NF–FF Transformation with Bi-Polar Scanning From Nonuniformly Spaced Data

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ABSTRACT
An efficient probe compensated near-field–far-field transformation technique from irregularly spaced bi-polar samples is developed in this paper. The singular value decomposition method is applied to recover the uniformly distributed data, whose position is fixed by a nonredundant sampling representation of the electromagnetic field. Then an optimal sampling interpolation algorithm is used for reconstructing the plane-rectangular samples required to carry out the standard near-field–far-field transformation. This last step is required to benefit by the use of FFT algorithm. Numerical examples are reported to assess the effectiveness of the proposed technique.

1. INTRODUCTION
The method of constructing antenna radiation patterns from near-field (NF) measurements has been widely investigated in the last two decades and used for applications ranging from cellular phone antennas to large phased arrays and complex multi-beam communication satellite antennas. It has been proved to be an efficient and attractive alternative to conventional far-field (FF) range and compact range measurements. There are diverse methods for the FF evaluation depending on the ways data are acquired. Among them, that employing the bi-polar scanning [1-3] is particularly attractive for its mechanical characteristics. The antenna under test (AUT) rotates axially, whereas the probe is attached to the end of an arm which rotates around an axis parallel to the AUT one. This allows one to collect the NF data on a grid of concentric rings and radial arcs (see Fig. 1). The bi-polar scanning maintains all the advantages of the plane-polar one [4, 5] while providing a compact, simple and cost-effective measurement system. In fact, only rotational motions are required and this is convenient since rotating tables are more accurate than linear positioners. Moreover, since the arm is fixed at one point and the probe is attached at its end, the bending is constant and this allows one to hold the planarity.

An efficient probe compensated NF–FF transformation technique with bi-polar scanning has been developed in [3] by taking advantage of the nonredundant sampling representations of electromagnetic (EM) fields [6], properly extended to the probe voltage (the voltage measured by a nondirective probe has the same effective bandwidth of the field [7]). An optimal sampling interpolation (OSI) algorithm has been applied to recover the plane-rectangular data from the bi-polar ones, thus enabling the FFT use in the NF–FF transformation. Such a technique allows one to lower the number of needed NF data in a significant way with respect to the approach in [1, 2], without losing the efficiency.

Fig. 1 - Geometry of the problem.
Unfortunately, due to an inaccurate control of the positioning systems, it may be practically impossible to get regularly spaced NF measurements. On the other hand, their position can be accurately read by optical devices. Moreover, the finite resolution of the positioning devices prevents the possibility to locate exactly the receiving probe at the points fixed by the sampling representation. According to these considerations, the development of an accurate and stable reconstruction process from the knowledge of nonuniformly distributed data becomes relevant. It must be stressed that, in this context, the formulas available in literature for the direct reconstruction from nonuniform samples are not user friendly, unstable and valid only for particular sampling points arrangements.

A convenient strategy is to recover the uniform samples from those irregularly spaced and then determine the value at any point of the scanning surface by an accurate and stable OSI formula. In this framework, the approach proposed in [8, 9] is based on an iterative technique which has been found convergent only if it is possible to build a biunique correspondence, which associates at each uniform sampling point the nearest nonuniform one. With reference to the field reconstruction on a plane, this restriction has been overcome in [10] by developing an approach based on the use of the singular value decomposition (SVD) method [11] for reconstructing the uniform plane-polar data. This latter approach is preferable to that based on the iterative technique, since it is more flexible and allows one to take advantage of data redundancy for increasing the algorithm stability [10].

The aim of this paper is just the extension of the NF-FF transformation technique with bi-polar scanning developed in [3] to the case of irregularly spaced NF data.

2. THEORETICAL BACKGROUND

A point on the scanning plane can be specified by the bi-polar coordinate system using the AUT angle \( \alpha \), the angle \( \delta \) and the arm length \( L \) (see Fig. 1). The polar coordinates \( \rho, \varphi \) are related to them by the following relations:

\[
\rho = 2L \sin(\delta/2) \quad ; \quad \varphi = \alpha - \delta/2 \quad .
\]

To cover the circular scanning region with a bi-polar scanner, the probe passes from one acquisition ring to another by travelling along the arc described by the end of the arm. During this movement, the AUT stays fixed. Once the probe is located on the ring to be considered, the AUT rotation allows one to perform the data acquisition at the sampling points. The choice of the distance from one ring to another and the angular sampling rate on them can be fixed according to a nonredundant sampling representation, which uses radial lines instead of radial arcs, thus remarkably reducing the number of rings and sampling points on them as shown in [3]. Moreover, if the AUT is quasi-planar, an effective source modelling [6] is obtained by choosing the surface \( \Sigma \) enclosing it coincident with the smallest oblate ellipsoid having major and minor semi-axes equal to \( a \) and \( b \) (see Fig. 1). Note that, because of the rotational movement of the scanner arm, the positions of the samples on the \( n \)-th ring are shifted by \( \varphi_0(\xi_n) = -\delta_n/2 \) with respect to the corresponding ones in the plane-polar grid.

According to [6], when considering an observation curve \( C \) described by an analytical parameterization \( r = r(\xi) \), the “reduced electric field”

\[
E(\xi) = E(\xi') e^{j\gamma(\xi')},
\]

//gamma(\xi') being a phase function to be determined, can be closely approximated by a spatially bandlimited function. For electrically large antennas, the bandlimitation error becomes negligible as the bandwidth exceeds a critical value \( W_2 \) and can be effectively controlled by choosing a bandwidth equal to \( \chi W_2 \), \( \chi > 1 \) being an excess bandwidth factor. When considering a radial line, by adopting \( W_\xi = \beta \ell'/2\pi \) (\( \beta \) being the wavenumber and \( \ell' \) the length of the ellipse \( C' \), intersection curve between \( \Sigma \) and the meridian plane), we get:

\[
\gamma = \beta \alpha \left[ v \sqrt{v^2 - 1} - E \cos^{-1} \left( \frac{1 - \varepsilon^2}{v^2 - \varepsilon^2} \right) \right],
\]

\[
\xi = \frac{\pi}{2} \left[ E(\sin^{-1} u \varepsilon)^{2}/E(\pi/2 \varepsilon)^{2} \right],
\]

where \( E(\cdot) \) denotes the elliptic integral of the second kind [12], \( \varepsilon = f/a \) is the eccentricity of \( C' \), \( f \) is its focal distance, \( u = (r_1 - r_2)/2f \), \( v = (r_1 + r_2)/2a \) are the elliptic coordinates, \( r_{1,2} \) being the distances from the observation point \( P \) to the foci of \( C' \). Moreover, \( \sin^{-1} u = \Theta_{\infty} \), \( \Theta_{\infty} \) being the polar angle of the asymptote to the hyperbola through \( P \).

When the observation curve is a ring, it is convenient to utilize the azimuthal angle \( \varphi \) as parameter and the corresponding bandwidth

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where \( n \) is the number of retained samples along the radial line fixed by \( \varphi \) and can be evaluated from the measured voltages \( V_{n} \) and \( V'_{n} \) via the OSI expansion:

\[
\tilde{V}(\xi, \varphi) = \sum_{n=-n_{o}+p+1}^{n_{o}+p} \tilde{V}(\xi_{n}, \varphi) \Omega_{N}(\xi_{n} - \xi) D_{N}(\xi - \xi_{n})
\]

(6)

where \( n_{o} = \text{Int}(\xi / \Delta \xi) \), \( 2p \) is the number of retained samples and \( \xi_{n} = n \Delta \xi = \frac{2n \pi}{2N_{n}+1} \); \( N'' = \text{Int}(\chi N') + 1 \)

(7)

\( \chi > 1 \) being an oversampling factor needed to control the truncation error. Moreover,

\[
D_{N}(\xi) = \frac{\sin((2N''+1)\xi/2)}{(2N''+1)\sin(\xi/2)}
\]

(9)

\[
\Omega_{N}(\xi) = \frac{T_{N}[2(\cos(\xi/2)/\cos(\xi_{0}/2))^{2} - 1]}{T_{N}[2/\cos^{2}(\xi_{0}/2) - 1]}
\]

(10)

are the Dirichlet and Tschebyscheff Sampling (TS) functions, respectively, \( T_{N}(\cdot) \) is the Tschebyscheff polynomial of degree \( N \), and \( \xi_{0} = \pi \Delta \xi \).

The intermediate samples \( V(\xi_{n}, \varphi) \) are given by:

\[
\tilde{V}(\xi_{n}, \varphi) = \sum_{m=-m_{o}+q+1}^{m_{o}+q} \tilde{V}(\xi_{n}, \varphi_{m,n}) \Omega_{M_{n}}(\varphi - \varphi_{m,n}) D_{M_{n}}(\varphi - \varphi_{m,n})
\]

(11)

where \( \tilde{V}(\xi_{n}, \varphi_{m,n}) \) are the uniformly spaced samples on the ring specified by \( \xi_{n} \), \( 2q \) is the number of retained samples along \( \varphi \), \( m_{o} = \text{Int}((\varphi - \varphi_{0}(\xi_{n}))/\Delta \varphi_{n}) \) and

\[
\varphi_{m,n} = \varphi_{0}(\xi_{n}) + m \Delta \varphi_{n} = \varphi_{0}(\xi_{n}) + \frac{2m \pi}{2M_{n}+1}
\]

(12)

\[
M_{n} = \text{Int}(\chi M_{n}) + 1; \quad M_{n} = \text{Int}(\chi W_{n}) + 1
\]

(13)

\[
M_{n} = M_{n}^{*} - M_{n}; \quad \chi' = 1 + (\chi - 1)[\sin(\Delta \varphi_{n})(\xi_{0})]^{2/3}
\]

(14)

The basic theory of the classical probe compensated NF measurements on a plane as proposed in [13] is based on the application of the Lorentz reciprocity theorem. According to such a theory, the AUT far-field components are related to: i) the two-dimensional Fourier transforms of the output voltages \( V_{V} \) and \( V_{H} \) of the probe for two independent sets of measurements (the probe is rotated by 90° in the second set); ii) the far-field components radiated by the probe and the rotated probe, when used as transmitting antennas. The key relations in the reference system used in the present work are explicitly reported in [3, 10]. However, these equations are valid whenever the probe maintains its orientation with respect to the AUT and this requires its co-rotation with the AUT. Obviously, the scanning equipment is remarkably simplified when this is avoided. Probes exhibiting only a first-order azimuthal dependence in their radiated far-field (i.e., an open-ended cylindrical waveguide excited by a TE_{11} mode) can be used without co-rotation, since \( V_{V} \) and \( V_{H} \) can be evaluated from the measured voltages \( V_{\varphi} \) and \( V_{\varphi}' \), through simple trigonometric relations [3]. According to the above considerations, an efficient probe compensated NF–FF transformation from a nonredundant number of bi-polar data is achieved by recovering the values of \( V_{V} \) and \( V_{H} \) in the plane-rectangular grid needed to perform the described NF–FF transformation.

### 3. NF DATA RECONSTRUCTION FROM NONUNIFORM SAMPLES

Let us now assume that, apart from the sample at the origin of the coordinate system on the scan plane, the irregularly distributed samples lie on \( K \geq N_{us} \) nonuniformly spaced rings (see Fig. 2), where \( N_{us} \) is the number of the rings uniformly spaced according to the nonredundant sampling representation considered in the previous section. This hypothesis can represent the spatial distribution of the NF measurements. In fact, since the scanning procedure fixes each ring by means of a rotational movement of the arm and collects the data on it by rotating the AUT, errors can occur on the ring location and on the position of the samples on it. As a consequence, the starting two-dimensional problem is reduced to find the solution of two independent one-dimensional problems. In this framework, let us assume to know the probe voltage at

\[
W_{\varphi}(\xi) = \beta a \sin \theta_{a}(\xi).
\]

(5)
Let us now tackle the problem of evaluating the probe voltage at a generic point \( P(\xi, \varphi) \) on the plane from the knowledge of the recovered uniform samples on the irregularly spaced rings. To this end, the OSI expansion (11) can be employed to determine the intermediate samples \( \tilde{V}(\xi_k, \varphi) \) (crosses in Fig. 2) on the radial line through \( P \). Since the intermediate samples are nonuniformly distributed on the considered radial line, the voltage at \( P \) can be found in analogous way by recovering the regularly spaced intermediate samples again via SVD and then interpolating them via the OSI expansion (6). The overdetermined linear system to be considered is now:

\[
\tilde{V}(\xi_k, \varphi) = \sum_{n=n_0 - p+1}^{n_0 + p} \tilde{V}(\xi_n, \varphi) \Omega_N(\xi_k - \xi_n) D_N(\xi_k - \xi_n)
\]

which can be expressed in matrix form as (15).

It must be stressed that, in order to minimize the computational effort for computing the plane-rectangular data needed to perform the probe compensated NF–FF transformation described in the previous paragraph, it is convenient to determine on each ring the same number \( N_0 \) of uniform samples with plane-polar distribution. This number is fixed according to the sampling rate on the outer ring. In such a way, although the so recovered NF data are redundant in \( \varphi \), the number of SVD on the radial lines is minimized since these samples are radially aligned. It is worthy to note that the overall number of SVD required to recover these samples is \( K + N_\varphi \). Once these latter have been determined, the plane-rectangular data can be evaluated by using the corresponding OSI expansion in [5].

## 4. NUMERICAL RESULTS

The validity of the developed technique has been assessed by many numerical tests. The following simulations refer to the field radiated by a uniform planar circular array having diameter equal to 33.6 \( \lambda \), \( \lambda \) being the wavelength. Its elements, symmetrically placed with respect to the plane \( y = 0 \), are elementary Huygens sources linearly polarized along the \( y \) axis and are radially and azimuthally spaced at 0.7 \( \lambda \). Accordingly, this antenna can be modelled as enclosed in an oblate ellipsoid having \( a = 17 \lambda \) and \( b = 2.2 \lambda \). An open-ended cylindrical waveguide with radius equal
to 0.338 λ is considered as measurement probe. The scanning plane is 20 λ away from the AUT center and the bi-polar measurement system is characterized by $L = 80 \lambda$ and $\delta_{\text{max}} = 53^\circ$, so that the NF data lie in a circular zone of radius $= 72 \lambda$. The nonuniform samples have been generated by imposing that the distances along $\xi$ and $\phi$ between the position of each nonuniform sample and the associate uniform one are random variables uniformly distributed in $[-\Delta \xi / 4, \Delta \xi / 4]$ and $[-\Delta \phi / 2, \Delta \phi / 2]$. It must be stressed that this is a very pessimistic occurrence in a real scanning system.

The process for recovering $V_V$ and $V_H$ from the nonuniformly distributed bi-polar samples of $V_x$ and $V_y$ has yielded fast and accurate results. Figure 3 shows a representative reconstruction example of $V_V$ on the radial line at $\phi = 90^\circ$. As can be seen, there is an excellent agreement between the exact and the reconstructed probe voltage save for the peripheral region, where an unavoidable truncation error occurs due to the lack of needed guard samples. The algorithm performances have been assessed in a more quantitative way by evaluating the maximum and mean-square errors occurring in the reconstruction of the uniform plane-polar samples of $V_V$. These errors (see Figs. 4 and 5) are normalized to the voltage maximum value on the plane and have been obtained by comparing the aforementioned reconstructed uniform samples and the exact ones. Note that this comparison has been made in the central zone of the scanning plane, so that the existence of the required guard samples is assured. Obviously, even better results are to be expected when the nonuniform samples are closer to the uniform ones.

The stability of the algorithm has been investigated by adding random errors to the exact data. Both a background noise (bounded to $\Delta a$ in amplitude and with arbitrary phase) and uncertainties on the data of $\pm \Delta a_r$ in amplitude and $\pm \Delta \phi$ in phase have been simulated. As shown in Fig. 6, the algorithm is stable. In any case, it is possible to take advantage of the data redundancy for improving the stability (see Fig. 7).

The developed algorithm has been employed to determine in a fast and accurate way the plane-rectangular data required for the probe compensated NF–FF transformation [13]. The E-plane pattern, reconstructed from the recovered plane-rectangular data lying in a 100λ × 100λ square grid, is shown (crosses) in Fig. 8. The pattern reconstructed (via the uncompensated NF–FF transformation) from the exact plane-rectangular field samples lying in the same grid is also reported as reference (solid line). As can be seen the FF reconstruction is very accurate, thus assessing the effectiveness of the proposed technique.

A further example of simulated NF–FF transformation from nonuniformly distributed bi-polar data is shown in Fig. 9. It refers to an AUT obtained from the previously considered array by changing the excitations of its elements in order to obtain a Tschebyscheff-like behaviour with sidelobe ratio (SLR) $= 40$ dB in the FF region. Also in this case the FF reconstruction is resulted to be very accurate.
5. CONCLUSIONS

The problem of an efficient AUT pattern reconstruction from the knowledge of irregularly spaced bi-polar data has been tackled and solved in this work. The developed method takes advantage of a nonredundant sampling representation of the probe voltage and of the use of the corresponding OSI expansion for interpolating the samples. This has allowed the building of linear systems whose best solution in least squares sense has been obtained by applying the SVD technique. The reconstruction process has yielded accurate and stable results even in presence of very large position errors.

REFERENCES


Fig. 8 - FF pattern in the E-plane. Solid line: reference. Crosses: reconstructed from probe compensated nonuniform NF measurements.

Fig. 9 - FF pattern in the E-plane of a Tschebyscheff-like planar circular array. Solid line: reference. Crosses: reconstructed from probe compensated nonuniform NF measurements.


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