Abstract — In this paper, theory of characteristic modes (TCMs) is used to perform modal analysis of a trapezium-shaped PEC plate. So it follows that a wide-spread overview about the electromagnetics behavior of the plate is provided. The first seven characteristic currents and corresponding resonance frequencies of the structure are obtained. All modes are orthogonal to each other. By applying the reactive loading, the first two modes are forced to be resonating close to each other at frequency of about 3.5 GHz, which is suitable for WiMax applications. To excite aforementioned modes on the structure, an appropriate configuration of microstrip feed lines is utilized. Due to the orthogonality property of current modes, the created radiation patterns are also orthogonal to each other. Since two characteristic currents are excited on the structure’s body, it is expected to achieve two orthogonal radiation patterns too, which could result in a dual-mode antenna. To carry out the above-mentioned modal analysis, a specific method of moments (MoM) code has been developed and applied. The proposed antenna is simulated by well-known full-wave package Ansoft HFSS and results clarification of the expectations about the orthogonality in patterns.

Index Terms — Dual-mode antenna, method of moments (MoM), and theory of characteristic modes (TCMs).

I. INTRODUCTION

Design of antennas for wireless communication systems has attracted lots of interests. Obviously, the inclusion of multiple technologies in wireless devices will significantly increase the antenna’s functionality. The more progress in system design, the much more attention in antenna selection. Since the advent of numerical based electromagnetic simulators, the approach of antenna design has been totally altered. At the moment, some huge amount of functions are performed by these simulators, which may results in vague understanding of the antenna performance especially whenever self optimizer tools in these simulators are utilized.

In many occasions the designer requires some wide spread insight about the antenna operation in different frequencies or may be the response of radiating part to various configurations of feedings. Making a profit of mentioned information enables the designer to choose the best conceivable option among all possibilities of an antenna to offer. This cannot be easily achieved by simulators and of course is quite time consuming.

Modal analysis could be an effective solution to cope with this problem due to the wide spread insight, which is provided by about the behavior of analyzed body. For a long time ago while now, modal analysis has been used in electromagnetics for the analysis of close structures (e.g., waveguides and cavities) in which it is quite simple to reach close solutions by applying boundary conditions [1, 2]. Nonetheless, it is not that easy to calculate different modes in open structures (e.g., antennas or scatterers) and it is usually quite time-consuming. Probably, this is one of the reasons that why modal analysis is not commonly used for antenna design at the present time. However, getting involved with the computation’s difficulties of modes on open
structures has its own values since it results in a wide-spread insight about the overall electromagnetic behavior of the analyzed body. Fulfillment of modal analysis in electromagnetic structures can be obtained via some limited number of approaches, for instances, which are spherical modes [3], modal expansion method [4], and eigen functions of conducting bodies. One subdivision of the classical eigen function analysis, termed as characteristic modes, was introduced first by Harrington in the early 1970s [5, 6]. The characteristic modes are real current modes that can be computed numerically for conductive structures of arbitrary shapes and are commonly named characteristic currents. Since characteristic currents form a set of orthogonal currents on an analyzed body, they create a set of orthogonal radiation patterns as well. As a direct result of this phenomenon, if one can form some characteristic currents on an analyzed body, achieving orthogonal patterns is expected [7, 8].

Generally, resonance frequency of different modes is dependent on physical characteristics of the analyzed structure and is the natural behavior of that body. However, utilizing reactive loading approach [9] enables us to have a control on almost every intended mode’s resonance frequency.

The core objective of this paper is to indicate that the modal analysis of a structure can orient the whole design procedure. Accordingly, as an arbitrary structure, a trapezium-shaped conducting plate is analyzed by the theory of characteristic modes (TCMs) and its first seven modes are obtained in the first step. Then, an overview of the general modal behavior of the structure is given. After that, oriented reactive loading is carried out on the plate to make some changes on the resonance frequency of two intended modes among all those seven. At last, a feeding structure for the excitation is suggested. By exciting those mentioned specific modes, two orthogonal radiation patterns are achieved, which results in a dual-mode antenna.

II. MODAL ANALYSIS OF A TRAPEZIUM-SHAPED PEC PLATE

In this section, to provide a better understanding, at first a brief study on theory of characteristic modes is presented, and after that, first seven characteristic modes of a trapezium-shaped conducting plate are calculated using a developed proper MoM [10] code. Based on the obtained results, reactive loading on the analyzed body is performed to alter the resonance frequency of the first two modes. This reactive loading is fulfilled to have a dual-mode antenna (in this case, dual-mode means dual orthogonal radiation patterns) and it is also essential to make both modes similar from resonance frequency point of view. Hence, reactive loading is used to set the resonance frequency of first two modes at about 3.5 GHz so that make the antenna (which will be designed based on the mentioned modal analysis) suitable for WiMax applications.

A. Brief study on theory of characteristic modes (TCMs)

By the method explained in [5, 6], characteristic modes or characteristic currents can be calculated by the eigen functions of the following particular eigen value matrix equation,

\[ [X]\vec{J}_n = \lambda_n [R]\vec{J}_n \]  

(1)

where \( \lambda_n \) is eigen value, \( \vec{J}_n \) is nth eigen vector or characteristic current, and \([R]\) and \([X]\) are the real and imaginary parts of the generalized impedance matrix \([Z]\), which is produced in traditional MoM analysis of a structure. In fact equation (1) is derived from a particular weighted eigen value operator equation [5, 6]. The traditional MoM applies the inversion of \([Z]\) (with considering the excitation point) to compute the created current on the body of analyzed structure. However, the result which is derived from equation (1) is a set of creatable currents on that body without considering any excitation. To put it in other words, equation (1) introduces the inherent behavior of an electromagnetics structure apart from the way of excitation and its own problems; i.e., impedance matching, soldering point, and etc.

One of the most important things, which should be taken into consideration in equation (1) is how eigen values \( \lambda_n \) respond to alteration of frequency. \( \lambda_n \)'s variation range is from \(-\infty\) to \(+\infty\), and \( \lambda_n \) s of smallest magnitude are more important from radiation and scattering problems point of view. As given in [5, Eq. (20)], the modes with positive \( \lambda \), predominantly store magnetic energy, whereas those with negative \( \lambda \), mainly store electric energy. The mode having \( \lambda = 0 \) is called the resonant mode.
In other words, at a specific frequency, the eigen value of a particular mode becomes zero and the mode is at resonance. Another representation of \( \lambda_n \)'s is \( \alpha_n \)'s, which are called characteristic angles \([11]\). The formulation is as follows,

\[
\alpha_n = 180^\circ - \tan^{-1}(\lambda_n) .
\]

(2)

It is obvious that at resonant frequency, characteristic angle \( (\alpha_n) \) becomes 180°. Furthermore, due to \([12]\) the frequencies related to angles between 135° and 225° represent the mode’s bandwidth. To study the modal behavior of a conducting body, the first seven modes would suffice due to the weakness of amplitude and intense oscillation of the modes with higher number.

**B. Applying TCMs on a trapezium-shaped PEC plate and reactive loading**

Before getting involved with mode’s calculation, pointing out to some prerequisites would be instrumental. At first, to meet all TCMs properties, it is a must that, the analyzed body is surrounded by air \( (\varepsilon_r = 1) \) \([5]\). Under this circumstance, TCMs guarantees that each characteristic current \( (\overline{J_n}) \) results in corresponding characteristic field \( (\overline{E_n}) \), which is orthogonal to all other characteristic fields \( (i.e., \text{orthogonal radiation patterns}) \). As a result, at the beginning, a trapezium-shaped plate close to an infinite PEC plate (ground plane) is placed in an air surrounded area far away from any object or scatterer. Afterward, the modal behavior of this structure is carried out by utilizing TCMs.

Secondly, in this work, modal analysis of the plate is performed to express some operational information, which are supposed to result in a proposal of an antenna \( (\text{existence of the ground plane, which is mentioned above is also for this reason}) \). Therefore, a way to make it possible to excite appropriate modes on the plate’s body should be considered at the outset of design procedure. Consequently, three stands have been put under the trapezium-shaped plate to make it possible to push the current via them to the plate.

Owe to the above-mentioned issues, by developing a MoM code that uses 133 RWG \([13]\) basis functions, the first seven characteristic currents and corresponding characteristic angles of the discussed plate with dimensions of \( l_1 = 30 \text{ mm} \), \( l_2 = 20 \text{ mm} \), \( h_1 = 10 \text{ mm} \), \( h_2 = 2 \text{ mm} \), \( s_1 = 4 \text{ mm} \), \( s_2 \)
= 11 mm, and \( s_2 = 2 \text{ mm} \) have been calculated, and the results are illustrated in Figs. 1 and 2, respectively.

According to Fig. 1, there are two modes with horizontally direction currents (\( J_1 \) and \( J_2 \)), one vertically direction current mode (\( J_3 \)), three loop shaped current modes (\( J_4, J_6, \) and \( J_7 \)), and one special mode (\( J_5 \)) in which the current shape is to some extent a mixture of horizontal and loop shaped. It is important to point out that, the outer current’s loop in \( J_3 \) is closed via the ground plane. Generally, loop-shaped currents result in creation of passive modes [14] so that the corresponding characteristic angle never hits the 180° point; Fig. 2 vividly represents this issue (\( \alpha_3, \alpha_6, \) and \( \alpha_5 \)). The exact resonance frequency of other modes is as follows; \( J_1: 6.1 \text{ GHz}, J_2: 5.6 \text{ GHz}, J_4: 7.9 \text{ GHz}, \) and \( J_5: 12 \text{ GHz} \). Commonly the resonance frequency of first mode is less than the second mode, but in this specific structure, ground plane shifts the resonance frequency of the second mode to a value less than the first mode’s resonance: with the aid of image-theory, it is possible to eliminate the ground plane and replace the symmetrical shape of the plate instead. By doing so, it becomes apparent that two slots (the empty rooms between plate’s body and ground plane in Fig. 1), which are actually two reactive loads, are blocking the \( J_5 \)’s movement and shifts this mode’s resonance to less values. These slots have no considerable effects on \( J_1 \)’s resonance due to the current direction of this mode.

To decrease the resonance frequency of the first mode, a vertical slot would be effective according to this fact that the direction of this current mode is horizontal. Therefore, a slot at the middle of the plate is created from its top till the middle stand (Fig. 3), the slot width is \( l_{s1} = 2 \text{ mm} \). This slot makes the first mode \( J_1 \) to be exactly resonated at frequency of 3.5 GHz (Fig. 4). However, there is no considerable change on resonance frequency of the second mode due to the direction of its current. \( J_1 \) and \( J_2 \) in modified structure are presented in Fig. 3. The other vertically current mode is \( J_4 \), so the resonance frequency of this mode is also reduced to less value (5.2 GHz) in this new shape of plate.

At last, it is worth mentioning that slot’s creation on the plate makes the seventh mode active in contrary with Fig. 1’s structure. As it can be observed from Fig. 3, according to the existence of the slot, \( J_7 \) changes its configuration from loop shaped one (Fig. 1) to vertically current and this makes the mode to be an active mode, which resonate at about 12 GHz.

The final step is to make the second mode to resonate close to the first mode at about 3.5 GHz. To meet this condition, the proposed structure alongside with first and second characteristic currents on the body’s structure are presented in Fig. 5 (\( l_{s2} = 2 \text{ mm}, h_3 = 4 \text{ mm} \), other dimensions are same as previous structures in Figs. 1 and 3). Correspondingly, Fig. 6 represents the behaviour of characteristic angles in this new structure. As it is clear from the figure, new added horizontal slots result in a reduction at resonance frequency of all modes that their current directions are vertical. There are still two reactive modes (\( J_2 \) and \( J_6 \)) and the resonance frequency of the other modes are as follow; \( J_1: 3.5 \text{ GHz}, J_2: 3.65 \text{ GHz}, J_4: 4.1 \text{ GHz}, J_5: 4.8 \text{ GHz}, \) and \( J_7: 5.1 \text{ GHz} \). So at this moment our predefined goal is achieved, i.e., two resonating modes at about 3.5 GHz. Now if some configuration of currents similar to those ones presented in Fig. 5 can be shaped on the structure, it is expected to obtain two orthogonal radiation patterns.
III. ANTENNA DESIGN

Once the modes $J_1$ and $J_2$ on the structure presented in Fig. 5 have been extracted, the next step is to find optimum feeding scheme to excite these modes on the antenna. To do this, the proposed antenna structure is shown at a glance in Fig. 7. As it can be observed, there are four separated ports, which connected to the radiating part (the trapezium-shaped plate) via four microstrip transmission lines, printed on a Rogers RT/duroid 5870 substrate with thickness of $h = 0.787$ mm and $\varepsilon_r = 2.33$. To consider an infinite ground plane (which have taken into consideration the numerical modal analysis) in the antenna structure, other dimensions of substrate are $l_x = l_y = 80$ mm, which result in an adequately large plan in

Fig. 5. Normalized first and second characteristic currents of a reactively loaded trapezium-shaped conducting plate near the ground plane.

Fig. 6. First seven characteristic angles of the reactively loaded trapezium-shaped conducting plate of Fig. 5.

Fig. 7. Proposed dual-mode antenna with orthogonal radiation patterns.

Fig. 3. Normalized first, second, and seventh characteristic current of a reactively loaded trapezium-shaped conducting plate near the ground plane.

Fig. 4. First seven characteristic angles of the reactively loaded trapezium-shaped conducting plate of Fig. 3.
Fig. 7. Proposed antenna structure and its characteristics.

comparison with the wavelength. In order to be able to excite first and second modes on the structure’s body in a better way, a thin slot is created at the middle of the radiating part with width of \( d_6 = 0.1 \) mm (Fig. 7 (d)) to decrease the unwanted current’s follow from \( \text{port} 2 \) to \( \text{port} 3 \). Both this slot and the substrate’s thickness have not been analyzed in TCMs and of course their existence shifts the corresponding resonance frequency of the desired modes. However, owing to the fact that without these mentioned elements, the antenna structure cannot be completely achieved, they have been kept on the structure and instead, their undesirable effects have been compensated by using of four stubs that can be seen in Fig. 7. Other antenna’s parameters, which are presented in Fig. 7 (c) are as follow: \( d_1 = 40 \) mm, \( d_2 = 14.1 \) mm, \( d_3 = 4 \) mm, \( d_4 = 34 \) mm, \( d_5 = 13 \) mm, \( d_7 = 8 \) mm, and \( w = 2 \) mm. The antenna physical structure is symmetric with respect to \( xz \) plane and the dimensions, which are not specified in the figure are the same as those represented in Figs. 1, 3, and 5.

IV. RESULTS

To obtain the dual-mode operation, some alternations in phases of the ports have been performed. In other words, to create \( J_2 \) (Mode2) on the trapezium-shaped plate, all ports are excited with the same initiate phase. Furthermore, creation of \( J_1 \) (Mode1) on the body is achievable by producing 180° phase difference in \( \text{ports} 2 \) and \( 4 \) with respect to \( \text{ports} 1 \) and \( 3 \). Table I presents the feeding configuration that can be employed at ports.

Table I: Feeding configuration (amplitude and phase) for the excitation of 1st and 2nd modes.

<table>
<thead>
<tr>
<th>( \text{port} 1 )</th>
<th>( \text{port} 2 )</th>
<th>( \text{port} 3 )</th>
<th>( \text{port} 4 )</th>
<th>( \text{Excited current} )</th>
</tr>
</thead>
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<tr>
<td>( J_1 )</td>
<td>( J_1 )</td>
<td>( J_1 )</td>
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<td>( J_1 )</td>
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<td>( J_2 )</td>
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The proposed antenna is simulated by the full-wave well known package Ansoft HFSS. The current distributions of first and second modes on the trapezium-shaped plate are obtained as illustrated in Fig. 8, which are both acceptably similar with those presented in Fig. 5. Apart from this, Fig. 9 represents the \( S \)-parameters of the antenna, which revealed that the antenna can work well at 3.5 GHz.

At this moment, with regard to the fact that two characteristic currents are created on the analyzed structure, it is expected that the corresponding radiation patterns will be orthogonal to each other and Fig. 10 clarifies the correctness of this issue. As it can be observed from Fig. 10, the orthogonality...
condition of patterns is obtained very well at both \( \phi = 0^\circ \) and \( 90^\circ \) planes, which makes the antenna to operate in two modes, which are different in radiation pattern. As it has been cleared before, associated with each mode, there is a specific configuration of feeding. So it follows that changing the initiate phase of feeding’s ports as presented in Table. I, results in switching between modes in the proposed dual-mode antenna.

![Fig. 8. The excitation of first and second modes on the proposed antenna structure.](image)

![Fig. 9. Scattering parameters of the proposed antenna structure.](image)

**V. CONCLUSION**

Modal analysis of a structure is exploited to design an antenna whose advantage is operating in two modes. Each mode has a radiation pattern, which is orthogonal to the other mode. The first seven characteristic modes of a trapezium-shaped radiator are obtained by a specific MoM code. With the aid of a general overview about the electromagnetic behavior of the analyzed body, reactive loading is performed in order to set the resonance frequency of first two modes to be identical. From the TCMs, it is expected that each characteristic current leads to a radiation pattern that is orthogonal to the radiation pattern of other characteristic current. As a result, a specific feeding structure is proposed to excite the discussed first two modes on the structure’s body so that a dual-mode antenna is obtained, which benefits from two orthogonal radiation patterns. By choosing an appropriate initiate phase of the feeding’s ports, switching between modes is achievable.

![Fig. 10. Normalized radiation patterns of the proposed antenna in first and second modes.](image)

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