Canonical
Twodimensional Inverse Scattering
Problem

The following twodimensional electromagnetic TE- or TM-scattering problem is considered: A circular cylindrical scatterer with cross-section in the $xy$-plane and infinitely long in $z$-direction is embedded in vacuum and is composed of 3 circular cylindrical concentric layers ($n = 3$) with radii

$$a_1 = 2 \lambda_0$$
$$a_2 = 4 \lambda_0$$
$$a_3 = 6 \lambda_0$$

with $\lambda_0$ being the vacuum wavelength. The relative permittivities $\varepsilon_{rn}$, permeabilities $\mu_{rn}$ and conductivities $\sigma_n$ are given as follows

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\varepsilon_{rn}$</th>
<th>$\mu_n$</th>
<th>$\sigma_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.02</td>
<td>$\mu$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.08</td>
<td>$\mu$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>$\mu$</td>
<td>0</td>
</tr>
</tbody>
</table>

Notice: This is not a weak scatterer as it is assumed within the first-order Born approximation.

Either plane wave TE- or TM-excitation with wavenumber

$$k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0}, \quad c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

and propagation direction perpendicular to the cylinder axis is considered, i.e. the unit-vector of propagation $\hat{k}_i$ is located in the $xy$-plane.
\[ \varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0 \varepsilon_{rn} \]
\[ \mu = \mu_0 \mu_r = \mu_0 \mu_{rn} \]
\[ \sigma = \sigma_n \]
with \( n = 1, 2, 3 \)
Definition of the inverse problem:
Suppose the scattered field $\mathbf{E}_s, \mathbf{H}_s$ is known with "arbitrary" accuracy on a measurement surface $S_M$ not necessarily in the far-field, i.e. it is supposed to be known as a function of the variables $\varphi_M, k_0, k_i$ for fixed but arbitrary $r_M$. Determine $\varepsilon_{r1}, \varepsilon_{r2}$ and $\varepsilon_{r3}$, together with $a_1, a_2, a_3$. The following "data acquisitions" might be considered:

<table>
<thead>
<tr>
<th>Method</th>
<th>$\varphi_M$</th>
<th>$k_0$</th>
<th>$k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>angular diversity</td>
<td>$\varepsilon[0, 2\pi)$</td>
<td>fixed</td>
<td>$\varepsilon[0, 2\pi)$</td>
</tr>
<tr>
<td>frequency diversity</td>
<td>$\varepsilon[0, 2\pi)$</td>
<td>$\varepsilon[0, \infty)$</td>
<td>fixed</td>
</tr>
<tr>
<td>angular and frequency</td>
<td>$\varepsilon[0, 2\pi)$</td>
<td>$\varepsilon[0, \infty)$</td>
<td>$\varepsilon[0, 2\pi)$</td>
</tr>
</tbody>
</table>

Due to the complexity of the problem, discretization errors and noise are not yet to be considered.

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