ON THE IMPLEMENTATION OF THE EQUIVALENCE THEOREM IN THE HYBRID FDTD-MoMTD TECHNIQUE

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ABSTRACT. The hybrid FDTD-MoMTD technique is a powerful tool for analysing the transient excitation of inhomogeneous permeable bodies by arbitrary thin-wire antennas. This technique is based upon the calculation of the equivalent electric and magnetic currents on a Huygen’s surface that encloses the antenna. This paper studies, by means of several numerical experiments, the effects on the accuracy of the results of the size of the Huygen’s box, the size of the spatial increment in the FDTD algorithm and the distance from the Huygen’s surface to the observation point.

1 INTRODUCTION

A hybrid technique that efficiently combines two powerful numerical methods, the Finite Difference Time Domain (FDTD) and the Method of Moments in the Time Domain (MoMTD), was recently described in [1]. This technique, which is applicable to complex geometries comprising thin-wire structures and arbitrary inhomogeneous dielectric bodies, has been successfully used to study linearly and non-linearly loaded broadband antennas near arbitrary inhomogeneous bodies and to simulate a three dimensional Ground Penetrating Radar (GPR) [2]-[3].

The hybridization is based upon the surface equivalence theorem (Huygen’s principle) [4] and it is implemented, using a time-stepping procedure, as follows [1]:

1) An imaginary closed Huygen’s surface $S_H$ is located around the thin-wire antenna. At each time step, the equivalent sources on $S_H$ are derived from the fields radiated by the antenna in free space by applying the MoMTD.

2) Next, the FDTD algorithm is applied to compute the fields in the entire computational domain, removing the antenna inside $S_H$ and replacing it by the equivalent sources. Thus, according to the equivalence principle, we obtain the total field outside $S_H$ and just the field scattered by the inhomogeneous part of the configuration inside $S_H$.

3) The FDTD solution inside $S_H$ when the antenna is not present is the extra incident field on this antenna needed to compute, by the MoMTD, the currents induced on its surface and subsequently the radiated fields and the equivalent sources on $S_H$.

This paper studies the effect of the size of the FDTD cell used to model $S_H$ and the size of $S_H$ itself on the precision of the method. To this end, a Hertzian (infinitesimal) dipole, placed at the center of $S_H$, was chosen as a radiation source. This choice allowed us to use the known closed-form solution for the fields created by the Hertzian dipole to calculate their exact values at the points of interest and to make $S_H$ arbitrarily small around the radiation source. The difference between the values of the fields derived using their analytical expressions and the ones obtained using the hybrid method are calculated as a function of the size of the FDTD cell on $S_H$ and of the size of $S_H$ itself. The next section describes the numerical experiments that have been carried out and the results obtained.

2 NUMERICAL EXPERIMENTS AND RESULTS

The basic geometry used for the numerical experiments is shown in Figure 1, where the arrow represents a Hertzian dipole oriented along the $z$ axis, positioned at the center of the Huygen’s surface $S_H$ and radiating in free space. The external surface $S_B$ is where the FDTD algorithm is truncated by applying suitable absorbing boundary conditions. The FDTD cells, as well as $S_H$ and $S_B$, are assumed to be cubic and the volume $V_H$ inside $S_H$ is uniformly discretized using a cell size $\Delta$. If the Hertzian dipole radiates in free space, according to the equivalence theorem, there will be no scattered field inside $S_H$ and the total field should be zero. We chose three observation points: $P_1$ and $P_2$ outside $S_H$ and $P_3$ at the center of the Huygen’s volume. $P_1$ is located at a fixed position, $(0.48,-0.1,-0.36)$ m, while the coordinate of the observation point $P_2$ depends on the size of $S_H$ in such a way that it always remains a distance of two FDTD cells from $S_H$. The fields at all these points are calculated...
both numerically, using the hybrid method, and analytically from the expressions [5].

\[
\begin{align*}
E(r,t) &= \frac{1}{4\pi} \left[ \frac{1}{r} \int_0^t \left\{ \frac{1}{c^r \, \Delta t} \right\} (\hat{z} \times \hat{r}) \right] + \frac{1}{c^r} \int_0^t \left\{ \frac{1}{r^2} \right\} (\hat{z} \times \hat{r}) \\
H(r,t) &= \frac{1}{4\pi} \left[ \frac{1}{c^r} \int_0^t \left\{ \frac{1}{r^2} \right\} (\hat{z} \times \hat{r}) \right]
\end{align*}
\] (1)

where \( i = i(t) \) is the transient current excitation, \( r \) is the radial distance from the center of the Hertzian dipole to the observation point, \( \hat{r} \) and \( \hat{z} \) are unit vectors, and the rectangular brackets denote that the variables contained within them are to be evaluated at the retarded time \( t' = t - r / c \). The expressions in (1) are also used to calculate the equivalent electric and magnetic currents from the tangential component of the fields on the surface \( S_H \).

![Figure 1](image1.png)

**Figure 1.** a) Geometry for the Hertzian dipole radiating in free space; b) Projection on the plane x=0.

Henceforth the transient current excitation will be particularized for a differentiated Gaussian pulse current defined as

\[
i(t) = -2g^2 \left( t - t_{\text{max}} \right) \exp \left[ -g^2 (t - t_{\text{max}})^2 \right]
\]

where \( g = 10^9 \text{ s}^{-1} \) and \( t_{\text{max}} = 4g \).

At the points \( P_1 \) and \( P_2 \) (outside \( S_H \)) an error factor, \( Q_1 \), is defined as the sum of the normalized root-mean-square error of each component of the electric field, that is:

\[
Q_1 = \sum_{a=x, y, z} Q_{1,a} = \sqrt{\sum_{a=x, y, z} \left( \frac{\int_0^t [E_a(t) - E_a^N(t)]^2 \, dt}{\int_0^t [E_a^N(t)]^2 \, dt} \right)^2}
\]

where \( N \) is the total number of time intervals and \( E_a(t) \) and \( E_a^N(t) \) represent the values, at time step \( n \), of the \( a \) electric field component calculated using expression (1) and the hybrid method respectively.

At point \( P_3 \) (inside \( S_H \)), as previously stated, the total field should be null unless a scattered field exists. To calculate an error factor similar to \( Q_1 \) inside \( S_H \), a known scattered electromagnetic field is required there. To this end, a perfect electric conducting (PEC) plate is introduced into the original geometry at a fixed distance, \( d_c = 0.2 \text{ m} \), from the center of the Huygens’s zone as shown in Figures 2a-c. The expression of the field inside \( S_H \), scattered by the PEC, can be also obtained from (1) by replacing the PEC surface by the image of the original radiation source (see Figure 3). Then, an error factor, \( Q_2 \), inside \( S_H \) is defined as

\[
Q_2 = \sum_{a=x, y, z} Q_{2,a} = \frac{\sum_{a=x, y, z} \left[ |E_a(t) - E_a^N(t)|^2 \right]}{\sum_{n=0}^{N} |E_a^N(t_n)|^2}
\]

where \( E_a(t) \) and \( E_a^N(t) \) represent the values of the electric field components at \( P_1 \) calculated using the image of the radiation source and the hybrid method respectively. To calculate \( Q_2 \), the effect of the undesired return from \( S_H \) due to the staggered position of the equivalent electric and magnetic currents [5] has been eliminated. This was carried out by time gating when there is no overlapping between the undesired and the desired signals or, if overlapping exists, calculating the field inside \( S_H \) with the Hertzian dipole radiating in the free space and subtracting these results from those obtained including the PEC.

![Figure 2](image2.png)

**Figure 2.** a) Hertzian dipole in front a PEC interface; b) Projection on the x=0 plane; c) Equivalent problem including the image source.

To study the effects of the size of the FDTD cell and \( S_H \), the error factors \( Q_1 \) and \( Q_2 \) have been evaluated for three different cases:

i) Keeping the value of \( \Delta \) constant and changing the size of \( S_H \) (Figure 3).

ii) Changing the value of \( \Delta \) and maintaining the number of cells inside \( S_H \) constant which, in consequence, does not remain constant (Figure 4).
iii) Changing the value of $\Delta$ while the size of $S_H$ is unchanged (Figure 5).

Figure 3:- Error factors at the observation points as a function of $l_H$ and the number of cells inside $S_H$ for a fixed value of $\Delta = 5$ mm.

Figure 4:- Error factors at the observation points as a function of $l_H$ and the cell size $\Delta$ keeping $p=8$.

Figure 5:- Error factors at the observation points as a function of the cell size $\Delta$ and the number of cells inside $S_H$ for a fixed size of $S_H$.

The parameter $p$ in Figures 3 and 5 is an integer number, such that, the half length $l_H$ of one side of $S_H$, is given by $l_H=p\Delta$ and the surface of $S_H$ equals $24(p \Delta)^2$ (see Figure 2). Moreover, in all cases, the principal spectral components have been resolved with at least 15 cells per wavelength.

Figure 3 shows how the error factors $Q_1$ and $Q_2$ increase as $l_H$ decreases while the value of $\Delta = 5$ mm is kept constant. This can be explained from the dependence upon $r$ as $1/r^2$ and $1/r^4$ of the near-field contributions to the total field in (1). When $l_H$ decreases the influence of the near fields increases and a smaller value of $\Delta$ would be necessary to get a greater number of points on $l_H$ in order to take into account the strong dependence of the near fields upon $r$. On the contrary, when $l_H$ increases, the smoother variation of the amplitude of the fields allows us to use a greater value of $\Delta$. Another way of illustrating this effect is given in Figure 4, which shows that $Q_1$ and $Q_2$ increase when $l_H$ decreases even though $\Delta$ also decreases. The method used to decrease $l_H$ was to keep the parameter $p$ constant ($p=8$) in $l_H=p\Delta$ while $\Delta$ decreases.

For a fixed value of $l_H=40$ mm, the dependence upon $\Delta$ of the error factors $Q_1$ and $Q_2$ is shown in Figure 5. As expected, in the three cases the error factors decrease as $\Delta$ decreases because the number of field samples on $l_H$ increases.

Figures 6a-b show the temporal evolution of the three electric field components calculated numerically and analytically in the cases where greater values of the error factors, represented in Figures 3 and 4, were found. These correspond to point $P_2$ for $l_H=20$ in Figure 4 and point $P_1$ for $l_H=40$ in Figure 5. It can be observed that even in these cases the agreement between the numerical and analytical results is very good. All these results show that the method works properly even when the Huygen’s surface is small and the observation point is near the surface. With regard to this, the smallest $S_H$ that we were able to implement (in a 600 MHz PC with 768 Mbytes of RAM) had $l_H=10$ mm and was modeled with $\Delta =1$ mm ($l_H=10 \Delta$) with $P_2$, being at 2 mm from $S_H$ or equivalently at 1.2 cm from the radiation source. In this case, not included in the previous plots, the value obtained for $Q_1$, at point $P_2$, was $Q_1=0.0673$. Figure 7 shows the three electric field components calculated numerically and analytically at this point. Again, it is observed that there is very good agreement. It should be pointed out that similar conclusions to the ones included in this paper were obtained for narrower gaussian excitations.
Figure 6.- Temporally evolution of the electric field calculated numerically and analytically. a) Point $P_2$ for $l_H$ = 20; b) Point $P_1$ for $l_H$ = 40.

Figure 7.- Temporally evolution of the electric field calculated numerically and analytically to point $P_2$ for $l_H$ = 10 mm.

3 CONCLUSIONS

This paper studies the effects of the following parameters on the accuracy of the results obtained by the hybrid FDTD-MoM technique: the size of the Huygen's box around the antenna, the size of the spatial increment in the FDTD algorithm and the distance from the Huygen's surface to the observation point. By means of several numerical experiments the method is shown to be robust and gives very good results even when the Huygens's surface is very near the radiation source.

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4 REFERENCES


