Reducing the Numerical Calculation in the Wave Iterative Method by Image Processing Techniques

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Abstract — The wave iterative method is a numerical method used to model electromagnetic circuits. It is based on the concept of waves in the place of electromagnetic fields. To study the electronic circuits having complex structures, this method requires much time. We propose in this article to improve this method by using techniques of image processing. That’s why the structure of the studied circuit is considered as an image. The objective is to reduce computing time by reducing dimensions of the calculation matrices. The reduced matrices are built containing only the important part of the information. Our goal is to prove that the most important zones in the structure are located in the contour with small steps in the vicinity of the contour.

Keywords — Image processing techniques, reducing calculation matrices, reducing computing time, WCIP method, R-WCIP method.

I. INTRODUCTION
The wave concept iterative process (WCIP) method is a numerical method [1-8] used to ensure electromagnetic modeling of high frequency (HF) electronic circuits. This method is characterized by its stability and its convergence towards good results. In the case of complex structures, this method is also stable but it takes much time to converge to the optimal result. A big number of iterations are necessary to model complex structures requiring a fine mesh. We have a problem of numerical complexity because the number of cells describing the circuit is important. That’s why the WCIP method takes much time to give reach the convergence to the optimal values. In order to improve performances of this method, techniques of image processing are used. Thus, a rapid convergence to the optimal result is ensured.

Our study is based on the principle of image segmentation [9-14], especially the contour detection technique [15-18]. We choose this technique in order to focus on the important information part of the studied structure. The contour represents the place of the important points in the electronic structure because the values of the electromagnetic fields are important on contours and they are weak elsewhere. The contour of a digital image corresponds to a brutal change of the luminous intensity and the image properties. The important values of the electromagnetic fields are in the zones of the structure discontinuity between metal and dielectric in the structure. To locate these zones of interest, we have resort to a method based on contour detection. We focus on the important information while the rest will be neglected. Thus, we build new calculation matrices containing only the useful information. So the new matrices have reduced sizes compared to the original matrices in the classical WCIP method. The calculation time is reduced because we calculate on these reduced matrices that’s why a large gain in computing time is realized and the improved method is noted Reduced-WCIP (R-WCIP).

II. THEORETICAL STUDY
A. Outline on the wave iterative method
The WCIP method is an integral method used in the electromagnetic modeling of HF electronic circuits. The wave iterative method is developed in detail in [1-8]. It is called WCIP because it is based on waves instead of electromagnetic fields and it establishes a recurrent relation between incident and reflected waves. The iterative method is used to study a frequency selective surface (FSS) having a complex structure represented in Fig. 1.
The height of the substrate is \( h = 3 \text{mm} \) and its permittivity \( \varepsilon_r = 3.8 \), \( a = b = 30 \text{mm} \), \( p = 28 \text{mm} \), \( G = 2.8 \text{mm} \), \( D = 22.2 \text{mm} \), \( w_1 = 0, 2 \text{mm} \), and \( w_2 = 3.4 \text{mm} \), the total structure is synthesized using a grid of 150x150 pixels.

The amplitudes of the incidental and reflected waves are expressed in function of electromagnetic fields by the following relations:

\[
\begin{bmatrix}
  A_i \\
  B_i 
\end{bmatrix} = \frac{1}{2 \sqrt{Z_0 i}} \begin{bmatrix}
  
  E_i \\
  J_i 
\end{bmatrix}.
\]

(1)

\( A_i \) and \( B_i \) are respectively the incidental and reflected waves in the plan \( i \). “\( i \)” is an indication of medium (\( i = 1, 2 \)), \( Z_0 \) indicates the wave impedance of the medium. The iterative process consists in establishing a recurrent relation between incident and reflected waves in two different domains as indicated in the following equations:

\[
\begin{align*}
  \vec{E}_{(m,n)} &= \Gamma A_{(m,n)} + \vec{E}_0 \\
  \vec{A}_{(m,n)} &= \Gamma^* \vec{E}_{(m,n)}
\end{align*}
\]

(2)

Two operators are defined: one is in the space domain and the other one is in the spectral domain. The fast Fourier mode transformation (FMT) and the reverse transformation (FMT^-1) ensure the transition from one domain to another. In general, the algorithm of the iterative process is summarized in Fig. 2 with \( \alpha = TE \) or \( TM \).

B. Image processing technique

The image processing technique used in this work is the technique of segmentation [9-14]. It consists in cutting out an image in related areas presenting homogeneity according to a certain criterion, such as the color. The union of these areas must give again the initial image. The segmentation technique allows the extraction of qualitative information in the image. The image segmentation is based on the technique of contour detection which defines the borders between distinct homogeneous zones. It is characterized by the fact that they take into account only information on the contour of the objects. This technique detects all the image contours. The algorithms of contour detection are characterized by the use of derivations operators, the gradient or the Laplacian, from which we respectively seek for the maxima or the passage by zero. In general, the variation of intensity in an image corresponds to relevant zones that contain important information. It ensures the extraction of the useful part in the image. This information corresponds to
borders of homogeneous areas. The contour detection [15-18] is a preliminary stage to many applications of image analysis. Contours constitute rich indices, as well as the points of interests, for any later interpretation of the image. They correspond to discontinuities of the intensity function. For that, the first derivative of the intensity function is studied in order to define local maxima of the intensity gradient function and passages by zero of the Laplacian. That’s why the contour is the place of maximum first derivative and the crossing points by zeros of the second derivative for the intensity function. The difficulty consists in the presence of noise in the image that’s why the derivative calculation requires an image pre-filtering. In a continuous image \( f(x, y) \), a contour appears as a line where the very strong variations of \( f(x, y) \) are localized.

We consider \( \vec{G} \) the gradient of \( f(x, y) \), the module of \( \vec{G} \) is defined in the following relation:

\[
G = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2}.
\]  

The direction of the gradient is defined by:

\[
\vec{g} = \frac{\vec{G}}{G}.
\]  

The contour is the place of the maximum of the gradient \( G \) in the direction \( \vec{g} \), a point of contour is defined by:

\[
\frac{\partial G}{\partial \vec{g}} = 0.
\]  

The second derivative of \( f(x, y) \) in the direction \( \vec{g} \) gives that:

\[
\frac{\partial G}{\partial \vec{g}} = \frac{\partial^2 f}{\partial \vec{g}^2} = 0.
\]  

Finally, the passage by 0 of the Laplacian is obtained and it is given by:

\[
\Delta f = 0 \approx \frac{\partial^2 f}{\partial \vec{g}^2} = 0.
\]

This property is used in the approaches known as contour detectors by passage by zero of the Laplacian. Thus, we conclude that it exists a great number of methods of contour detection in an image. The majority of them can be gathered in two categories. The first category seeks the extremum of the first derivative of the intensity function especially the local maxima of the gradient intensity. The second method seeks the cancellations of the second derivative which means the passage by zero of the Laplacian or the resolution of a non-linear differential expression. In order to guarantee the double derivation, the image is at least pretreated by convolution with a function twice derivable. For this reason, we use the Gaussian function. The approach used in our study is based on the method of “Canny” which has a very good quality of contour detection [15].

C. Reducing matrices by the new algorithm

The important zones of the studied structure are located in the vicinity of contours and they are defined by the technique of contour detection. The useful information is extracted from the original matrix “MxN”, then reduced matrix “Mr x Nr”is built having smaller sizes as in Fig. 3 so that a big part of calculation matrices is neglected.

![Fig. 3. Reducing the dimensions of the original matrices towards reduced matrices.](image)

In the new proposed algorithm, the calculation is carried out on the reduced matrices “Ar” and “Br” which represent the incidental and reflected waves as shown in Fig. 4. The new process starts with the classical WCIP algorithm which calculates a certain number of iterations (Nmin) applied on original matrices having basic sizes “MxN”. After that, the dimensions of the original matrices are reduced using the algorithm of contour detection. The WCIP algorithm is applied to continue the remaining iterations but in this case we operate on the matrices having reduced sizes “Mr x Nr” until reaching the convergence after “Nmax” iterations. Finally, original matrices are rebuilt. Thus, this new approach is called Reduced WCIP (R-WCIP).
since we apply a mechanism of reduction of matrices dimensions by the technique of contour detection. The R-WCIP algorithm operates first on original matrices then on reduced matrices in order to reduce the computing time and to obtain a good result in a minimum of time as in Fig. 5.

III. SIMULATION RESULTS

A. Reduction ratio in the calculation matrices

The contour of the basic studied structure is represented in Fig. 6. The technique of contour detection specifies the zones where the electromagnetic field values are important. In an image in gray levels, a contour is characterized by a brutal change of the intensity values. The goal is to transform this image into another in which contours appear by convention in white on black bottom. Thus, we consider only the contour because it contains the useful information and the remaining zones are neglected.

The zones that are close to contour are called the vicinity of contour. For more precision, the vicinity is detected with different steps (a step of 1 pixel in Fig. 7 and a step of 3 pixels in Fig. 8).

![Diagram](https://via.placeholder.com/150)

**Fig. 5.** The new proposed algorithm R-WCIP.

Our goal is to prove that the most important part in the structure is in the contour with small step in the vicinity of the contour. From the original matrix that its dimensions are equal to 150x150 pixels, a reduced matrix is formed containing only useful information represented in white in the image of the contour vicinity detection. What is represented in black is neglected because it contains information that is not important. In the case of one step in the contour vicinity, a reduction ratio around 76% is realized as described in the following:

- Discretization 150x150 pixels
- Original matrix 150x150 pixel
- Reduced matrix 72x72 pixels
- Reduction ratio = 76%

In the case of three steps in the contour vicinity of the structure, the reduction ratio is around 59% as described in the following:

- Discretization 150x150 pixels
- Original matrix 150x150 pixel
- Reduced matrix 96x96 pixels
- Reduction ratio = 59%

Fig. 6. Contour detection in the basic structure.

Fig. 7. Detection of contour vicinity (Step = 1).

Fig. 8. Detection of contour vicinity (Step = 3).

B. Variation of $S_{11}$ and $S_{21}$ calculated by the new method in function of frequency

In Figs. 9-12, the variation of the coefficients $S_{11}$ and $S_{21}$ is represented in function of frequency. These coefficients are calculated by our new R-WCIP method. These results are compared with those calculated by the classical WCIP method in order to prove that our results are closed to the best and optimal results. The maximum number of iterations calculated by the two methods is equal to 200 iterations ($N_{max}=200$). In the new R-WCIP method, two values of “$N_{min}$” are tested (25 and 50 iterations). Good results are obtained in comparison with the WCIP method. Thus, in our new R-WCIP method, the WCIP algorithm calculates all the 200 iterations on big matrices. Thus, our principal goal which is the reduction of calculation matrices is achieved so that the computing time is reduced. Finally, we have a fast convergence to the optimal result with minimum average error. Thus, good results are obtained by the new R-WCIP method with a best reduction in computing time.

Fig. 9. Parameter “$S$” in function of frequency, 50 basic iterations before reducing the dimensions of the matrices, $N_{max}=200$ iterations, contour vicinity with step=1.

Fig. 10. Parameter “$S$” in function of frequency, 50 basic iterations before reducing the dimensions of the matrices, $N_{max}=200$ iterations, contour vicinity with step=3.
Fig. 11. Parameter “S” in function of frequency, 25 basic iterations before reducing the dimensions of the matrices, Nmax=200 iterations, contour vicinity with step=1.

From the last results, we conclude that the new R-WCIP method provides good results either in the case of one step or three steps in the contour vicinity. This proves that one step near the contour is sufficient to give good results which means that the important information part is located on the contour and not far from the contour. Also, we notice that the number of iterations applied on basic matrices before reducing the dimensions of matrices is 25 or 50 and the remaining iterations are calculated on reduced matrices. Thus, a big gain in computing time is realized and good results are found with the new R-WCIP method.

C. Gain in computing time

In the next paragraph, the classical WCIP method is used to calculate a maximum number of iterations (Nmax). To make a comparison between the two methods, the new R-WCIP method calculates the same number of iterations (Nmax) with two values of Nmin (25 and 50). In Table 1 and Table 2, we observe an important gain in convergence time when calculating S_{11} and S_{21} values after Nmax iterations by the two methods. This gain of time is provided by the new R-WCIP method because we add an algorithm of matrices reduction based on contour detection. The mesh used in this structure is 256x256 cells. The used machine has a microprocessor Intel(R) Pentium(R)

Fig. 12. Parameter “S” in function of frequency, 25 basic iterations before reducing the matrices dimensions, Nmax=200 iterations, contour vicinity with step=3.

Table 1: Gain in computing time given by “R-WCIP”, Fr=11GHz, Grid 150 x 150 pixels, (Nmax=1000 iterations)

<table>
<thead>
<tr>
<th>Nmin</th>
<th>WCIP (Nmax=1000) Time (s)</th>
<th>“R-WCIP” (Nmax=1000) Time (s)</th>
<th>Gain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>86</td>
<td>19</td>
<td>77%</td>
</tr>
<tr>
<td>50</td>
<td>86</td>
<td>24</td>
<td>72%</td>
</tr>
</tbody>
</table>

Table 2: Gain in computing time given by “R-WCIP”, Fr=11GHz, Grid 150 x 150 pixels, (Nmax=200 iterations)

<table>
<thead>
<tr>
<th>Nmin</th>
<th>WCIP (Nmax=200) Time (s)</th>
<th>“R-WCIP” (Nmax=200) Time (s)</th>
<th>Gain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>17</td>
<td>5.9060</td>
<td>65%</td>
</tr>
<tr>
<td>50</td>
<td>17</td>
<td>7.5320</td>
<td>55%</td>
</tr>
</tbody>
</table>
D. Comparison in terms of the average error

In Table 3, we represent the values of the average error calculated on the transmission coefficients $S_{21}$. The band of frequency is from 10GHz to 15GHz. The error is calculated when using the new R-WCIP method in comparison with the classical WCIP method. Two different values of "Nmin" are chosen (25 and 50). The number "Nmin" represents the minimum number of iterations calculated by the classical WCIP algorithm before the reduction of the matrices dimensions in the new R-WCIP method. Then, the remaining iterations are carried out after the mechanism of reduction until the convergence with “Nmax” iterations. The maximum number of iterations “Nmax” is equal to 200 iterations. The average error in comparison with the classical WCIP method is limited. This proves the effectiveness and robustness of our new approach. Finally, we notice that the convergence to the good result very close to the desired value with a minimum average error in each frequency. Thus, the new R-WCIP method gives good results with minimum error.

Table 3: Comparison between WCIP and «R-WCIP» in terms of the average error ($S_{21}$), band of frequency from 10GHz to 15GHz, (Nmax=200 iterations)

<table>
<thead>
<tr>
<th>Nmin</th>
<th>Error(DB)</th>
</tr>
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<tbody>
<tr>
<td>25</td>
<td>0.6077</td>
</tr>
<tr>
<td>50</td>
<td>0.6064</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

In our research work, the wave iterative method has been improved especially when modeling complex structures. Image processing techniques have been used to reduce the dimensions of calculation matrices and to focus on zones in the contour vicinity where the electromagnetic fields are important. The new R-WCIP method operates on reduced matrices so that the computing time is reduced. Finally, a fast convergence to the optimal result is realized in spite of the complex structures of the studied electronic circuits.

REFERENCES


