BROADBAND ANTENNA RESPONSE USING HYBRID TECHNIQUE COMBINING FREQUENCY DOMAIN MoM AND FDTD

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Abstract— A hybrid technique is proposed for computation of broadband characteristics of an antenna in the presence of a dielectric scatterer. The technique links the frequency domain Method of Moments (MoM) and the Finite Difference Time Domain (FDTD) method. The coupling of these methods is achieved by using the Equivalence Principle, applied over the intervening surface. A great reduction in computation time is achieved in the MoM domain by using an impedance interpolation technique. The Gaussian pulse (GP) and Derivative Gaussian pulse (DGP) are investigated as the excitation sources for the MoM. Examples are given and the results found to be in good agreement with alternative methods.

Index Terms—Method of Moments (MoM, Finite Difference Time domain (FDTD), Hybrid computational, Equivalence surface.

I. INTRODUCTION

Hybrid techniques linking the frequency-domain Method of Moments (MoM, i.e. as applied to an integral equation formulation) and the Finite Difference Time Domain (FDTD) method provide a powerful and flexible approach to the numerical solution of a complex antenna structure in the presence of a relatively large lossy dielectric volume [1-4]. If broadband analysis is required, such as the transient response for sub-surface radar, this can be accurately investigated by computing the field propagation over the two different domains and on the equivalence-principle surface (the surface that couples the two domains), over the entire bandwidth.

Previous work in the literature includes Bretones et al’s [5] use of a time-domain integral equation (TDIE) method in a hybrid approach for studying the transient excitation of a thin wire antenna located in the proximity of an inhomogeneous dielectric scatterer and above a PEC ground plane. Cerri et al. [6] also used a TDIE method in developing a hybrid technique. This method has the advantage of generating information over a wide frequency band: it does not require an iterative procedure to couple with FDTD, but it requires very large run-times when treating a junction with more than two wires [7], unlike the frequency-domain version, in which the complex metallic structures may be modelled accurately in shorter run-times and with more flexibility for modelling complex geometries. Huang et al. [8] employed a hybrid technique for modelling the interaction of ground-penetrating radar (GPR) with complex ground using a combination of frequency domain MoM, Fourier transformation and iterations; however, the reaction of the back-scattered field and the source is not discussed in detail and the run time for the MoM part is time-consuming for large source regions. Another hybrid MoM/FDTD method [9] was applied for the specific case of numerical simulations of SAR and the magnetic field of shielded RF coils loaded with a human head for a biomedical application. In another case [10] the source antenna is modelled as a stack of Hertzian dipoles. However, in both [9] and [10] the authors neglect the effect of the back-scattered field on the source. The same approximation is used in [11], which is oriented towards two-dimensional UHF/VHF propagation problems: the FDTD domain is excited just by a vertical slice near the problem area.

Research is still proceeding and more groups have become interested in the hybrid method. Bretones et al have recently published a method to combine the NEC with FDTD [12]. Unfortunately, the back-scattered field on the wire is calculated in a non-optimal way, as the algorithm entails running the FDTD code Nt times (where Nt is the number of the basis functions on the wire antenna). This requires extensive computational time, which could be prohibitive in the case of complex mobile phone handsets with (for example) helical antennas using hundreds of basis functions. Some interesting comparisons between the MoM and FDTD numerical methods are published in [13] for modelling electrically small antennas and in [14] for radiation and scattering involving dielectric objects. The advantages of each technique are discussed: this shows how a hybrid method combining the advantages of the two is a powerful and effective technique.

In this paper, the fields are computed over a wide bandwidth using the method of interpolation of impedance/admittance matrices on the antenna side, applied to frequency-domain MoM [15-17]. This saves execution time and memory requirements for the MoM, while retaining the power and maturity of FDMoM (or
FDIE). Basically, the Inverse Discrete Fourier Transform (IDFT) is performed on the equivalence-principle surface within the FDTD domain and, on the other hand, the Discrete Fourier Transform (DFT) is used to account for the back-scattered fields for the MoM domain. The number of frequency samples required to predict the coupling fields between the two domains over the entire bandwidth was investigated. The positions of these samples over the selected bandwidth were also studied in relation to their effect on recovery of the time-domain fields for different resonant structures. If the conducting structure is of a modest electrical size, such that it obviously cannot support more than one resonant mode, at a frequency that can be approximately estimated, as few as three interpolation points can be sufficient to cover the required mode bandwidth.

**Fig. 1. Basic geometry of the hybrid combination of frequency domain MoM and FDTD.**

**II. SUMMARY OF THE METHOD**

Consider Fig. 1, which shows two different regions. One contains a source and the other a scatterer. The coupling domains between the two regions have been addressed fully in refs. [1-3]. Usually, the coupling is implemented using a single excitation frequency with iteration across the equivalence-principle surface. The forward fields from the source to the scatterer \( E(j\omega), H(j\omega) \) are computed using the frequency domain MoM. The induced surface currents \( J_{si}(j\omega) \) and \( M_{si}(j\omega) \) that represent the FDTD excitation over the closed surface \( S_{ci} \) can be given by:

\[
M_{si}(j\omega) = E(j\omega) \times \hat{n} \quad (1)
\]

\[
J_{si}(j\omega) = \hat{n} \times H(j\omega) \quad (2)
\]

where \( \hat{n} \) is the normal vector directed outward from the closed surface \( S_{ci} \).

The back-scattered fields \( (E_b(t), H_b(t)) \) on the enclosing surface are derived from the FDTD computation. Then the reaction of these fields on the source region can be stated as:

\[
R_B = < E_{ts}, J_{ib} > = < H_{ts}, M_{ib} > = \int_S (E_{ts} \cdot J_{ib} - H_{ts} \cdot M_{ib}) dS_{ib} \quad (3)
\]

where \( E_{ts} \) and \( H_{ts} \) are the test electric and magnetic fields from the source. \( M_{ib} \) and \( J_{ib} \) are the induced magnetic and electric surface currents on the closed surface \( S_{ib} \). "< >" and "•" represent the inner product and the vector dot product respectively.

The procedure is repeated until a steady state solution for this single frequency is reached. In general, if broadband antenna analysis is required, many frequency samples are required to cover the entire bandwidth on the FDMoM side to predict the required time variations of the induced surface current on the enclosed equivalent surface (for use by the FDTD side). Using IDFT, these samples can be combined as the excitation of the FDTD domain. The main problem is the computational time required to evaluate these samples on the MoM side of the problem because the MoM has to be executed once for each sample.

Thus, an impedance/admittance interpolation method on the MoM side was developed to predict these fields on the equivalence-principle surface. The method requires storage of impedance matrices for a few different frequencies over the specified bandwidth. Then, using the quadratic interpolation method (requiring three selected points), the impedance can be found at any frequency between these points, as follows.

The interaction \([Z]\) matrices for the three selected frequencies are directly computed by FDMoM [1-3]. The elements of \([Z]\) for the intermediate frequencies are approximated by a quadratic function:

\[
Z_{mn}(f) = A_{mn}f^2 + B_{mn}f + C_{mn} \quad (4)
\]

where \( f \) denotes frequency and \( A_{mn}, B_{mn}, \) and \( C_{mn} \) are the \( mn \)th elements of the complex coefficient matrices \([A], [B], \) and \([C]\). Equation (4) can be cast into a system of three equations and three unknowns. These equations, together with the elements of the directly computed \([Z]\) matrices that are calculated at three selected frequencies, are used to determine the coefficient matrices. If the frequency band of interest is especially wide, it may be necessary to divide the band into several interpolation frequency ranges and implement a process of stepping through them.

The frequency characteristics of the \([Z]\) matrix elements can be determined by the Electric Field Integral Equation (EFIE) and the form of the basis and test functions used [15]. These equations reveal that the term \( e^{jkR} \) dominates the frequency behaviour of the \([Z]\)
elements. For matrix element $Z_{mn}$, $R$ equals $r_{mn} = |r_m - r_n|$, where $r_m$ is the observation location and $r_n$ is the source location. When the observation and source points are close to each other, $r_{mn}$ is small and $e^{-j\sqrt{\mu \sigma}}$ varies slowly with frequency. When they are far from each other, $r_{mn}$ is large and $e^{-j\sqrt{\mu \sigma}}$ fluctuates rapidly with frequency and thus dominates the frequency variations of the $[Z]$ matrix elements. An improved method for computation of $[Z]$ elements in this case can be implemented by direct interpolation of the $[Z]$ elements divided by the factor $e^{-j\sqrt{\mu \sigma}}$ and then the resultant interpolation value is multiplied by the same factor [15,16].

The interpolation function can also be cast in a form that accurately models the behaviour of the large-amplitude terms in $[Z]$, i.e. the self-terms and those quantifying the coupling between very close segments. This has been implemented for thin wire antennas [16]. However, this was done to increase the accuracy in evaluating the terms of $[Z]$ at the intermediate frequencies or to allow larger interpolation frequency ranges.

Thus, the currents for a particular frequency, resulting from a wideband excitation (e.g. a Gaussian Pulse) can be found once the impedance matrix at that frequency can be predicted. Therefore, the scattered fields on the equivalence-principle surface due to the predicted currents can be computed. Once these fields are computed, the finite difference equations at the equivalent surface can be stated as follows:

$$
E = E_{FDTD} + iDFT(\hat{n} \times H(j\omega))/(\varepsilon/\Delta t + \sigma/2) \quad (5)
$$

$$
H = H_{FDTD} - iDFT(\hat{n} \times E(j\omega))/((\Delta / \mu) \quad (6)
$$

where $E_{FDTD}$ and $H_{FDTD}$ are the normal electric and magnetic field finite-difference updating equations. $\Delta t$ is the time step, $\varepsilon$, $\mu$, and $\sigma$ are the absolute permittivity, permeability and conductivity of the surrounding medium, respectively. The above equations are executed for each FDTD iteration and hence simultaneously cover all of the frequency samples, taking into the account the frequency spectrum of the pulse applied, obtained from MoM and interpolation by performing the IFDT on the transferred excitation data. Using the above technique and the theory presented in [2] it can be deduced that the computation time required for the hybrid method depends predominantly on the execution time of FDTD method.

III. RESPONSES AND NUMBER OF SAMPLES

Consider Fig. 2, which shows the frequency responses of two different forms of the real part of the antenna input impedance. Usually the selected points shown in both of them can be used for interpolation, to predict the entire response. This response will be used to obtain the time-domain performance of the antenna using the inverse Fourier transform. For the case shown in Figure 2a, uniform frequency sampling can be used since the rate of change of the curve at resonance is slow. However, for the case shown in Figure 2b, a very large number of frequency samples would be required to resolve the detail in the resonance peaks. Since the location of the resonances cannot normally be known beforehand, there is no convenient way to quickly determine the order of the samples required for interpolation. For a limited known bandwidth the resonances can usually be predicted with sufficient accuracy for some extended set of interpolation points by applying approximated analytical approaches: confidence in the choices can be strengthened by repeating the simulation two or three times with differing choices to test for stability.

To save time, and use the same number of frequency samples to predict the frequency response, the non-uniform inverse Fourier transform was used:

$$
x(t) = 2 \sum_{i=1}^{n} \Delta \omega_i \text{Re}(X(j\omega_i)e^{j\omega_i t}) \quad (7)
$$

where $\omega_i$ (for $i=1,2,\ldots,n$) is the $i^{th}$ frequency sample.

\[\Delta \omega_i\] is the frequency width from the preceding sample to the frequency sample $\omega_i$. A substantially higher density of frequencies $\omega_i$ (for $i=1,2,\ldots,n$) is chosen around the sharp resonance regions of the frequency response. This method must be used when a strong resonance is present, either inherently in one part of the system (e.g. a helical antenna) or as a result of strong coupling between the source and a scattering element. Since the time-domain behaviour of the antenna can be found, the scattered fields in the time domain can be found easily, subject to the proper selection of frequency samples that adequately predict the approximated antenna response. Therefore, these fields can be used to replace the actual source (the antenna) by a chosen equivalent surface. This surface can then be coupled to any time-domain method, such as FDTD, in a hybrid technique.
IV. EXCITATION SOURCE

The input voltage source considered was either a Gaussian pulse (GP) or a derivative Gaussian pulse (DGP). The temporal behaviour of this impressed voltage source can be given as follows. For GP excitation:

\[ v(t) = Ae^{-g^2(t-t_d)^2} \]  

(8)

where A is the amplitude of the pulse, g is the inverse of the rise time, and \( t_d \) is the time delay.

The frequency domain version of this source can be found using Fourier transformation, thus:

\[ v(j\omega) = \frac{\sqrt{\pi}}{g} e^{-\frac{\beta \omega}{g}} e^{-\frac{\omega^2}{4g^2}}. \]  

(9)

A pulse with parameters \( A=1V, g=1.5 \times 10^9 s^{-1} \) and \( t_d = 2.5ns \) was chosen as an example having a 3dB bandwidth somewhat less than 1 GHz. This provides excitation across the operating band for antennas with a bandwidth of 1 GHz. The time-recovered GP, reconstituted from the frequency domain using 128 selected frequency samples uniformly distributed over the frequency spectrum, was found to be close to the shape of the original pulse. The working bandwidth of this pulse was taken to be 1.5 GHz.

The second excitation source was represented by a DGP. Originally, this pulse was used because there is no DC component in its frequency spectrum. It is given by:

\[ v(t) = -Ag^2(t-t_d)e^{-g^2(t-t_d)^2} \]  

(10)

where A is the amplitude. The frequency domain version of the above pulse can be found, using a Fourier transform, as:

\[ v(j\omega) = -j\frac{A\omega}{g} \sqrt{\pi} e^{-\frac{\beta \omega}{g}} e^{-\frac{\omega^2}{4g^2}}. \]  

(11)

The time-domain version of the DGP, its spectrum and phase are shown in Fig. 3. The time reconstruction of this pulse over 128 frequency samples, uniformly distributed over 1.5 GHz, is presented in Fig. 3d. This was obtained using the inverse discrete Fourier transform.

V. SIMULATION AND RESULTS

Several examples are given to highlight the features of the method. The results are compared with those from some homogeneous time-domain methods such as pure FDTD and TDIE [18]. In all cases, the dielectric volume was considered to be free space. This is adequate to test the stability of the method and its agreement with the results of other methods (which may not be able to handle dielectrics).

Example 1. A straight wire antenna of length 0.5m and radius 1mm was analysed when driven by a DGP with \( g=1.5 \times 10^9 s^{-1} \) and \( t_d = 2.5 ns \). The number of frequency samples for the Z-matrix interpolation used here was 9. The proposed method was executed over the entire bandwidth of 1.5 GHz. The numbers of frequency samples over the bandwidth were chosen to be 64 and 128, in order to match the inverse discrete Fourier transforms: a uniform discretisation technique was used in this case. The source current versus time is shown in Fig. 4 and the results are compared with those from standard packages implementing the FDTD and TDIE methods. The results of the proposed method are in good overall agreement with those obtained from FDTD and TDIE.

Fig. 3. Example of a differential Gaussian pulse for \( g=1.5 \times 10^9 s^{-1} \) and \( t_d = 2.5 ns \). (a) vs. scaled time. (b) Spectrum magnitude vs. frequency. (c) Phase vs. frequency. (d) Recovered pulse for 128 frequency samples, vs. scaled time.

Fig. 4. The input source current versus scaled time for a dipole excited by a DGP.
Example 2. The same wire antenna geometry of example 1 was used, but excited by a GP having \( g = 1.5 \times 10^9 \text{ s}^{-1} \) and \( t_d = 2.5 \text{ ns} \). The number of frequency samples used for Z-matrix interpolation was 14. The lowest frequency sample was chosen at 25Hz to exclude DC effects, which would require a different treatment. The proposed method was run over the rest of the entire bandwidth of 1.5 GHz. The number of frequency samples used to cover the bandwidth was chosen to be 128, uniform discretisation again being used. The source current versus time is shown in Fig. 5. The results are in good agreement with those obtained from FDTD alone and TDIE.

Example 3. A source consisting of a helical wire antenna was analysed, using the same excitation voltage as in Example 1. The helix had the following geometry: pitch distance 0.01m; helix radius 0.015m; wire radius 0.5mm; no. of turns 6. The excitation was located at the centre of the helix. Fifteen points were used for Z-matrix interpolation to cover the entire 1.5 GHz bandwidth. 256 frequency samples were predicted, applying the non-uniform discretisation technique. The source current is presented in Fig. 6: compared to the results obtained from TDIE, the proposed method is quite acceptably accurate for such a complex geometry as the helix. The FDTD results are not presented in Figure 6 since no real model for the exact geometry of the antenna can be developed using this method, due to its curved wire nature.

Example 4. A near field due to a dipole, having similar configuration and voltage excitation as in Example 1, was investigated using the proposed technique. The dipole was replaced by the closed equivalence-principle surface inside the FDTD region that represents the scattered fields, as shown in Fig. 7. The size of the closed surface was \( 4 \times 4 \times 61 \text{ cells} \text{ (cell size = 1cm)} \). A suitable number of frequency samples used inside the FDTD zone, as predicted from the selected frequency points, was 256. The memory required for field transformation was around 4Mbyte. The interval time considered in this example was double the time taken by the previous examples, to illustrate the stability of the method. The near fields at 0.05m from the antenna are shown in Fig. 8. The results show a good agreement of the present method with that computed from the pure FDTD and the TDIE methods, both in the decay of the amplitude and the periods of the following pulses, for interval times of most interest. A maximum of less than 5% difference was detected between these field amplitudes. The computation time was two hours on a Sun Sparc 10 workstation.

Example 5. A two-arm square-spiral wire antenna with a scatterer formed of two crossed wire dipoles (Fig. 8) were investigated. The square spiral was selected here simply to compare the results with FDTD, as its shape is conformal to the rectangular grid. The radius of all of the antennas was chosen to be 0.01m. The dipoles were excited at the centre of the spiral. The size of the FDTD zone was \( 6 \times 6 \times 61 \text{ cells} \). A suitable number of frequency samples used inside the FDTD zone, as predicted from the selected frequency points, was 256. The memory required for field transformation was around 4Mbyte. The interval time considered in this example was double the time taken by the previous examples, to illustrate the stability of the method. The near fields at 0.05m from the antenna are shown in Fig. 9. The results show a good agreement of the present method with that computed from the pure FDTD and the TDIE methods, both in the decay of the amplitude and the periods of the following pulses, for interval times of most interest. A maximum of less than 5% difference was detected between these field amplitudes. The computation time was two hours on a Sun Sparc 10 workstation.
the wires was chosen to be 1mm. The spiral was located 3cm above the cross point (origin point) of the crossed dipoles. The source excitation was placed at the centre of the spiral using the same DGP as in Example 1. The spiral antenna was then replaced by the closed equivalence-principle surface inside the FDTD region that represents the scattered fields, as shown in Fig. 8. The size of the closed surface was $16 \times 18 \times 4$ cells (cell size = 1cm). The number of frequency samples predicted from the selected frequency points was 256 and the memory required to store the fields on the equivalent surface was 2Mbyte. The near electric field was recorded at $x = 0.0m$, $y = 0.08m$, $z = 0.03m$, as shown in Fig. 9. There is good agreement between the two methods although the peak values differ by approximately 3%. The total simulation time was two hours on a Sun Sparc 10 workstation.

![Fig. 8. The electric field component $E_z$ versus the scaled time at 0.05m from a source dipole excited by a DGP.](image)

VI. CONCLUSIONS

A method to obtain broadband antenna responses from the hybrid technique using the frequency-domain Method of Moments (IE Method) and the Finite Difference Time Domain method has been presented. This has the advantages that complex arbitrarily-shaped metallic antenna structures can be simulated with the mature FDMoM method, in combination with large complex penetrable dielectric bodies that can be efficiently modelled with the FDTD method. A [Z]-matrix interpolation methodology on the MoM side was used in order to significantly reduce the computational time required for wide-band performance evaluation of antennas. Quadratic interpolation was found sufficient to predict the time-limited responses. The use of non-uniform frequency samples on sharp resonances has been shown to be a viable technique for minimisation of computational and memory requirements. The results were in reasonable agreement with available data from relevant (but less flexible) single-algorithm methods: it is likely that the disagreement in amplitude (of less than 5%) with the FDTD method could be due to deficiencies in the representation of wires in the latter: this does not invalidate the fundamental merits of the hybrid technique. Particularly, the computation time for the present method depends mostly on the running time of the FDTD method.

An enhancement to the technique can be achieved by implementation of the method in Ref. [2]. In a single-frequency example, this showed how current crossing the boundary between the two domains can be represented: this has great advantages for the present work because it can reduce the memory requirements and the computation time since only the most complex part of the antenna need to be modeled.

![Fig. 9. The antenna geometry of example 5 inside the hybrid method domains.](image)
Fig. 10. The electric field component versus the scaled time at position $x = 0.0m$, $y = 0.08m$, $z = 0.03m$ of Fig. 8; (a) $E_x$ component, (b) $E_y$ component.

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