USING NONUNIFORM SEGMENT LENGTHS WITH NEC TO ANALYZE ELECTRICALLY LONG WIRE ANTENNAS

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ABSTRACT

In this paper an electrically long monopole is analyzed using the Numerical Electromagnetics Code, version two (NEC-2). Due to the electrical length of the monopole and the segment length requirement established for the NEC-2 program, the computer memory requirement and computational time become excessive. By successively increasing the segment length with distance from the source, very large structures can be analyzed accurately and efficiently. Various schemes for selecting the segment lengths are considered, and the results are compared to those obtained by using a large number of electrically short segments. This technique of grading segment lengths allows one to employ NEC-2 to analyze other electrically long wire antennas.

INTRODUCTION

When modeling long wires at high frequencies, the use of sub-domain current basis functions (such as used with NEC-2) poses problems with regard to excessive computer storage space and computation time. In this paper an electrically long monopole is analyzed using NEC-2. Various schemes for choosing segment lengths are investigated to reduce the computer memory and computation time requirements, while maintaining sufficient accuracy. By successively increasing the segment length with distance from the driving point, it is found that very large structures can be analyzed accurately and efficiently.

In general, and as stated in Reference [1], NEC-2 requires that the length of each segment be less than 0.1λ and larger than 0.001λ at the frequency of operation. When wires are relatively long, with no abrupt changes or bends, segments somewhat longer than 0.1λ may be permissible. Segments less than 0.05λ are advisable for modeling critical regions of an antenna. It is shown here that accurate results can be obtained using electrically short segments in the source region with much longer segments away from the source. Nevertheless, there are obvious limitations to segment length. The main thrust of this paper is
to illustrate an optimum segmentation scheme for electrically long wires in order to maximize the efficiency of NEC-2.

SEGMENTATION SCHEMES

Consider as a test case a monopole antenna 20 meters in length and 3.175 millimeters in diameter which is operated at a frequency of 299.8 MHz (i.e., the wavelength is 1 meter) over a perfect ground plane. This structure is used to illustrate the capabilities, accuracy, and limitations of using NEC-2 with various segmentation schemes to model electrically long wires. The ground plane is located on the x-y plane of a cartesian coordinate system while the monopole antenna is directed along the z-axis. The monopole is driven by a 1 volt source (applied E-field type) located at its base.

Note that the electrical length of the test monopole is exactly 20 wavelengths (20\(\lambda\)). Thus, the monopole is an electrically long thin wire, but the electrical length is short enough that a reference case using equal length segments can be used to provide highly accurate results for comparison purposes. At excessively high frequencies, reference cases of high accuracy are impossible to run, since they would exceed the computer memory available. For the data presented here a DEC VAX 11/785 minicomputer with 16 megabytes of core memory was used.

The electrically long monopole is modeled with three segmentation schemes in order to determine which model, with the smallest number of wire segments, leads to results that are comparable with those of the reference case. Before the three different models of the monopole antenna are discussed, a description of the current modeling technique is necessary. The complex-valued current along segment \(i\) is defined as \(I_i(s)\) and may be written as (Reference [2]),

\[
I_i(s) = A_i + B_i \sin [k(s-s_i)] + C_i \cos [k(s-s_i)]
\]

(1)

where,

\(k\) is the propagation constant (2\(\pi/\lambda\)),
\(s\) is the distance parameter along the segment axis

and

\(s_i\) is the center point of segment \(i\).

Three specific segment configurations are considered with various maximum segment lengths. In order to make direct comparisons of the wire currents, Equation (1) is used to generate the monopole
current data given the coefficients computed using NEC-2. This was done because the NEC-2 program allows the current to be calculated only at the center of each segment which can make comparisons difficult when the segments are relatively long and their centers are far apart.

The first model considered is the reference model. In this model, the monopole is divided into 2000 equal segments of length $0.01\lambda$. This reference case, according to the NEC-2 user handbook [1], is sufficiently accurate to provide standard data for verifying nonstandard segment configurations. Figures 1 and 2 show the resulting current magnitude and phase along the length of the monopole antenna, respectively.

The second model utilizes graded segment lengths over the entire wire structure, an option within NEC-2 designated here as "entire grading". The length of each segment is defined as a constant ratio (RDEL) times the length of the previous segment as shown in Equation (2):

$$S_i = \text{RDEL} \ S_{i-1} \quad \text{for} \quad 2 \leq i \leq \text{NS}$$

(2)

where NS is the total number of segments on the wire and $S_i$ defines the total length of the $i^{th}$ segment. Utilizing this model, the user may choose the length of the first segment ($S_1$) and the length of the last segment ($S_{NS}$), which determines the values of RDEL and NS. The length of the first segment is expressed in terms of RDEL, NS and the total length (L) of the monopole by Equation (3), taken from Reference [1], while the resulting expression for $S_{NS}$ is given by Equation (4).

$$S_1 = \frac{L \ (1 - \text{RDEL})}{1 - (\text{RDEL})^{NS}}$$

(3)

$$S_{NS} = (\text{RDEL})^{NS-1} \ S_1$$

(4)

Using Equations (3) and (4), equations for RDEL and NS may be obtained in terms of the specified parameters $S_1$, $S_{NS}$, and L. Equations (5) and (6) represent the resulting expressions for RDEL and NS, respectively.

$$\text{RDEL} = \frac{1 - (S_1/L)}{1 - (S_{NS}/L)}$$

(5)
\[ NS = 1 + \frac{\ln(S_{NS}/S_1)}{\ln(\text{RDEL})} \] (6)

Using Equations (5) and (6), three examples illustrating the entire grading technique are considered. In each of the cases \( S_1 \) is chosen to be 0.01\( \lambda \), while \( S_{NS} \) is chosen to be 0.25\( \lambda \), 0.5\( \lambda \), and 0.75\( \lambda \), respectively. From the given values of \( S_1, S_{NS}, \) and \( L \), the values of RDEL and NS are calculated. Since the value of NS must be an integer, and since Equation (5) leads in general to a non-integer value, the value of NS is then rounded up or down, as needed. The integer value of NS is then used in Equation (7) to find the value for RDEL. Using the calculated values of NS and RDEL, new values for \( S_1 \) and \( S_{NS} \) are then determined which may vary slightly from the originally chosen values.

\[ \text{RDEL} = (S_{NS}/S_1)^{1/(NS-1)} \] (7)

Using the entire grading technique, the current along the monopole was computed for three different cases. Figures 3 and 4 represent the current magnitude and phase, respectively, with \( S_{NS} = 0.25\lambda \) (the longest segment is one-quarter wavelength). Similar plots were obtained for \( S_{NS} = 0.5\lambda \) and \( S_{NS} = 0.75\lambda \). Figures 5 and 6 correspond to the case where \( S_{NS} = 0.5\lambda \) while Figures 7 and 8 correspond to the case of \( S_{NS} = 0.75\lambda \). In order to make comparisons between the reference current and the current obtained for the entire grading technique, a quantitative measure of the current deviation must be defined. The rms deviation of the current \( I(z) \) from the reference value \( I_{\text{REF}}(z) \) is designated as \( \Delta I_{\text{RMS}} \) and defined by

\[ \Delta I_{\text{RMS}} = \sqrt{\frac{1}{L} \int_0^L |I_{\text{REF}}(z) - I(z)|^2 \, dz} \] (8)

A percentage rms deviation of the current may be defined as the percentage of \( \Delta I_{\text{RMS}}/I_{\text{RMS}} \) where

\[ I_{\text{RMS}} = \sqrt{\frac{1}{L} \int_0^L |I_{\text{REF}}|^2 \, dz} \] (9)

Table 1 gives a comparison between the reference case results and the entire grading technique results in terms of the total number of segments used, the program run time, the computed input impedance and the percentage rms deviation. A significant reduction in run time is realized with the entire grading modeling technique with a relatively small reduction in solution accuracy.
Figure 1. Current Magnitude Along a Monopole of Length 20\( \lambda \). Using 2000 Equal Segments of Length 0.01\( \lambda \).

Figure 2. Current Phase Along a Monopole of Length 20\( \lambda \). Using 2000 Equal Segments of Length 0.01\( \lambda \).

Figure 3. Current Magnitude Along a Monopole of Length 20\( \lambda \). Using Entire Grading with \( S_{NS} = 0.25\lambda \).

Figure 4. Current Phase Along a Monopole of Length 20\( \lambda \). Using Entire Grading with \( S_{NS} = 0.25\lambda \).
Figure 5. Current Magnitude Along a Monopole of Length $20\lambda$ Using Entire Grading with $S_{NS} = 0.5\lambda$.

Figure 6. Current Phase Along a Monopole of Length $20\lambda$ Using Entire Grading with $S_{NS} = 0.5\lambda$.

Figure 7. Current Magnitude Along a Monopole of Length $20\lambda$ Using Entire Grading with $S_{NS} = 0.75\lambda$.

Figure 8. Current Phase Along a Monopole of Length $20\lambda$ Using Entire Grading with $S_{NS} = 0.75\lambda$. 
The technique of grading the segment lengths over the entire wire length significantly reduces the number of required segments when compared to the equal segment model (See Table 1). Yet, this reduction may not be sufficient to treat another antenna of much larger dimensions due to the limitation on the longest segment. Hence, a segmentation scheme which further reduces the number of required segments while yielding relatively accurate results, is needed.

The third segmentation scheme considered here utilizes a partial grading technique, i.e. grading of segment lengths over a portion of the monopole antenna (source end), while using long segments of equal length for the remainder of the monopole. For the graded portion of the monopole antenna, the previously defined equations are used with $L$ being replaced by $L_1$ (the length of the graded portion), $NS$ being replaced by $NS_1$ (the total number of segments on the graded portion), and $S_{NS}$ being replaced by $S_{NS_1}$ (the length of the longest segment on the graded portion). A value of $RDEL=1.1$ is chosen to yield a ten percent increase in the length of successive segments. With $RDEL$, $S_1$, and $S_{NS_1}$ specified, an integer value for $NS_1$ is evaluated, then $RDEL$ is reevaluated using Equation (7). The new value of $RDEL$ is then used to evaluate $L_1$, which in turn is used to find $L_2$, the length of the portion of the monopole with equal segments. With $L_2 = L - L_1$, a new value of $L_2$ must be chosen to yield an integer number of segments ($NS_2$) on the antenna region containing segments of equal length. Once that is accomplished, $L_1$ is recalculated using $L_1 = L - L_2$. From the new value of $L_1$, the final value for $RDEL$ is evaluated using Equation (5). Then $RDEL$ and $L_1$ are used to re-estimate the values of $S_1$ and $S_{NS_1}$, whose values may vary slightly from the originally defined values.

Using the partial grading technique, nine cases are considered. In each case, the length of the first segment of the weighted portion of the monopole is chosen to be approximately $0.01\lambda$, while the length of the last segment is chosen to be approximately $0.25\lambda$, $0.35\lambda$, $0.45\lambda$, $0.5\lambda$, $0.55\lambda$, $0.65\lambda$, $0.75\lambda$, $0.85\lambda$, and $0.95\lambda$, respectively. The long segments of equal length are chosen to be exactly $0.25\lambda$, $0.35\lambda$, $0.45\lambda$, $0.5\lambda$, $0.55\lambda$, $0.65\lambda$, $0.75\lambda$, $0.85\lambda$ and $0.95\lambda$ in length, respectively. For each of the nine cases, the current magnitude and phase along the monopole were plotted and are shown in Figures 9 through 26. The percent rms deviation of the current with regard to the reference current is defined as shown in Equations (8) and (9). The quantities of interest for the nine partial grading examples are shown in Table 1. The results of the partial grading examples with $S_{NS} = 0.25\lambda$ and $S_{NS} = 0.35\lambda$ (see Figures 9 through 12) have results that compare quite well with the results of the reference model. The partial grading example with $S_{NS} = 0.45\lambda$ yields acceptable results (see Figures 13 and 14), especially in the region away from the source. Due to the huge reduction in the number of segments, 74 total segments as opposed to the 2000 segments used in the reference case, this model would be ideal when quick results are more important than highly accurate results. The partial grading example with $S_{NS} = 0.5\lambda$ produced poor results (see Figures
Figure 9. Current Magnitude Along a Monopole of Length 20λ Using Partial Grading with RDEL = 1.1 and S_{NS} = 0.25λ.

Figure 10. Current Phase Along a Monopole of Length 20λ Using Partial Grading with RDEL = 1.1 and S_{NS} = 0.25λ.

Figure 11. Current Magnitude Along a Monopole of Length 20λ Using Partial Grading with RDEL = 1.1 and S_{NS} = 0.35λ.

Figure 12. Current Phase Along a Monopole of Length 20λ Using Partial Grading with RDEL = 1.1 and S_{NS} = 0.35λ.
Figure 13. Current Magnitude Along a Monopole of Length 20$\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.45\lambda$.

Figure 14. Current Phase Along a Monopole of Length 20$\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.45\lambda$.

Figure 15. Current Magnitude Along a Monopole of Length 20$\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.5\lambda$.

Figure 16. Current Phase Along a Monopole of Length 20$\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.5\lambda$. 

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Figure 17. Current Magnitude Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.55\lambda$.

Figure 18. Current Phase Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.55\lambda$.

Figure 19. Current Magnitude Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.65\lambda$.

Figure 20. Current Phase Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DEL} = 1.1$ and $S_{NS} = 0.65\lambda$. 
Figure 21. Current Magnitude Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DE} = 1.1$ and $S_{NS} = 0.75\lambda$.

Figure 22. Current Phase Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DE} = 1.1$ and $S_{NS} = 0.75\lambda$.

Figure 23. Current Magnitude Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DE} = 1.1$ and $S_{NS} = 0.85\lambda$.

Figure 24. Current Phase Along a Monopole of Length $20\lambda$ Using Partial Grading with $R_{DE} = 1.1$ and $S_{NS} = 0.85\lambda$. 
Figure 25. Current Magnitude Along a Monopole of Length 20\(\lambda\) Using Partial Grading with RDEL = 1.1 and \(SNS = 0.95\lambda\).

Figure 26. Current Phase Along a Monopole of Length 20\(\lambda\) Using Partial Grading with RDEL = 1.1 and \(SNS = 0.95\lambda\).

15 and 16) due to numerical instabilities caused by the large number of 0.5\(\lambda\) segments. Using a large number of 0.5\(\lambda\) segments seems to affect the boundary conditions used in determining the current expansion coefficients defined in Equation (1). The current results from the cases with long segments of 0.55\(\lambda\) and 0.65\(\lambda\) (see Figures 17 through 20) are somewhat worse than those of the 0.45\(\lambda\) case. The three cases with \(SNS = 0.75\lambda, 0.85\lambda,\) and 0.95\(\lambda,\) respectively, seem to yield acceptable results (see Figures 21 through 26), when compared to the reference case. However, this may be coincidental. Long segments over 0.65\(\lambda\) are not recommended, because there is no guarantee that they would yield similar percentages of accuracy for general long wire antennas.

CONCLUSIONS

Three segmentation schemes for electrically long antennas were investigated and compared using a monopole over a perfect ground plane as an example. The first model, characterized by equal segments of 0.01\(\lambda\), provided accurate results based on the general segment length recommendations of the NEC-2 code, i.e., segments should be longer than 0.001\(\lambda\) but shorter than 0.1\(\lambda\) [1]. The entire grading technique,
<table>
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<tr>
<th>Modeling Procedure</th>
<th>$S_{NS}$</th>
<th>Total Number of Segments</th>
<th>Run Time (Seconds)</th>
<th>Input Impedance ($\Omega$)</th>
<th>Percent rms Deviation</th>
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<tr>
<td>Reference Case</td>
<td>0.01(\lambda)</td>
<td>2000</td>
<td>99182.84</td>
<td>305.48–j233.22</td>
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<td>267</td>
<td>431.97</td>
<td>334.09–j204.65</td>
<td>6.632</td>
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<tr>
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<td>159</td>
<td>137.78</td>
<td>332.62–j176.87</td>
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<td>0.75(\lambda)</td>
<td>115</td>
<td>67.73</td>
<td>334.22–j182.63</td>
<td>14.49</td>
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<tr>
<td><strong>Partial Grading</strong></td>
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<td></td>
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<tr>
<td>0.25(\lambda)</td>
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<td>48.72</td>
<td>323.85–j194.41</td>
<td>6.921</td>
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<tr>
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<td>34.07</td>
<td>309.68–j191.79</td>
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<tr>
<td>0.45(\lambda)</td>
<td>74</td>
<td>25.55</td>
<td>315.99–j123.05</td>
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<tr>
<td>0.50(\lambda)</td>
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<td>24.63</td>
<td>54.58–j6.28</td>
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<tr>
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<td>68</td>
<td>22.88</td>
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<td>21.05</td>
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<tr>
<td>0.75(\lambda)</td>
<td>61</td>
<td>19.27</td>
<td>376.01–j198.12</td>
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<td>0.85(\lambda)</td>
<td>59</td>
<td>18.55</td>
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<td>0.95(\lambda)</td>
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<td>17.13</td>
<td>393.51–j261.64</td>
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Table 1. Comparison of the Results Obtained Using: (1) Electrically Short Segments of Equal Length, (2) Entire Grading, and (3) Partial Grading.

where successive segment lengths are increased over the entire wire length (moving away from the source), produced sufficiently accurate results using significantly fewer segments. The third modeling procedure, referred to the partial grading technique, allows for even fewer segments in the long wire model by successively increasing the segment lengths over a portion of the wire and using "long" segments of constant length over the remaining portion of the wire. The partial grading technique provided accuracy equivalent to the entire grading technique with a further reduction in computation time. The use of many segments exactly one-half wavelength long was shown to generate numerical instabilities in the solution process. Segments longer than one-half wavelength should be avoided as they were shown to yield neither
accurate nor dependable results. Although the results for some cases with segments lengths greater than one-half wavelength yielded reasonable results, this agreement seemed to be coincidental. Segments less than $0.5\lambda$ in length led to the most accurate results, among the cases considered for the partial grading technique. Consequently, when using the partial grading modeling procedure, it is recommended that the long segments of equal length be kept at or below $0.45\lambda$ to achieve reliable results.

Comparisons of results from the three segmentation schemes show that a significant reduction in the number of unknowns (and resulting computation time) is realized by using the partial grading and entire grading techniques as opposed to the modeling scheme with equal length segments over the entire wire structure. Yet, the resulting sacrifice in accuracy is relatively minor in all the cases, except for the partial grading case with equal segments of exactly $0.5\lambda$. Generally, the greatest error occurs in the current near the source. Consequently, the error in input impedance is significantly greater than the overall error in the computed current distribution.

In all three modeling schemes, the segment lengths in the neighborhood of the source were kept very small (approximately $0.01\lambda$). This condition is necessary, as indicated in Reference [1], when any wire structure is modeled. For a loaded wire, the segment lengths in the neighborhood of the load must be kept small. Similar grading schemes can be implemented for wire structures having sharp bends by using electrically short segments in the vicinity of the bend and graded segments away from the bend.

As a general conclusion, this study accomplished its major goal of developing efficient modeling schemes for electrically long antennas using NEC-2. The entire grading and partial grading techniques have been shown to be effective in modeling antennas quickly and accurately with a small number of unknowns. An application of the segmentation schemes developed here are found in References [3] and [4].

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REFERENCES


