Polarimetric Scattering from a 3-D Rectangular Crack in a PEC Covered by a Dielectric Layer

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Abstract — A novel direct approach for calculation of the polarimetric scattering fields from a narrow 3-D rectangular crack in an infinite ground plane underneath a dielectric layer is presented. Since the electromagnetic fields are directly calculated and thus the approach in invertible, this technique is suitable for microwave NDT applications where cracks of narrow width, arbitrary length and depth under a dielectric layer are frequently encountered. A set of coupled field integral equations (FIE) with logarithmic and hypersingular kernels are derived and then discretized by a collocation method based on Chebyshev polynomials. The results of this direct approach are in good agreement with non-invertible full numerical FEM and MoM results.

Index Terms — 3-D rectangular crack, Chebyshev polynomials, dielectric layer, integral equation, and polarimetric scattering.

I. INTRODUCTION
To detect surface cracks in metals, several electromagnetic techniques are suggested [1-5]. Recently, far field polarimetric scattering measurements are proposed where common NDT techniques may not be practical such as in blast furnaces [6]. Oil, paint, electrical, and thermal coatings on cracks alters the scattering signature. For practical purposes, a solution that takes the dielectric coating effect into consideration is in demand.

Electromagnetic scattering from a dielectric coated slot in conductors is an ongoing research using various computational techniques. Initially, Knops and Cohn studied the effects of a dielectric layer on top of an aperture [7]. Later, Chen solved the integral equation for a waveguide ended with a dielectric slab and provided some physical and mathematical explanations [8]. Nevels and Butler used electric vector potential and Sommerfeld integrals to model the diffraction from a slot covered by a dielectric layer [9-10]. Electromagnetic problems are generally formulated by means of the electric and the magnetic potential integral equations (PIE) [11-13]. Weak singularity of the Green's function allows utilization of a variety of numerical methods [14]. On the contrary, the electric field integral equations (EFIE) and the magnetic field integral equations (MFIE) have strong second-order singularity. Hadamard introduced the hypersingular integrals for solving Cauchy's hyperbolic partial differential problems as the finite part of a divergent integral [15]. An exact solution of the Hadamard integrals exists only in particular cases, where the hypersingular integrals are solved by approximate methods. One approach is transformation of the hypersingular integrals into singular or weakly singular integrals by various regularization techniques [16-18]. Another method that avoids singular point restrictions is the direct numerical computation of the finite part integrals by a variety of quadrature techniques.

Recently, in applied mathematics, some innovative methods are suggested to solve high order singular integrals effectively [19-20]. Thus, a strong singular EFIE or MFIE can be solved directly without using potential vectors and consequently, some tedious numerical computations such as the curl operator are
eliminated. Here, an efficient approach based on finite part sense integrals is developed for calculation of electromagnetic scattering from a dielectric coated three-dimensional crack in a grounded slab.

In Section II, a set of coupled integral equations are formulated via the continuity of the tangential magnetic fields. In Section III, the calculation of the green’s functions and in Section IV, the extraction of the singular terms leaving an integrable equation is presented. In Section V, proper basis functions by considering the edge boundary conditions are presented. Then, the resultant linear matrix is solved. The scattered field due to the equivalent magnetic current under a dielectric layer is then obtained in Section VI. In Section VII, the validity and efficiency of the proposed method for calculating the scattering fields of 3-D cracks under a dielectric layer is demonstrated by comparison with MoM and FEM results.

II. THE SCATTERING PROBLEM

Assume a dielectric filled rectangular crack of \(a \times b \times c\) in an infinite ground plane is coated with a dielectric slab of height \(d\) and arbitrary permittivity and permeability of \(\varepsilon, \mu\). This crack is illuminated by an arbitrary polarized plane wave. Using the surface equivalence principle, we can write the scattered fields in terms of the equivalent magnetic current distribution, \(\hat{M}\) on the crack as:

\[\vec{E} = \left\langle \mu \vec{G} \cdot \hat{M} \right\rangle, \quad \vec{H} = \left\langle \mu \vec{G}^h \cdot \hat{M} \right\rangle, \quad (1)\]

where \(\mu \vec{G}\) and \(\mu \vec{G}^h\) are magnetic dyadic Green’s functions (DGFs). The \(\mu \vec{G}\) is the green’s function for the magnetic field \(\vec{H}\) generated by the magnetic current \(\hat{M}\). In addition, the notation \(\langle \cdot \rangle\) means the integration of products of the two functions. The integral equation is constructed by enforcing the continuity of the total tangential magnetic field on the aperture of the crack under slab that separates region 2 from region 3. Thus, we have:

\[\vec{H}_{\text{tan}}^\text{new} (2\hat{M}, \hat{J} = 0) + \vec{H}_{\text{tan}}^\text{ext} = \vec{H}_{\text{tan}}^\text{region2} (-\hat{M}, \hat{J} = 0), \quad (2)\]

where \(\vec{H}\) is the total tangential magnetic field in the absence of the crack. Equation (2) is broken into a pair of coupled Fredholm’s integral equations of the first kind:

\[-H' = \int \left( \frac{C_{\mu} G_{\mu} + D_{\mu} G_{\mu}}{C_{\mu} G_{\mu} + D_{\mu} G_{\mu}} \right) M ds + \oint \left( \frac{C_{\mu} G_{\mu} + D_{\mu} G_{\mu}}{C_{\mu} G_{\mu} + D_{\mu} G_{\mu}} \right) M ds, \]

where \(\frac{C_{\mu} G_{\mu} + D_{\mu} G_{\mu}}{C_{\mu} G_{\mu} + D_{\mu} G_{\mu}}\) are the magnetic green’s functions of the crack and the grounded dielectric slab when the source and the observation points are both on the ground plane (\(z=0\)). Here, a direct approach is used to convert the electric or magnetic integral equations into a simple system of linear equation based on finite part sense integrals [19]. Initially, the behavior of the Green’s functions is studied for extraction of the singular terms.

III. DERIVATION OF DGFs

The most common method of deriving the DGFs is by means of Fourier transform and differential methods. Closed-form formulations of the DGFs for multilayered media using complex image method are reported in [21-29]. The well-known TL model is also used in addition to complex image method to find the spectral components of the stratified medium [22]. In Fig. 2, a magnetic current source is assumed on the infinite ground plane underneath the multilayered media. Figure 3 depicts the circuit equivalent transmission line model where the dielectric impedance is terminated by the free space wave impedance. The space domain Green’s functions

\[\text{Fig. 1. Geometry of a narrow 3D-dimensional rectangular crack in an infinite ground plane underneath a dielectric layer.}\]
are obtained by applying the Sommerfeld transformation to the TL model as [26]:

\[
G_n(k_r, z, z' = 0) = \frac{\cos 2 \phi}{\rho} \left\{ \frac{e^{-k_x x'}}{k_x} - \sin \phi \left\{ \frac{e^{-k_x x'}}{k_x} \right\} \right. - \cos \phi \left\{ \frac{e^{-k_x x'}}{k_x} \right\},
\]

\[
G_0(k_r, z, z' = 0) = \frac{\sin 2 \phi}{\rho} \left\{ \frac{e^{-k_x x'}}{k_x} - \frac{1}{2} \sin 2 \phi \left\{ \frac{e^{-k_x x'}}{k_x} \right\} \right. - \cos \phi \left\{ \frac{e^{-k_x x'}}{k_x} \right\},
\]

\[
G_x(k_r, z, z' = 0) = G_n(k_r, z, z' = 0),
\]

\[
G_y(k_r, z, z' = 0) = \frac{\cos 2 \phi}{\rho} \left\{ \frac{e^{-k_x x'}}{k_x} - \sin \phi \left\{ \frac{e^{-k_x x'}}{k_x} \right\} \right. - \cos \phi \left\{ \frac{e^{-k_x x'}}{k_x} \right\},
\]

where \( \gamma = \sqrt{k_x^2 - \alpha^2} \) and \( k_r = \sqrt{k_x^2 + k_y^2} \) are cylindrical propagation constant and wave number, respectively. In the above equations, \( S_n \) are the Sommerfeld integrals that are solved by the proper selection of branch cuts and integration paths [30].

Fig. 2. The equivalence principle and image theory for a crack in an infinite ground plane.

Here, the spectral-domain Green’s function is approximated by a pole-residue term plus an asymptotic function [26–28]. Thus, \( \tilde{G}(k_r) \approx \sum_n a_n e^{-b_n k_r} + \sum_m 2 k_m^p R_m(k_m^p) \), \( R_m(k_m^p) \) represents the residues of \( \tilde{G} \) at real poles \( k_m^p \). The Prony coefficients \( a_n \) and \( b_n \) are found by using GPOF and choosing the path \( C \) by avoiding the poles \( k_i/k_i = \left[ 1 + j0, 10^{-1} \right], 0 \leq i \leq 1.2 k_{max}/k_i \) shown in Fig. 4 [26]. In our case, the cracks are not small but also close to the metal surface. Therefore, the lateral wave effects are small. Thus, on the conductor \( z=0 \), we could use the approximation of DGFs. Utilizing Hankel transform,

\[
\int e^{-i \phi} J_n(k_r) \phi^n dk_r = \frac{n(\alpha^2 + \rho^2)^{n/2} + (-i)\alpha n}{\rho(\alpha^2 + \rho^2)^{n/2 + 1/2}}, \quad n = 0, 1.
\]

Thus, \( S_n \) in (4)-(7) are approximated as:

\[
S_n = \frac{1}{2} \sum_m H_1(k_m^p)R_m + \frac{1}{2\pi} \sum_n \sqrt{\frac{\rho^2 + b_n^2}{\rho^2 + b_n^2}}, \quad n = 0, 1.
\]

\[
S_0 = \frac{1}{2} \sum_m k_m^p R_m H_0(k_m^p) + \frac{1}{2\pi} \sum_n \sqrt{\frac{\rho^2 + b_n^2}{\rho^2 + b_n^2}}, \quad n = 0, 1.
\]

Since the observation points are close to the source on the ground plane, the surface waves are dominant and the residues must be calculated precisely. The method of [31] is used for extraction of poles and calculation of residues.

For the computation of the Green’s functions of the cavity, various approaches such as Ewald’s method are available [32–34]. Here, we used the approach of [34] for deriving the dyadic Green’s functions of the cavity because the singular behavior is represented as a sum of infinite harmonics. Since in many applications including NDT, the crack width is much smaller than the wavelength and the crack length, the DGFs are approximated by the lowest order mode of the crack [35]. Thus, the space domain Green’s functions on the crack are [36]:

Fig. 3. Transmission line model of a magnetic source on a grounded slab.

\[
\tilde{G}_{i}^x = D(k_i^2 - k_i^2) \sin k_i \sin (y-a) \cos k_i \cos (y-b)\cos k_i \cos (y-b),
\]

\[
\tilde{G}_{i}^y = D(k_i^2 - k_i^2) \sin k_i \sin (y-a) \cos k_i \cos (y-b)\cos k_i \cos (y-b),
\]

\[
\tilde{G}_{i}^z = D(k_i^2 - k_i^2) \sin k_i \sin (y-a) \cos k_i \cos (y-b)\cos k_i \cos (y-b),
\]

\[
\tilde{G}_{i}^z = D(k_i^2 - k_i^2) \sin k_i \sin (y-a) \cos k_i \cos (y-b)\cos k_i \cos (y-b),
\]

where \( k_i = \frac{\pi}{2a}, k_i = \frac{\pi}{2b}, k_i = \sqrt{k_i^2 - k_i^2} \), \( D = 2/(j\omega \mu \rho) \) and \( n_p \) is the Neumann’s symbol(1 for \( p=0 \) and 2 for \( p > 0 \)).
IV. SINGULARITY EXTRACTION

The singularities in equations (4)-(7) are encountered when the observation point is on the source, i.e., \(x = x'\) or \(y = y'\). By expanding of the Hankel's functions around \(y = y'\), display second-order hypersingularity and logarithmic singularity. Next, (4)-(7) are rearranged as:

\[
\frac{\partial^2 G_x}{\partial x^2} = \frac{f_x^3(x,x)}{|x-x'|^3} + \frac{f_x^4(x,x')\log|x-x'| + f_x^5(x,x')}{|x-x'|},
\]

\[
\frac{\partial^2 G_y}{\partial y^2} = \frac{f_y^4(x,x')\log|x-x'| + f_y^5(x,x')}{|x-x'|},
\]

where \(f_1, f_2, \ldots, f_5\) are smooth functions obtained by the method of [6]. Note that the above procedure is repeated for all values of \(y\) and \(y'\) where \(x = x'\).

The complete harmonic series of (12)-(15) either converge very slowly or diverge [6, 37]. Additionally, any truncation of the series creates a large error due to the miscalculation of the remainder of the series at \(x = x'\) [6]. Therefore, the efficient approach of [37] is used to extract the singular terms of the harmonic series of (12)-(15). Considering a high enough number of modes (i.e., \(p > p_0\)), the series coefficients are approximated as:

\[
\frac{1}{k_r \tan k_r c} \approx \frac{1}{p} + a(p^3), \quad \frac{k_r}{k_r \tan k_r c} \approx \frac{1}{p} + a(p^3), \quad k_r^2 - k_r^2 \approx p + \frac{1}{p} + a(p^3).
\]

Finally, by substituting (19) in (12)-(15) and using the analytic simplifications of [6], the crack dyadic Green’s functions are derived as:

\[
\begin{align*}
\frac{\partial G_x^a + f_x^3(x,x)}{|x-x'|^3} + \frac{f_x^4(x,x')\log|x-x'| + f_x^5(x,x')}{|x-x'|},
\end{align*}
\]

\[
\begin{align*}
\frac{\partial G_y^a + f_y^4(x,x')\log|x-x'| + f_y^5(x,x')}{|x-x'|},
\end{align*}
\]

where \(g_1, g_2, \ldots, g_5\) are derivable nonsingular functions [6].

V. SOLUTION OF THE COUPLED INTEGRAL EQUATIONS

Direct integral equation solvers (DIES) straightforwardly solve the Integral equations with logarithmic or hypersingular kernels [38]. This method directly computes the finite part integral by numerical quadrature techniques that avoid the boundary singularities [19-20]. The magnetic currents at the edges of the crack are

\[
M_\gamma(-a,y') = M_j(a,y') = M_j(x',-b) = M_j(x',b) = 0.
\]

Please note that \(M_j(x',b), M_j(x',-b), M_j(-a,y')\) and \(M_j(a,y')\) are unknown magnetic currents on the crack and may tend to infinity at edges. By setting \(x,s = x, x'/a\) and \(t,s = y, y'/b\), the integral equation interval is transformed to \((-1,1)\). Next, the magnetic currents are approximated by finite series of products of two independent basic functions that satisfy the boundary conditions of (23). The first basis is a ‘pulse’ function and the other is a weighted Chebyshev polynomial of the second kind. Thus,

\[
M_j(s,t) = \sqrt{1 - s^2} \sum_{m=1}^{M} A_{m} U_{w,m}(s) P_{m}(t - t_0),
\]

and

\[
M_j(s,t) = \sqrt{1 - s^2} \sum_{m=1}^{N} B_{m} U_{w,m}(t) P_{m}(s - s_0).
\]

where \(A_{m}\) and \(B_{m}\) are unknown coefficients that must be calculated. \(P_{m}\) is the pulse basis functions of width \(s_0, M\), where \(s_0 = s_a - s_{a-1}, (M_a < 1)\), \(s_0 = s_{a-1} - s_{a-2}, (M_a < 1)\), Additionally, \(U_{w,m}\) is the m-th degree Chebyshev polynomial of the second kind. Using the zeros of \(U_{w,m}\), \(t_n\) and \(s_m\), the surface of the crack is discretized to \(M \times N\) non-equal elements. Hence,

\[
s_a = \frac{m\pi}{M+1}, m = 1, \ldots, M \quad \text{and} \quad t_0 = \frac{m\pi}{N+1}, n = 1, \ldots, N.
\]

By substituting (24) and (25) in (3) and collocating at each \(t_n\) and \(s_m\) on the crack we have:

\[
\frac{H(s,t)}{db} = \sum_{m=1}^{M} \sum_{n=1}^{N} \langle K_n(s,t)|t|O_{m}(s,t)|s\rangle \sum_{m=1}^{M} \sum_{n=1}^{N} \langle K_n(s,t)|t|O_{m}(s,t)|s\rangle
\]

\[
\frac{H(s,t)}{db} = \sum_{m=1}^{M} \sum_{n=1}^{N} \langle K_n(s,t)|t|O_{m}(s,t)|s\rangle \sum_{m=1}^{M} \sum_{n=1}^{N} \langle K_n(s,t)|t|O_{m}(s,t)|s\rangle
\]

where \(O_{m,s'}(s,t') = \sqrt{1 - s^2} U_{w,m}(s) P_{m}(t - t_0)\) and \(O_{m,s'}(s,t') = \sqrt{1 - s^2} U_{w,m}(t) P_{m}(s - s_0)\). Subsequently, the coupled integral equation of (27) is represented in a linear system as:

\[
[H] = [K_n|O_{m}] [K_n|O_{m}]^T [A],
\]

where \(A\) and \(B\) are \(1 \times MN\) unknown matrices that include unknown coefficients of (28) and are represented as \(A = [A_{11}, A_{12}, \ldots, A_{1N}, A_{21}, \ldots, A_{MN}]\) and
An arbitrary incident wave of Fig 1 can be decomposed into a parallel (E) and a perpendicular (H) Polarizations as:

\[
\begin{align*}
\hat{H} &= \hat{\phi} e^{jk(x\cos\phi+y\sin\phi+z)} H_{RL}, \\
\hat{E} &= \hat{\phi} e^{jk(x\cos\phi+y\sin\phi+z)} E_{RL},
\end{align*}
\]

where \(k = 2\pi/\lambda_o\) is the free space wave number and \(\phi_i\) and \(\theta_i\) are the incidence angles. The tangential magnetic field \((\hat{H}_x + \hat{H}_y)\) in the absence of the crack could be calculated by Fresnel’s laws [39].

VI. FAR FIELD SCATTERING

Upon solving (3), the equivalent magnetic current on the crack is calculated and then, the far field due to this embedded source in the grounded slab is obtained [13, 40-41]. Following the inverse Hankel transform. Thus:

\[
\begin{align*}
\hat{r}^t[H,E] &= \int G_t^{EH}(\phi,\theta) P(\phi,\theta) + \int G_t^{EH}(\phi,\theta) P(\phi,\theta),
\end{align*}
\]

where

\[
\begin{align*}
P_x(\phi,\theta) &= \pi ab \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} \Delta t e^{ik_0 z}, \\
P_y(\phi,\theta) &= \pi ab \sum_{m=1}^{M} \sum_{n=1}^{N} B_{mn} \Delta \alpha e^{k_0 \alpha Y}.
\end{align*}
\]

In derivation of \(P_x\) and \(P_y\) the following mathematical relation is used.

\[
\int_{-1}^{1} \sqrt{1-x^2} U_{m+1}(x) e^{\alpha x} dx = \pi(m+1) J_m(\alpha).
\]

The \(j^{th}\) order Bessel function of the first kind is denoted by \(J_{m}\) and where \(X = -k_0 a \cos \phi \sin \theta \) and \(Y = -k_0 b \sin \phi \sin \theta \).

VII. RESULTS

Here, few numerical examples that demonstrate the validity of this approach are presented. Assuming \(a = 0.1L, b = 0.8L, c = 0.25L, h = 0.1L, \varepsilon_j = 3.2-0.1j\) and \(\varepsilon_3 = 1\) in the configuration of Fig. 1, the calculated magnetic currents distribution \(|M_x|\) and \(|M_y|\) at the center of the crack \((x = 0 \text{ and } y = 0)\) for \(\phi = 0^\circ\) and \(\theta = 45^\circ\) are depicted in Fig. 5 and Fig. 6 for parallel (E) and perpendicular (H) polarizations, respectively.

Then, the bistatic polarimetric radar cross sections \(\sigma_{hi}, \sigma_{fv}, \sigma_{vV}\) at a constant observation elevation angle \(\theta = 45^\circ\) are compared with the fully numerical approaches of FEM and MoM for a crack with the dielectric cover (WD) and without the dielectric cover (WoD) as shown in Fig. 7.

The dielectric cover causes \(\sigma_{hi}\) to rise slightly; however, the other bistatic radar cross sections decrease by 8 dB. Figure 8 represents the same results for a constant observation azimuth angle \(\phi = 0^\circ\). \(\sigma_{vV}\) is almost constant while the bistatic cross polarizations increase and \(\sigma_{fi}\) decreases compared to uncovered crack. As shown, the results are in a good agreement with full numerical approaches.
Fig. 6. Comparison between magnetic current of DIES and MoM for the crack of Fig. 1 with: 
\[ a = 0.1 \lambda, \ b = 0.8 \lambda, \ c = 0.25 \lambda, \ h = 0.1 \lambda, \ \varepsilon_2 = 3.2 - 0.1 j \] and \[ \varepsilon_1 = 1 \] for vertical polarizations (E-Polarization) at \( \phi = 0^\circ \) and \( \theta = 45^\circ \).

Fig. 7. Bistatic radar cross sections (RCS\(_{bi}\)) of the crack in Fig. 1 at various observation angles \( \phi \) with 
\[ a = 0.1 \lambda, \ b = 0.8 \lambda, \ c = 0.25 \lambda, \ h = 0.1 \lambda, \ \varepsilon_2 = 3.2 - 0.1 j \] and \( \varepsilon_1 = 1 \) at \( \phi = 0^\circ \).

A rapid convergence of these integrals is very important in minimizing CPU time. For integrals that include Bessel and harmonic functions, an extended Levin’s collocation method of [43] is used that approximates the oscillatory integrals. On a 2 GHz Pentium4 PC of 1G RAM, the computation time of FEM (HFSS), MoM (FEKO) and our method (DIES) for the dielectric covered crack of Fig. 6 are 38.136, 32.751, and 18.225 minutes, respectively. Please note that the calculation of oscillatory integrals in (3) is the most time consuming computation of DIES.

Examination of the results shows that the crack dielectric cover alters the RCS signature significantly even for thin layers. Surface waves are also a contributing factor in RCS reduction where the dielectric layer acts as a waveguide that traps the wave and thus reduces the scattered energy.

Fig. 8. RCS\(_{bi}\) of the crack in Fig. 1 at various observation angles \( \phi \) with 
\[ a = 0.1 \lambda, \ b = 0.8 \lambda, \ c = 0.25 \lambda, \ h = 0.1 \lambda, \ \varepsilon_2 = 3.2 - 0.1 j \] and \( \varepsilon_1 = 1 \) at \( \phi = 0^\circ \) and \( \theta = 45^\circ \).

Table 1: Some material with their dielectric constants

<table>
<thead>
<tr>
<th>material</th>
<th>permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.2</td>
</tr>
<tr>
<td>Epoxy</td>
<td>3.5</td>
</tr>
<tr>
<td>Glass,Mica</td>
<td>6</td>
</tr>
<tr>
<td>GaAs</td>
<td>13</td>
</tr>
</tbody>
</table>

Figure 9 depicts the variations of RCS\(_{bi}\) for various dielectric constants of Table 1 for 
\[ a = 0.2 \lambda, \ b = 1 \lambda, \ c = 0.25 \lambda, \ h = 0.1 \lambda \] and \( \varepsilon_1 = 1 \) at \( \phi = 30^\circ \) and \( \theta = 30^\circ \). By increasing the dielectric constant, RCS\(_{bi}\) drops down at first and then slowly increases due to an increase in the electrical thickness of the substrate that excites additional surface wave modes. Thus, an increase in the excitation energy in the crack causes RCS\(_{bi}\) to rise. On the contrary, as the dielectric constant increases to a large value such as \( \varepsilon_2 = 13 \), the high reflectivity at the air-dielectric interface reduces the RCS\(_{bi}\). As the dielectric constant increases, the minimum RCS\(_{bi}\) as a function of elevation angle shifts to the left. In addition for \( \varepsilon_2 \leq 6 \), the RCS\(_{bi}\) is smoother and the scattering beam width widens for high permittivity. Referring to Fig. 9, the
energy density in the forward scattering region (180° < φ < 360°) is higher than other directions due to specula reflection.

As dielectric thickness increases, the RCSbi in the forward scattering region increases while in the exciting source region (0° < φ < 180°) decreases. The RCSbi dips for co-polarization at φ = 180° and φ = 360° and for cross-polarization at φ = 90° and φ = 270° are thickness independent. Similar to the influence of the dielectric constant, the dielectric thickness effect is not monotonic as well, noting that the surface waves are more prevalent in this case.

VIII. CONCLUSION
Most approaches in the literature use the electric and the magnetic potential integral equations to solve electromagnetic problems. Here, a direct field method is developed to solve the magnetic integral equations of a three dimensional rectangular crack in a grounded slab covered by a dielectric layer. This invertible solution is in demand in inverse scattering and NDT applications. The approach efficiently solves the integral equation by extraction of the hyper-singular terms and then discretizing the integral equation. The two-dimensional integrals include strong singularities that are approximated by ad hoc quadrature rules leading to a linear system of equations. In addition, the calculation of the oscillatory integrals is expedited by Levin’s method that is developed in applied mathematics. A good agreement is observed with MoM and FEM solutions that are full numerical and non-invertible. In addition, the sensitivity of the RCSbi to the permittivity and thickness of overlaying layer is investigated. In general, the dielectric layer alters the polarimetric scattering signatures of a crack in a non-monotonic manner.

Appendix: SOME QUADRATURE RULES
In equations (22-24) and (26-28), hypersingular, logarithmic and ordinary integral are present. An expression for the hypersingular integrals is in the form of [44]:

$$\int_{-1}^{1} \frac{(1-\tau)^{2}}{\sqrt{|x-s|}} f(t) dt = \pi \sum_{j=0}^{M} w_j(s) f(s_j),$$  \hspace{1cm} (A.1)

where $f(t)$ is a given regular function and $M$ is a integer. Here,

$$s_j = \cos j\pi/(M+2), (j=1,2,...,M+1),$$  \hspace{1cm} (A.2)
\[ w_j(s) = -\frac{2}{M+2} \sum_{n=1}^{\infty} (\alpha+1) (n+1) \sin \left( \frac{j\pi}{M+2} \right) \sin \left( \frac{(n+1)\pi}{M+2} \right) u_{s,n}(s) \]  
(A.3) \[
\int_{-1}^{1} u_{s,n}(t) (1-t^2)^{1/2} / |s-t| \, dt = -\pi T_m(s) \quad m \geq 0 \]  
(A.4) \[
\text{and } T_m \text{ is the } m^{th} \text{ degree Chebyshev polynomial of the first kind.}
\]  

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