Advanced Statistical 3D Models of Composite Materials for Microwave Electromagnetic Compatibility Applications

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Abstract — This article describes advanced tridimensional time domain tools for characterizing composite structures in EMC context. Because of current high interests from industrial areas (e.g., automotive, and/or aerospace), many domains are demanding for reliable and accurate numerical tools to characterize such materials. In this framework, a particular care is granted to mixing theories. This article will demonstrate the interest of trustworthy statistical tools to check at the same time the validity of electromagnetic mixing rules and to realistically calibrate these models.

Index Terms — Composite material, computational electromagnetics, electromagnetic compatibility, shielding effectiveness, time domain modeling.

I. INTRODUCTION

Composite materials are being extensively used for almost fifteen years in various areas such as transport (for instance automotive, aerospace, railway) [1, 2], communication [3], and energy [2]. Due to increasing environmental constraints (e.g., regarding sources of perturbations: crosstalk, grounding [1], lightning [2], wireless communication [3]), Electromagnetic Compatibility (EMC) requirements are more and more demanding for accurate and efficient characterization (with experimental and/or numerical tools) of these mixed materials. In this framework, many studies were proposed to model shielding performances of reinforced materials including enclosures with composite thin sheet: for instance based upon Finite Element Method (FEM) technique [4], Discontinuous Galerkin Method (DGM) [5], or experimental device with carbon-nanostructured composites in [6]. It is to be noted that relevant modeling processes are needed in that context, especially relying on time domain methods [7]. Indeed, on the one hand, since composites made of resin matrix and high strength fibers (e.g., carbon, graphite, glass) are widely spread in different industrial domains such as automotive [1], and aerospace and wind energy [2], they require particular care for electrical safety and functioning reasons. On the other hand, assuming the conductivity and sizes of inclusions leads to competitive numerical issues. Indeed, a priori weakly conducting particles are considered in comparison to purely metallic shields, and high scaling factors are observed since the sizes of inclusions are in between $10^5$ and $10^7$ times smaller than EMC systems. That is why mixing rules (part of Electromagnetic Mixing Theory, EMT) have been recently widely used for statistical developments [8] and numerical works [9-12] to characterize the effective permittivity of composite mixtures for different applications from scattering [8] to EMC [5-7, 9, 10] throughout metrology [11, 12]. They are mainly based upon classical unified mixing rule [8-10] with different variations as developed in Section II. The first aim of this contribution is to demonstrate the interest of a tridimensional (3D) time domain automated (with Matlab® tool based upon Finite Integration Technique (FIT, CST®) for the assessment of the electromagnetic attenuation (by computing Shielding Effectiveness, SE) of biphasic mixed materials. It is to be noted at that stage, that SE data will be computed from T-solver (time domain FIT) simulations; although final data are depicted in frequency domain (from Fast Fourier Transformation, FFT). The second objective is to provide first-orders statistical moments (i.e., mean and standard deviation, std) of SE to face equivalent homogeneous approaches (e.g., from Maxwell-Garnett, MG [8] or Dynamic Homogenization Method, DHM [9]) with proposed numerical modeling. This proposal is divided into four sections following introductory part I: Section II details theoretical foundations from mixing rules, whereas Section III provides information regarding the automated numerical procedure relying on time domain FIT method. Finally, numerical results are given in Section IV, both validating our methodology and successfully comparing it with advanced EMTs (e.g., DHM). Some concluding remarks are provided in last Section V.

II. THEORETICAL BASIS

Mixing equations such as Maxwell-Garnet (MG), Bruggeman-Rule (BR), or coherent potential rule [8, 9] are wide spread and useful formalisms to avoid costly 3D
numerical models. In this article, we will focus on particular case of biphase composite materials (two phases \(i=1\) or \(2\), respectively for matrix and conducting inclusions; volumetric fractions are respectively given in the following by \(\varphi=1-\nu\) and \(\nu\). For spherical inclusions, the classical formulation involves a depolarization 3x3 tensor \(N\), such as \(N\) is a diagonal matrix holding the Cartesian terms \((N_u)^T\), where \(N_u=1/3\) (\(u=x, y,\) or \(z\)) as discussed in [13]. Relying on terms in relation (1), the effective homogenized complex permittivity \(\varepsilon_u\) in Cartesian direction \(u\) (\(x, y,\) or \(z\)) is given in [9] by:

\[
\varepsilon_u = \frac{\varepsilon_1 N_{11} + \varepsilon_2 N_{22} + \varepsilon_3 N_{33}}{N_{11} + N_{22} + N_{33}},
\]

where \(\varepsilon_{in}\) represents the complex permittivity of the infinite medium. It is to be noted that according to numerical test case proposed in [9, 13], the material will be assumed to be low-loss (i.e., depending on real permittivity and conductivity of the material). Without any loss of generality, any kind of material may be taken into account through \(\varepsilon_{in}\) parameter (e.g., Debye frequency dispersive material as in [7]). The assumption \(\varepsilon_{in} = \varepsilon_\infty\) in relation (2) leads to the classical MG formulation, whereas DHM [9] offers to take into account the effect of the size of inclusions by defining \(\varepsilon_{in} = \varepsilon_\infty + \varepsilon_3(d/\lambda)^\alpha\), with \(\lambda\) standing for the wavelength in the effective medium, and \(d\) is the characteristic dimension of the inclusions (diameter here). Previous works in [9, 13] have demonstrated that for cylindrical fibers, \(\alpha = 2\) offers very accurate performances to predict effective properties (2D Finite Element Method, FEM, was considered as reference tool). Previous results were confirmed in [10], where the authors used 3D frequency solver FEKO® to model semi-infinite structure. In this context, few studies were proposed to model 3D configurations involving micro- or nano-inclusions; an interesting piece of work was given in [11] including Monte Carlo (MC) generation of fibers and simulation with FEM (COMSOL®) for extracting the medium conductivity.

III. NUMERICAL METHODOLOGY

As aforementioned, few works may be found regarding use of 3D electromagnetic tools to simulate random heterogeneous media. Due to the large frequency bandwidth required in this study (from 0.1 to 60 GHz), it is proposed to use time (T-) solver from CST® Microwave Studio. Baer et al. have proposed this solution for metrological work in [11] from 18 to 26 GHz, with periodical inclusions and waveguide.

The characteristics of time simulations (CST®) are summarized in Fig. 1. The details are given as follows: transient FIT solver (total duration is 400 ps); plane wave source from 100 MHz to 60 GHz, size of the composite (including spherical conducting inclusions as given in Fig. 2 (a)) parallelepiped is 1x1x6 mm³; minimum elementary meshes are 13 nm while the convergence of 3-D meshing was ensured (data not shown here); computing time for Intel Xeon 4 cores processor is less than 3 hours (including generation of random inclusions, launching simulations, and final extraction of SE results via FFT of E-field simulation, see Fig. 2 (b): 600 frequencies equally distributed from 0.1 to 60 GHz) per test case. From [8], SE is defined as the ratio between electric (E-) field at a given location \(z\) (see Fig. 1, E-field probe) and frequency \(f\) in absence \((E^0)\) and presence \((E^t)\) of composite structure. In this framework, the SE (i.e., attenuation here) of the modelled medium is defined as:

\[
\text{SE}(z, f) = \frac{E^0(z, f)}{E^t(z, f)}.
\]

![Fig. 1. Numerical setup of the 3D time domain model including source, boundary conditions, and composite sample (size, lossless epoxy resin, \(\varepsilon_1 = 5\varepsilon_0\); hidden spherical inclusions inside with \(\varepsilon_2 = \varepsilon_0\) and \(\sigma_2 = 1000S/m\).]

![Fig. 2. (a) Sectional view (\(x_0=0.1\) mm) of one random mixed structure; the volumetric fraction of inclusions is \(\nu = 0.10\) with constant radius of spheres (\(r=0.05\) mm). (b) Normalized E-field at sensor’s position (see Fig. 1) computed in time domain with CST®.]

IV. STATISTICAL ASSESSMENT OF SHIELDING EFFECTIVENESS

This section is devoted to the presentation of numerical results obtained. In a first subsection, the relevance of the model will be checked in comparison to MG and DHM formalisms for various volumetric fractions of inclusions (here \(\nu = 0.01, 0.05, 0.10\)). In a second step, SE statistics computed from the proposed technique will provide information about the impact of assuming random sizes of inclusions.
A. Relevance of the model from MG and DHM

Ten CST® models according to Figs. 1-2 were generated in this section, and SE mean and std were extracted for three volumetric fractions: $v_1=0.01$, $v_2=0.05$, and $v_3=0.10$.

Figures 3-4 show the results obtained considering $v_2$ and $v_3$ (see Fig. 5 for volumetric rate $v_1$ in Subsection IV.B). By assessing a limit of validity, the results validate MG formalism (bold crosses) in frequency. Thus, a good accordance is observed between SE statistics (plain curve and error bars) from numerical FIT modeling and MG mixing rule (transmitted E-field and SE are analytically computed from [14]). Results agree well respectively up to 25 and 15 GHz for cases $v_2$ and $v_3$ (case $v_1$ provides good agreement with MG almost over the entire frequency bandwidth, data not shown here). Based upon DHM [9] principles, an optimized value $\gamma=1.92$ was extracted (close to $\gamma=2$ proposed in [9, 10] for cylindrical infinite straight fibers). As depicted in Figs. 3-4, an excellent agreement is observed between homogenized SE from DHM and 3D time domain FIT models.

Fig. 3. SE of composite structure ($v_2=5\%$) respectively from MG (bold dark green crosses), DHM (thin black crosses), and statistical FIT modeling (plain green line).

Fig. 4. SE of composite material ($v_3=10\%$) respectively from MG (bold dark pink crosses), DHM (thin black crosses), and statistical FIT modeling (plain pink line).

B. Influence of random sizes and locations of spherical inclusions

This subsection is devoted to the presentation of results obtained when assuming both random locations and sizes of inclusions inside resin matrix. To this end, the automated procedure between Matlab® interface (randomly generating both sizes and locations of spherical radii with MC) and CST® was enriched: the complete automation of the procedure avoided any trouble when defining 3D models, launching time domain simulations, post treatments of data). Without any loss of generality, radii of spheres were assumed to be uniformly distributed ($r = 0.050 \pm 0.010$ mm, see caption in Fig. 5). Figure 5 shows the influence of randomizing sizes of inclusions (dashed blue line) in comparison to solely taking into account random location of inclusion inside resin matrix (plain red line). It is to be noted that the analytical results from DHM agree very well with time domain 3-D simulations.

Fig. 5. Impact of assuming random law (uniform distribution between 0.040 and 0.060 mm) for the size of spherical inclusions (volume fraction $v_f=1\%$); constant radius $r=0.050$ mm (bold red line), random distribution of sizes (thin blue line), and analytical homogenized DHM data (black circles).

Figure 6 provides qualitative and quantitative results by assessing the effect of random generation of inclusions for volumetric rate $v_f=0.01$. As it is the case in Fig. 5, SE std offers an overview of the statistical dispersion of results from 0.1 to 60 GHz. This is higher for test case including random dimensions of spherical inclusions (thin blue line) than where spheres’ radii are assumed constant ($r=0.050$ mm). Despite all, the reproducibility (and the relatively low coupling effect of inclusion due to their vicinity) leads to weak levels of SE std. Indeed, the averages over the whole frequency band of SE std are respectively 0.0047 and 0.0081 (far weaker than mean levels of SE around 1.2, see Fig. 5). Finally, Fig. 6 confirms the very good agreement between SE statistics from FIT modeling (CST®) and DHM (taken
from same optimization than in Section IV.A) mixing formalism.

Fig. 6. SE standard deviation (10 CST® simulations, see Fig. 4) with random locations of inclusions and: (red) constant spherical radius (bold line); (blue) random radii of spheres (thin line).

V. CONCLUSION
This article proposes an automated procedure relying on time domain modeling of composite biphasic mixture. It has been validated with regard to classical mixing rules (here Maxwell-Garnett) and offers a statistical overview of the SE of micro-structured composites. The automation of the entire procedure provides useful information about the statistical dispersion of results, jointly with numerical trustworthy data to calibrate advanced homogenization model, such as DHM. Some additional works are nowadays in progress to extend current study to: alternative shapes of inclusions, different electrical characteristics (e.g., conductivity, complex permittivity modeling) of particles and multi-physics issues (thermal and mechanical ones for instance), and modeling of realistic EMC issues including frequency dispersive materials as it is the case in [7]. The proposed methodology could be useful in different domains including antennas and propagation, plasma, metrology, and remote sensing.

REFERENCES