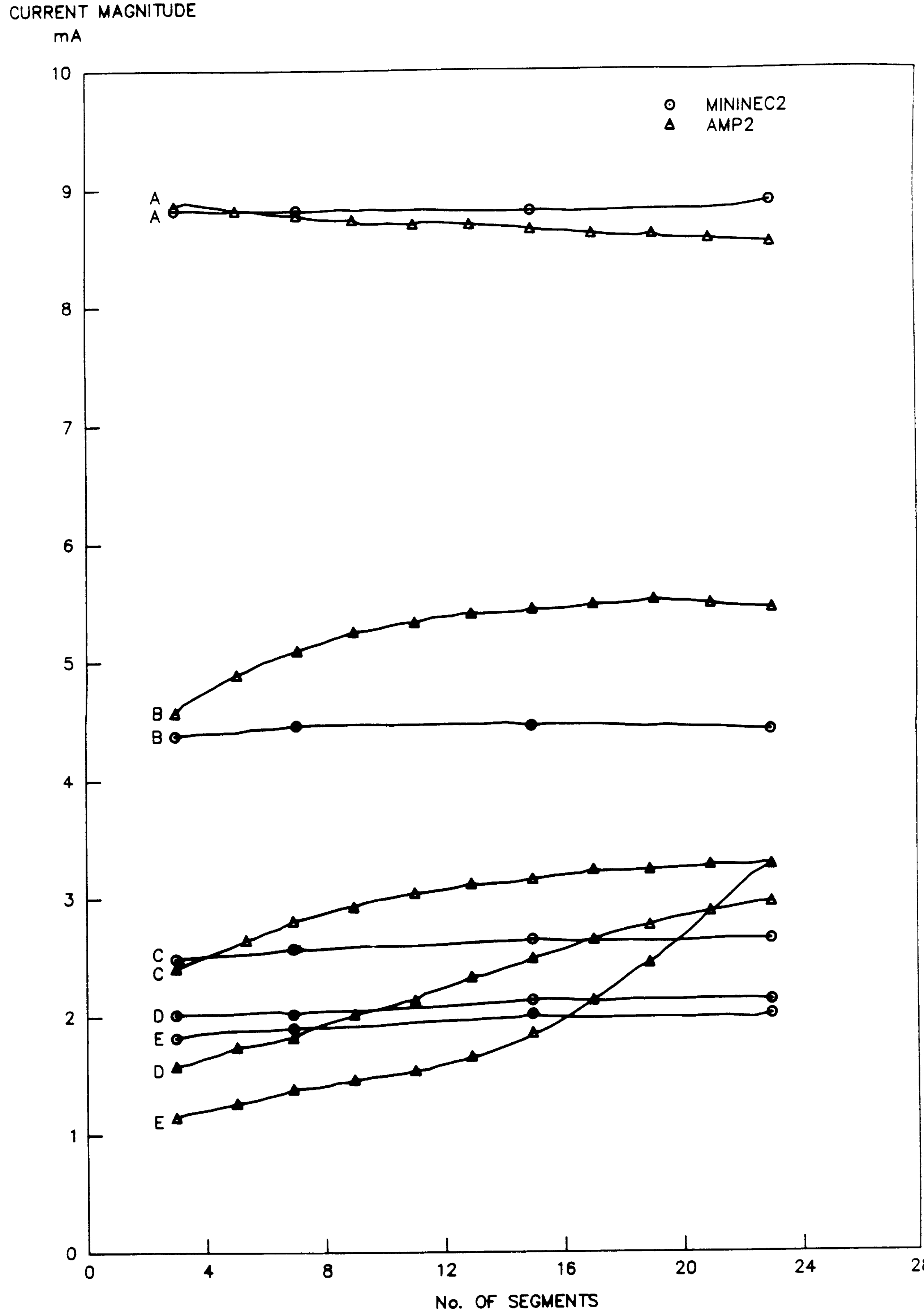

FIG.5a DISTRIBUTION OF CURRENT MAGNITUDE AT CENTRES OF HALF-WAVE SCATTERERS IN DIFFERENT MEDIA AT 114 MHz.

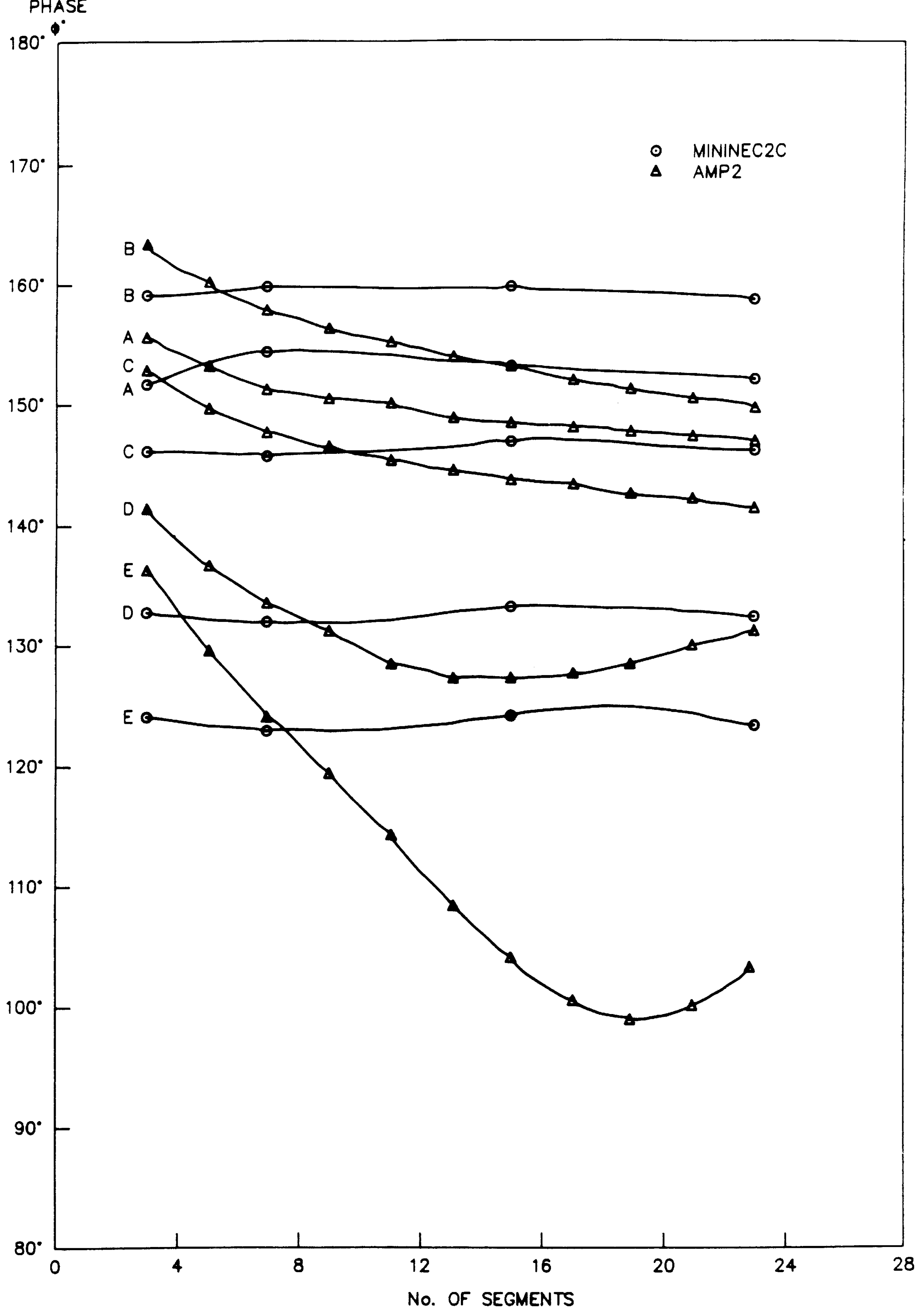

FIG.5b DISTRIBUTION OF CURRENT PHASE AT CENTRES OF HALF-WAVE SCATTERERS IN DIFFERENT MEDIA AT 114 MHz.

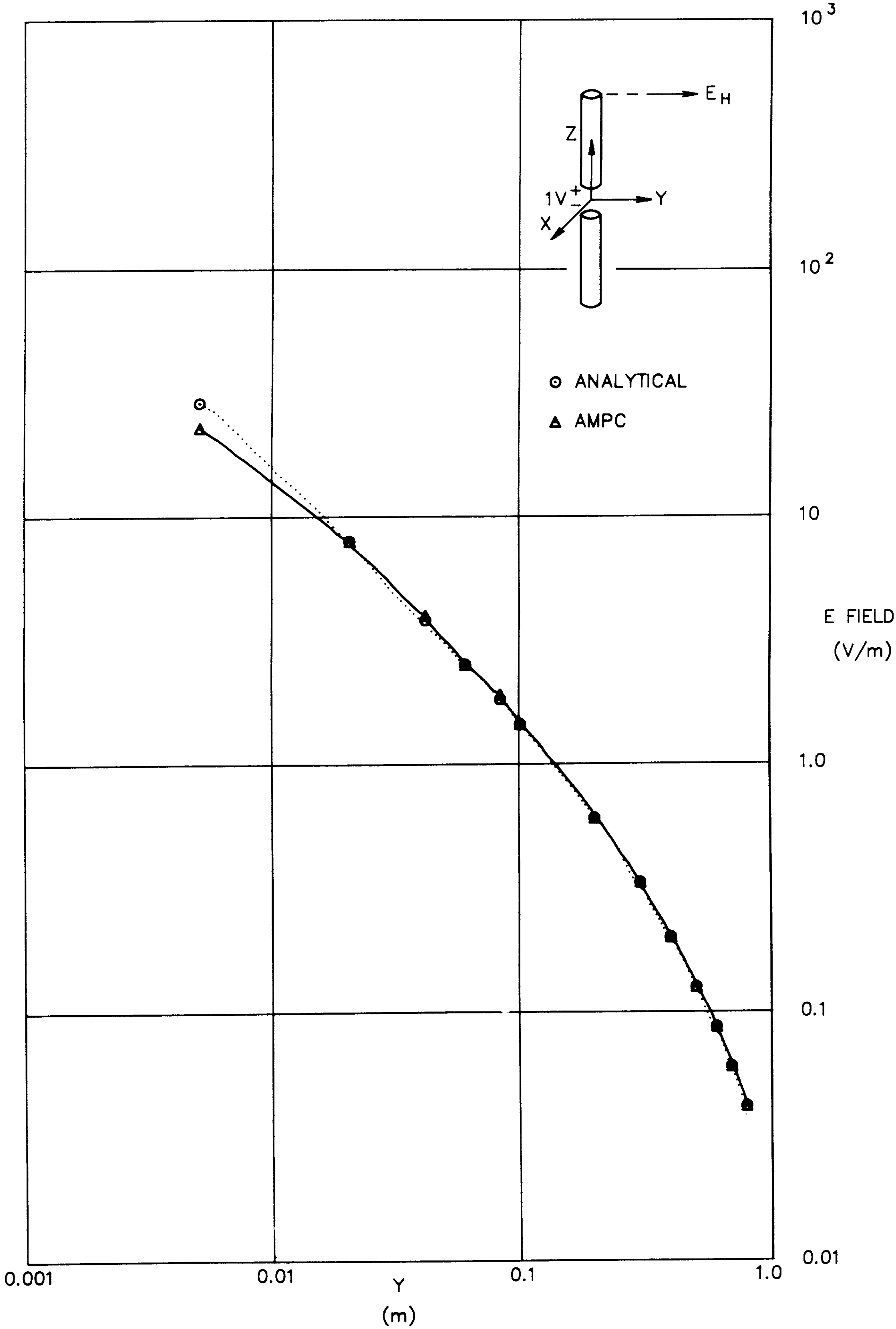

FIG.6a HORIZONTAL ELECRTIC FIELDS NEAR TIP OF RADIATING HALF WAVE DIPOLE AS Y IS VARIED, $\delta = 0.373$

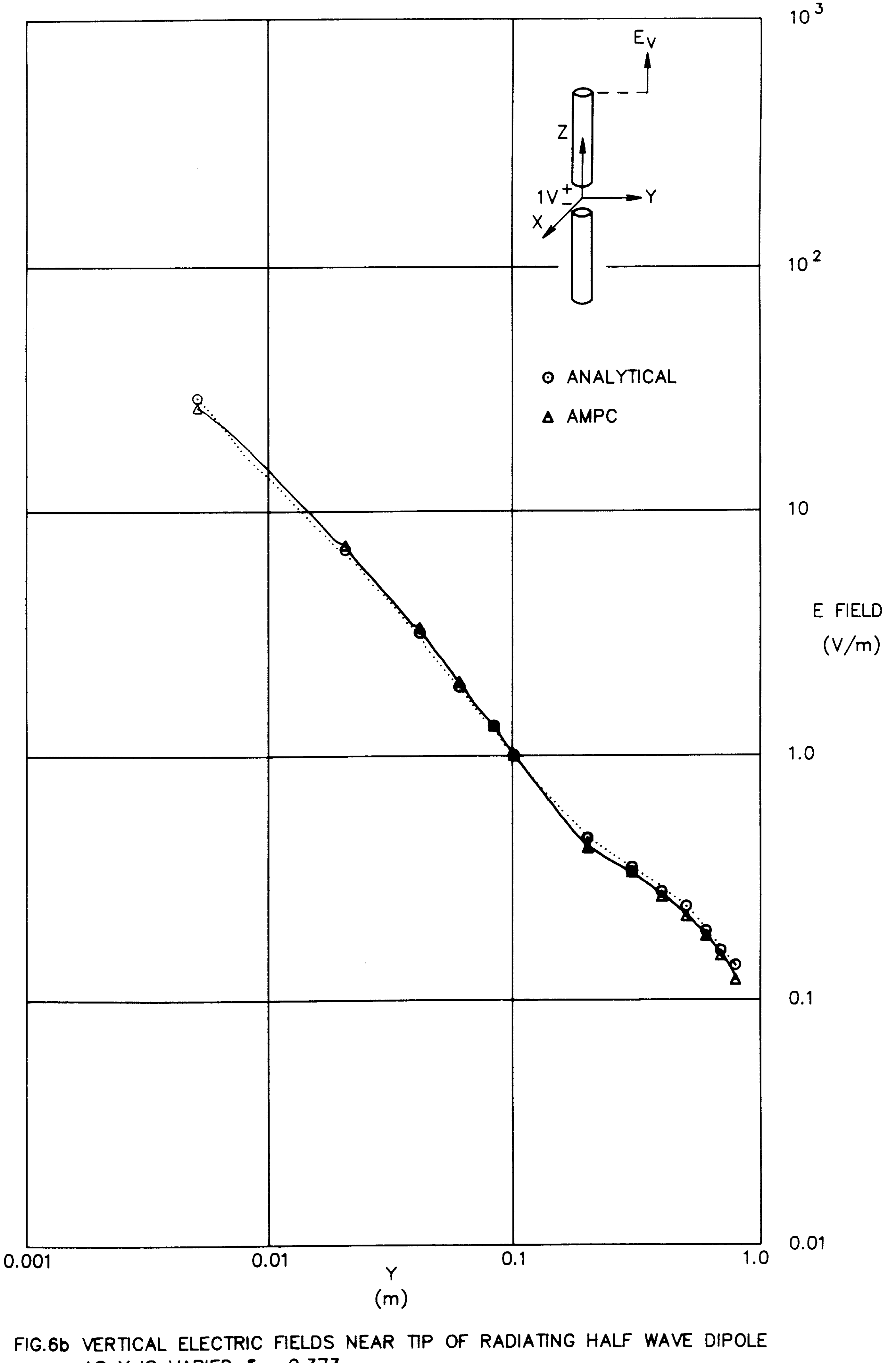

FIG.6b VERTICAL ELECTRIC FIELDS NEAR TIP OF RADIATING HALF WAVE DIPOLE AS Y IS VARIED, $\delta =$ 0.373