Improved Performance of FDTD Computation Using a Thread Block Constructed as a Two-Dimensional Array with CUDA

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Abstract — In a previous study, the authors proposed an finite-difference time-domain (FDTD) implementation for a compute unified device architecture (CUDA) compatible graphics processing unit (GPU) using a thread block constructed as a two-dimensional (2-D) array. However, it was found that the larger the computational domain of the 2-D FDTD simulation using the GPU, the slower the computational speed.

In the present paper, the authors investigated the computational performance with respect to the size of a thread block constructed as a 2-D array, and improved the performance of the implementation. Finally, regardless of the size of computational domain, the computational speed using a single GPU (NVIDIA GeForce GTX 280) achieved approximately 30.0 Gflops, which was approximately 20 times faster than that of a single core of a central processing unit (Intel 3.0-GHz Core 2 Duo). The improved performance was approximately 65% of the theoretical peak performance (47.23 Gflops) obtained by the theoretical memory bandwidth (141.7 GB/s).

Index Terms — Finite-difference time-domain method, GPU computing, graphics processing unit, high-performance computing.

I. INTRODUCTION

A graphics processing unit (GPU) is equipped with a large-memory graphics accelerator board for use in a personal computer (PC). The GPU has many processors for 32-bit floating-point calculations. The theoretical peak performance of recent GPUs is greater than 1 Tflops (floating point operations per second). High performance/cost has been reported for the hierarchical N-body simulation using a PC cluster equipped with 256 GPUs [1].

Programs can be developed that allow GPUs to perform general numerical calculations using a high-level shader language (HLSL) (Microsoft HLSL, NVIDIA Cg [2], etc.) or a programming environment (Brook [3], the NVIDIA compute unified device architecture (CUDA) programming environment [4], etc.). Implementation of the finite-difference time-domain (FDTD) method [5-7] on a GPU using various programming environments has been reported [8-16]. The development of the GPU code written in HLSL requires technical knowledge of computer graphics (CG) [8]. In the FDTD simulation using the GPU code written in NVIDIA Cg, the Euclidean normalized error increased monotonously with respect to the time step [9]. The GPU-FDTD code written in Brook has also been reported [10]. In three-dimensional (3-D) FDTD simulation, 3-D to 2-D translation has been reported [11]. Translation from 3-D to 2-D becomes very complicated because the 3-D computational domain of the FDTD simulation is allocated to 2-D texture as a CG technique. Programming tools for the GPU based on CUDA
have been available since 2007. In CUDA, the programmer does not need to be conscious of the CG technique. The advantages of CUDA over Cg and HLSL is that CUDA allows source code to be written in a C-like language and the memory on the GPU board can be used easily. The GPU implementation for LU decomposition solvers using CUDA for computational electromagnetics application has been reported [17]. More GPU implementations of the FDTD method using CUDA have been provided and the computational performances of these implementations have been discussed [12-16]. The FDTD computation using GPU has been implemented with data reuse of the electromagnetic field, and the computational performance has been reported [12]. In [13], a thread block [4] was constructed as a 2-D array. The performance of the 2-D FDTD implementation using GPU was investigated with respect to the four arrays considered (4 × 4, 8 × 8, 12 × 12, and 16 × 16). The FDTD computation was fastest for the 16 × 16 array among the four arrays. The GPU implementations of 3-D FDTD computation using CUDA have been reported [14, 15]. A thread block was constructed as a 2-D array [14], and the size of the 2-D array was 16 × 16. However, the performance of the implementation was not investigated the other 2-D arrays. In [15], a thread block was constructed as a 1-D array, and GPU implementations based on two thread-to-cell mapping algorithms were considered. The performances of the implementations were investigated with respect to the number of threads per thread block. Thus, a 1-D array or a 2-D array is used as a thread block.

In a previous study, the authors proposed a GPU implementation for FDTD computation using a thread block constructed as a 2-D array [16]. The computational domain of the FDTD simulation is divided into subdomains. The electromagnetic field data of a subdomain is stored in shared memory [4]. A subdomain is adjacent to four neighbor subdomains. In this case, a subdomain requires four overlapping areas that include the electromagnetic field data of four neighbor subdomains required to calculate the electromagnetic field on the boundaries of a subdomain. The proposed implementation uses two different subdomains for the calculation of the electric field and the magnetic field, and reduces the number of overlapping areas from four to two in order to reduce the number of branches in the CUDA program. In performance evaluation of the proposed implementation, NVIDIA GeForce GTX 280 was used as a GPU, and a 16 × 16 2-D array was used as a thread block. However, the larger the computational domain of the 2-D FDTD simulation using the proposed implementation, the slower the computational speed.

In the present paper, the authors investigated the performance of the proposed GPU implementation with respect to the size of a thread block constructed as a 2-D array and improved the performance of the proposed implementation. As a result, the computational speed of the implementation in a computational domain of 8,192 × 8,192 peaked when the size of the thread block was 32 × 4. Regardless of the size of the computational domain, the computational speed using a single GPU (NVIDIA GeForce GTX 280) was approximately 30.0 Gflops, which is approximately 20 times faster than that of a single core of a central processing unit (CPU) (Intel 3.0-GHz Core 2 Duo), where the Intel C compiler was used as C compiler.

The remainder of the present paper is organized as follows. The proposed GPU implementation for FDTD computation using a thread block constructed as a 2-D array in a previous study is described in Section II. In Section III, the performance of the proposed GPU-FDTD implementation is described in detail with respect to the size of a thread block constructed as a 2-D array, and the performance of the proposed implementation is improved. Finally, in Section IV, conclusions are presented and future research is described.

II. GPU-FDTD IMPLEMENTATION [16]

CUDA is a parallel computing architecture. NVIDIA GeForce GTX 280 has 30 streaming multiprocessors (SMs), each of which is composed of eight streaming processors (SPs) for 32-bit floating-point calculation, 16,384 registers, and 16 KB of on-chip memory. The CUDA program consists of the CPU code and the GPU code. The GPU code, which is written in a C-like language, includes data-parallel functions, referred to collectively as the kernel. A kernel is executed as a grid of thread blocks. A thread block is an array of threads that can cooperate. Threads within the same thread block are synchronized and share data.
in the shared memory. The CPU code is written in the C language, and the CPU launches the GPU kernel.

In the case of the 2-D FDTD method, the equations in the transverse magnetic (TM) case are as follows:

\[
H^{n+1/2}_{x}(i, j + 1/2) = H^{n-1/2}_{x}(i, j + 1/2)
- \Delta t / \mu \Delta y \left\{ E^{n}_{y}(i, j + 1) - E^{n}_{y}(i, j) \right\},
\]

\[
H^{n+1/2}_{y}(i + 1/2, j) = H^{n+1/2}_{y}(i + 1/2, j)
- \Delta t / \mu \Delta x \left\{ E^{n+1}_{x}(i + 1, j) - E^{n}_{x}(i, j) \right\},
\]

\[
E^{n+1}_{x}(i, j) = E^{n}_{x}(i, j)
- \Delta t / \varepsilon \Delta y \left\{ H^{n+1/2}_{x}(i, j + 1/2) - H^{n+1/2}_{x}(i, j - 1/2) \right\}
+ \Delta t / \varepsilon \Delta x \left\{ H^{n+1/2}_{y}(i + 1/2, j) - H^{n+1/2}_{y}(i - 1/2, j) \right\},
\]

where \( E^{n+1}_{x}(i, j) \) is the required value of the electric field at grid point \((i, j)\) and the \((n+1)\)-th time step, \(\Delta x\) and \(\Delta y\) are the sizes of the spatial division in the \(x\) and \(y\) directions, respectively, and \(\Delta t\) is the time increment. Parameters \(\varepsilon\) and \(\mu\) are the electric permittivity and the magnetic permeability in the medium, respectively. A large quantity of electromagnetic field data in the computational domain for FDTD simulation is stored in the global memory as off-chip device memory on a CUDA-compatible graphics accelerator board. The CPU allocates the data of the electromagnetic fields to a global memory on the GPU board. The memory size of each electromagnetic field array in the program must be an integer multiple of 16 for coalesced global memory access [4]. If the memory size of each required electromagnetic field array in the computational domain is not an integer multiple of 16, the memory size, which is larger than that of each required array, is allocated in order to be equal to an integer multiple of 16. Shared memory enables faster data access than global memory and accounts for 16 KB of on-chip memory in the role of CPU cache memory. In the proposed implementation, a thread block is constructed as a 2-D array, and the computational domain of the FDTD simulation is divided into a small subdomain. The electromagnetic field data in each subdomain are stored in each shared memory as shown Fig. 1. Calculating the electric field data \( E^{n+1}_{x}(i, j) \) in Region 5 requires the magnetic field data \( H^{n+1/2}_{y}(i, j + 1/2) \) in Region 6 and the magnetic field data \( H^{n+1/2}_{y}(i, j + 1/2) \) in Region 8. The required data of the magnetic field, which overlap neighboring subdomains as shown in Fig. 1, are also stored in each shared memory. Each subdomain of the proposed implementation includes two overlapping areas. In the CUDA
program, the size of each subdomain, excluding overlapping areas, is \( N_x \times N_y \) when the size of the thread block constructed as a 2-D array is \( N_x \times N_y \).

In calculating the magnetic field data \( H_x \) and \( H_y \) (Equations (1) and (2)), all of the threads in each \( N_x \times N_y \) thread block first store the data of electric field \( E_z \) of each subdomain in the shared memory, whereas no data of magnetic fields \( H_x \) and \( H_y \) of each subdomain are stored in the shared memory. Next, the same electric field data \( E_z \) required in Equations (1) and (2) are stored in the register only once [12]. After these procedures, all of the threads in a thread block are used to calculate Equations (1) and (2) in each subdomain. Finally, the calculated data of magnetic fields \( H_x \) and \( H_y \) are stored in the global memory, while the calculation of electric field \( E_z \) by Equation (3) at the following time step is performed in the same manner. The subdomain used to calculate the electric field \( E_z \) (Subdomain for the E field shown in Fig. 1) differs from that used to calculate the magnetic fields \( H_x \) and \( H_y \) (Subdomain for the H field shown in Fig. 1) in order to use the shared memory efficiently. Therefore, two kernels for the electric field and magnetic field calculations are required in the CUDA program. The number of time steps is counted and stored in the global memory by a particular SP in each kernel if the calculation of the electric field \( E_z \) or the magnetic field \( H_x \) or \( H_y \) requires the number of time steps for the boundary condition. The kernel codes of the proposed GPU-FDTD implementation are shown in Fig. 2.

**III. PERFORMANCE**

In the present paper, the authors used the NVIDIA CUDA programming environment for the GPU and a NVIDIA GeForce GTX 280 as the GPU board and timed the calculations required for a simple 2-D model, excluding for the absorbing boundaries, in order to investigate the basic performance of the proposed GPU-FDTD implementation. The propagation of electromagnetic waves from the line source in the TM case was used as the calculation model. The line source was located in the center of the 2-D computational domain. The authors compared the GPU implementation with the conventional CPU implementation. In the GPU implementation, the authors developed a GPU-FDTD code written in the C language and a kernel written in a C-like language for the instruction set of the GPU using the CUDA programming environment. The propagation of electromagnetic waves from the line source in the TM case was used as the calculation model. The line source was located in the center of the 2-D computational domain. The authors compared the GPU implementation with the conventional CPU implementation. In the GPU implementation, the authors developed a GPU-FDTD code written in the C language and a kernel written in a C-like language for the instruction set of the GPU using the CUDA programming environment. A kernel can be embedded in the code written in the C language and a kernel written in a C-like language for the instruction set of the GPU using the CUDA programming environment.
same PC equipped with an Intel Core 2 Duo E8400 (3.0 GHz) as the CPU, 2.0 GB of memory (DDR3-1333), and Fedora 9 as the Linux operating system and timed 1,000 iterations of the calculation by Equations (1) through (3) for the GPU and CPU-only computation.

In Equations (1) through (3), the authors replace $\Delta t/\mu \Delta x$, $\Delta t/\varepsilon \Delta x$, and $\Delta t/\varepsilon \Delta y$ with constants. As a result, the number of operations in Equations (1) through (3) is estimated to be 12. The theoretical peak performance of the FDTD simulation using the NVIDIA GeForce GTX 280 as a GPU is obtained as $47.23 \text{ Gflops} = 141.7 \text{ GB/s} ÷ 4 \text{ byte/word} × 12 \text{ operations} ÷ \text{nine words}$, where the theoretical memory bandwidth is 141.7 GB/s, and the number of load/store data in Equations (1) through (3) is estimated to be nine words. On the other hand, the theoretical peak performance of the GPU is obtained as $933.12 \text{ Gflops} = \text{three operations/SP} × 240 \text{ SP} × 1.296 \text{ GHz}$, and the theoretical peak performance of the FDTD simulation using the GPU (47.23 Gflops) is smaller than in the latter example (933.12 Gflops). Therefore, the bottleneck of 2-D FDTD computation using the GPU is the memory bandwidth.

Here, $T_{\text{GPU}}$ is the GPU computation time (s) in the computational domain of $L \times L$, and $N_{\text{itr}}$ is the number of time steps of the FDTD simulation. Subsequently, the actual computational speed (flops) can be obtained as 12 operations $× L × L × N_{\text{itr}}/T_{\text{GPU}}$ ($N_{\text{itr}} = 1,000$). When the size of the subdomain is $16 × 16$, the two computational speeds of the GPU-FDTD simulation using shared and non-shared memory are shown in Fig. 3. In Fig. 3, the ‘non-shared memory’ indicates the basic GPU-FDTD computation without the subdomain for using shared memory, while ‘shared memory’ indicates the proposed GPU-FDTD implementation using shared memory. The larger the computational domain of the 2-D GPU-FDTD simulation, the slower the computational speed. In the computational domain of $8,192 \times 8,192$, the authors investigated the performances of two GPU-FDTD implementations with respect to the size of a thread block constructed as a 2-D array (Table 1). In ‘shared memory’, the thread block of size $32 \times 4$ achieved a peak speed (Table 1(a)). In the ‘non-shared memory’, the thread block of size $64 \times 4$ achieved a peak speed (Table 1(b)). In Table 1, the computation time using the proposed GPU-FDTD implementation was very long when $N_y$ was larger than or equal to $N_x$. The authors analyzed the performance of the proposed GPU-FDTD implementation using the NVIDIA CUDA Visual Profiler. Bank conflicts of shared memory occurred when $N_x \leq 8$ for all cases of the total number of threads per thread block considered herein. Therefore, the performance of the global memory overall throughputs, which is the sum of the global memory write throughput and the global memory read throughput, decreased markedly in the GPU computation of the electric field and the magnetic field. For all cases of the total number of threads per thread block, the number of divergent branches within a warp increased in the GPU computation of the electric field when $N_x \leq 16$. In the ‘shared memory’, the performance of the global memory overall throughputs was the best for the case in which the total number of threads per thread block is 128. In Table 1(a), a thread block of size $32 \times 4$ achieved a peak speed. In the case of the $32 \times 4$ thread block, the global memory overall throughput of the GPU computation of the electric field was 99.12 GB/s, while the global memory overall throughput of the GPU computation of the magnetic field was 126.70 GB/s. The
computational speeds using the 32 × 4 thread block in the ‘shared memory’ and the 64 × 4 thread block in the ‘non-shared memory’ are shown in Fig. 3. Regardless of the size of the computational domain, the computational speed of the proposed GPU-FDTD implementation achieved approximately 30.0 Gflops. The authors improved the performance of the GPU-FDTD simulation by using the optimum size of a thread block constructed as a 2-D array and compared the computation time of the GPU-FDTD simulation with that of CPU-only simulation. In Fig. 4, the speedup factor shows the ratio of the computation time of the CPU only (T_{CPU}) to that of GPU (T_{GPU}). The FDTD simulation using a single GPU was approximately 20 times faster than that using a single CPU core.

The authors estimated the effective performance (Fig. 5). As a result, the effective performance achieved approximately 65% of the
theoretical peak performance (47.23 Gflops) obtained using the theoretical memory bandwidth.

IV. Conclusion

The authors proposed GPU-FDTD implementation using a thread block constructed as a 2-D array in a previous study. However, in a 16 × 16 thread block, the larger the computational domain of 2-D GPU-FDTD simulation, the slower the computational speed. In the present paper, the authors investigated the computational performance with respect to the size of a thread block constructed as a 2-D array. As a result, the computational speed of the GPU-FDTD simulation peaked when the thread block size was 32 × 4. Regardless of the size of the computational domain, the computational speed of the GPU (NVIDIA GeForce GTX 280) was approximately 30.0 Gflops, which is approximately 20 times faster than using a single core of the central processing unit (Intel 3.0-GHz Core 2 Duo). Finally, after improving the performance of the proposed GPU-FDTD implementation, the effective performance was approximately 65% of the theoretical peak performance of GPU-FDTD computation using an NVIDIA GeForce GTX 280 as a GPU.

In the future, the authors intend to apply the proposed method to 3-D FDTD simulation.

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REFERENCES


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