Simulation of Non Linear Circuits by the Use of a State Variable Approach in the Wavelet Domain

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Abstract — A method for the simulation of complex circuits with nonlinear elements is proposed. The method is based on wavelet expansion of the state variable description, and leads to a compact representation of the nonlinear problem which is characterized by accuracy and computational efficiency.

Keywords — Circuit Simulation, Nonlinear Loads, Wavelet Expansion.

I. INTRODUCTION

The most common approach to design and optimize microwave devices is to represent them through equivalent circuits, which are in general composed by linear, non linear, lumped, and distributed elements.

The main difficulty of the problem is the mixed nature between linear components (best treated in the frequency domain) and nonlinear components (best treated in the time domain). Some authors proposed the impulse response and convolution (IRC) technique [1] – [3], in which a great computational effort is dedicated to the process of recording and convolving the quantities related to the nonlinear elements. A different approach consists of approximating the frequency response by a Padé approximation, so called asymptotic waveform evaluation method, and has been used for both linear and nonlinear circuits [4] – [6]; the main drawback of the method is the sometimes low approximation (due to the reduction in the number of poles) in case of very complex circuits. In [7] a further approach is presented: the numerical inversion of Laplace transform technique, which is characterized by several advantages with respect to IRC and AWE, but suffers from the series approximations and the nonlinear iterations involved.

Recently wavelet based techniques have been proposed also for the analysis of nonlinear circuits (in transient or steady state mode), showing good potential [8], [9]. A basis of Daubechies wavelets on the interval is here used to expand the unknown quantities and the circuit equations are obtained by the application of the modified nodal analysis. The nonlinearities are treated by the use of the substitution theorem, and the problem is solved by the application of a standard Newton – Raphson algorithm with an analytical calculation of the Jacobian.

This particular formulation makes the method efficient from a computational point of view, since the matrices involved in the calculation are sparse (i.e. characterized by a small number of non zero elements), hence the number of multiplications required in the solution is low. Furthermore the characteristics of the chosen wavelet basis reduce the size of the matrix.

The method has been tested in several cases, here is reported the calculation of voltages and currents in a complex circuit, and the results are compared with the results coming from a SPICE simulation.

II. MATHEMATICAL FORMULATION

A. Modified Nodal Analysis in the Wavelet Domain

Let us consider a complex circuit, composed by a set of lumped and distributed parameters, connected to independent voltage generators and linear loads. It is possible to divide the circuit into two interconnected parts: a subnetwork \( \phi \) composed by lumped linear and nonlinear elements, for which we are interested in the calculation of voltages and currents, and a linear subnetwork \( \pi \), composed by lumped and/or distributed elements, seen as a multiple port circuit. By the used of the MNA it is possible to write the circuit equations in the Laplace domain in the following form [10]

\[
W_\phi x_\phi (s) + G_\phi x_\phi (s) + L_\pi (s)i_\pi (s) = b(s)
\]

where \( x_\phi \) is the vector of unknowns (nodal voltages, whose dimension is the total number of MNA variables); \( W_\phi \) and \( G_\phi \) are constant matrices describing the lumped elements of the network \( \phi \) and \( b \) is a constant vectors whose entries are the independent voltage and current sources (together with the initial condition sources, if present). \( L_\pi \) is a matrix whose entries are zeroes or ones, mapping the vector \( i_\pi (s) \) of currents entering the linear subnetwork \( \pi \) into the node space of
the network \( \phi \). The linear multiterminal subnetwork \( \pi \) can be described by the standard approach as

\[
Y_\pi(s) V_\pi(s) = i_\pi(s)
\]

where \( Y_\pi(s) \) is the \( y \) parameter matrix and \( V_\pi(s) \) is the vector of terminal voltage nodes connecting the subnetwork to the network \( \phi \). By substituting (2) in (1) it is possible to write

\[
\tilde{W}_\phi x_\phi(s) + \tilde{G}_\phi x_\phi(s) = b(s)
\]

where \( \tilde{G}_\phi \) and \( W_\phi \) take also into account the contribution of the matrix \( Y_\pi(s) \).

As an example, we apply the MNA to the simple network represented in Fig. 1.

![Simple network](image)

In this case equations (1) and (2) respectively become

\[
\begin{bmatrix}
G_1 & -G_1 & 0 & 0 & 1 & 0 \\
-G_1 & G_1 & 0 & 0 & 0 & 0 \\
0 & 0 & G_2 & -G_2 & 0 & 0 \\
0 & 0 & -G_2 & G_3 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
i_{i_1} \\
i_{i_2}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
i_i = \begin{bmatrix} sC \\ -(1/sL + G) \end{bmatrix} v_2 \\
-1/sL + G \]

We now consider a wavelet basis \( b(t) = [b_1(t), ..., b_n(t)] \) on the interval \([0, T_m] \); by performing the Wavelet Expansion (WE) of a generic function \( f(t) \), we obtain a vector of coefficients of dimension \( n \); the notation used is the following:

\[
f(t) = b_f = \sum_j b_j(t) f_j, \text{ where } f = [f_1, ..., f_n]^T \]

is the vector of the wavelet coefficients. By using the differential operator \( D \) and the integral operator \( I \) in the wavelet domain (for wavelets on the interval, introduced in [11]) the differentiation (or integration) of a function is simply performed by a matrix – vector product, i.e.,

\[
h(t) = \frac{df(t)}{dt} \Rightarrow h = Df.
\]

Formally, this means that it is possible to obtain the equation in the wavelet domain by using the Laplace domain equations and substitute the variables with the vectors of coefficients and the operator \( s \) with the differential matrix \( D \). According to this, equation (3) can be expressed as

\[
(W_\phi + G_\phi) x_\phi = T x_\phi = b
\]

i.e., an algebraic systems of the form \( T x_\phi = b \) in which the matrix \( T \) is straightforwardly calculated, the vector \( b \) contains the WE of the independent generators and \( x_\phi \) is the vector of unknowns (the wavelet coefficients of the expansion of the unknown voltages).

For the example of Fig. 1 we obtain for the matrix

\[
\begin{bmatrix}
G_1 U & -G_1 U & 0 & 0 & 0 & 0 \\
-G_1 U & G_1 U + CD & -CD - GU & 0 & 0 \\
0 & -I/L - GU & I/L + G_2 U & -G_2 U & 0 & 0 \\
0 & 0 & -G_2 U & G_2 U & 0 & 0 \\
U & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & U & 0
\end{bmatrix}
\]

\[(7)\]

where \( U \) is the identity matrix of the proper dimension. As it can be easily seen the matrix \( T \) is sparse and the system can be easily and conveniently solved by an iterative technique, requiring a low CPU time consumption.

### B. Treatment of the Nonlinearities

The presence of nonlinearities (connected at the output ports) leads to an additional term in equation (1)

\[
W_\phi x_\phi(s) + G_\phi x_\phi(s) + L_\pi(s) i_\pi(s) + L \left[ F(x_\phi(t)) \right] = b(s)
\]

(8)

where \( F(x_\phi(t)) \) represents the above mentioned nonlinearities.

For the sake of simplicity we here refer to a simple two port network, represented in Fig. 2a, where one port is connected to an independent generator, while the other port is connected to a nonlinear load, whose constitutive
relation is \( i = f(v) \). The extension to a more complex circuit (i.e. characterized by several ports, connected to generators, linear and nonlinear loads) is straightforward.

Fig. 2a. Circuit with a nonlinear load.

Fig. 2b. Application of the substitution’s theorem.

It is possible to substitute the nonlinear load with a voltage generator imposing the unknown voltage \( v_{NL}(t) \), as shown in Fig. 2b. Let us suppose that we are interested in calculating the current \( i_{NL} \) flowing through the nonlinear load. By applying the superposition effect (once the substitution has been performed the circuit is linear) we can write

\[
i_{NL} = i_{NL}^a + i_{NL}^b
\]

(9)

where the first term \( i_{NL}^a \) is related only to the presence of the independent generator \( E \) and can be calculated by simply solving a linear problem (of the kind reported in equation (6)), while \( i_{NL}^b \) is the current related to the effect of the unknown voltage generator and needs of course to be calculated. Under these assumptions we can write in the wavelet domain

\[
\mathbf{i}_{NL} = \mathbf{i}_{NL}^a + \mathbf{i}_{NL}^b = \mathbf{i}_{NL}^a + \mathbf{T} \mathbf{v}_{NL}
\]

(10)

where the vectors represent the WE of the currents, the matrix \( \mathbf{T} \) is the matrix solving the linear problem (see equation (6), and \( \mathbf{v}_{NL} \) is the wavelet expansion of the unknown voltage source. At this stage equation (10) is characterized by two vectors of unknowns: \( \mathbf{i}_{NL} \) and \( \mathbf{v}_{NL} \). The additional equation needed to solve the problem is the constitutive equation of the nonlinear load and it is enforced as follows.

By inverse transforming (10) we obtain the following time domain expression of the unknown current

\[
i_{NL}(t) = i_{NL}^a(t) + \sum_k \sum_j T_{kj} \psi_{NL,j} b_k(t)
\]

(11)

where the terms \( b_k(t) \) are the function of the wavelet basis, \( \psi_{NL,j} \) are the entries of the vector \( \mathbf{v}_{NL} \), while the terms \( T_{kj} \) are the entries of the square matrix \( \mathbf{T} \). Equation (11) must satisfy the constitutive equation of the nonlinear load \( i = f(v) \); in order to enforce it we impose the collocation at the discrete times \( t_n \), \( n \) equally spaced points in the interval \([0, T_m]\) (the ones which are characteristic of the definition of the wavelet functions [12]) obtaining a set of nonlinear equations in the unknown coefficients \( \psi_{NL,j} \).

The constitutive equation of the nonlinear load \( i - f(v) = 0 \) can be written as

\[
i_{NL}^a(t) + \sum_k \sum_j T_{kj} \psi_{NL,j} b_k(t) - f\left( \sum_j \psi_{NL,j} b_j(t) \right) = 0
\]

(12)

i.e., \( F(v_{NL}) = 0 \) in which we underline that the unknowns are the coefficients \( \psi_{NL,j} \). The analytical evaluation of the Jacobian is straightforwardly written as follows

\[
\frac{\partial F}{\partial v_{NL}} = \sum_k T_{kj} b_k(t_n) - \frac{\partial f}{\partial v} b_j(t_n)
\]

(13)

The solution of the system is performed by adopting a Newton – Raphson algorithm, with an analytical evaluation of the Jacobian. The convenience of the proposed method, in terms of low CPU time consumption in the presence of nonlinear loads, stands in the availability of the analytical form of the Jacobian; as a matter of fact its knowledge allows us to use a sparsification procedure which results in a reduction of the CPU time employed for the solution, as explained in the next section.

C. Computational Cost of the Proposed Method

The main advantage in using a wavelet basis stands in the fact that it is possible to represent very complex waveforms (typical of fast electrical transients, like the ones in microwave circuits) by a small number of wavelet functions (hence by a small number of coefficients). This leads to a reduced dimension of the matrices involved in the simulations, with respect to other standard techniques.
Furthermore it is well known that wavelets tend to concentrate in time zones where approximation needs to be high and have a reduced weight (low wavelet coefficients) in the other zones, a property known in the literature as self adaptive zooming. According to this observation, the use of a thresholding procedure is a common practice to reduce the computational cost \cite{12,13}. The coefficients of the wavelet matrix that are smaller than a fraction (typically a few thousandths) of the maximum are forced to zero.

This procedure does not significantly affect the accuracy of the computations (when reasonable values of the threshold are used) and produces sparse matrices that can be efficiently stored and computed. In particular in \cite{12} it is shown that for diagonal dominant matrices it is possible to obtain a percentage of non-zero elements of the order between 5\% and 15\% with solutions that are affected by extremely low error. The sparsification procedure is convenient because the number of multiplications required for the simulation is considerably lower with respect to the full matrix, leading to a significant CPU time reduction and a lower storage memory.

It is noteworthy that a lower approximation in the evaluation of the Jacobian (which is exactly calculated by the analytical computation as in \cite{13} and post processed by the sparsification procedure) does not affect the correctness of the solution, and may have effects only on the number of iterations necessary to reach convergence.

In the evaluation of the overall computation time reduction we have to consider the weight of the wavelet transform of the input quantities and the inverse wavelet transform of the results. These operations are efficiently performed via matrix vector products characterized by sparse matrices \cite{14}. As a consequence the reduction of the time needed to perform the matrix vector product obtained with the threshold procedure has a strong impact on the overall computation cost, as it involves the most time consuming activity.

### III. NUMERICAL APPLICATION

As an example of application of the proposed technique we considered the circuit schematically shown in Fig. 3. It is composed by four Multiconductor Transmission Lines (three conductors and ground each) represented by blocks B, C, E, and F and by the blocks A and D constituted by three longitudinally disposed resistors of 0.1 \( \Omega \) each. The per unit length parameters of the lines are the same for each line and are here reported

\[
R = \begin{bmatrix}
351.63 & 0 & 0 \\
0 & 366.11 & 0 \\
0 & 0 & 351.63
\end{bmatrix} \Omega/m
\]

\[
L = \begin{bmatrix}
4.3698 \times 10^{-7} & 7.675 \times 10^{-8} & 1.57 \times 10^{-8} \\
7.675 \times 10^{-8} & 4.35 \times 10^{-7} & 7.675 \times 10^{-8} \\
1.57 \times 10^{-8} & 7.675 \times 10^{-8} & 4.3698 \times 10^{-7}
\end{bmatrix} \text{H/m}
\]

\[
G = \begin{bmatrix}
7.5 \times 10^{-4} & 0 & 0 \\
0 & 7.5 \times 10^{-4} & 0 \\
0 & 0 & 7.5 \times 10^{-4}
\end{bmatrix} \Omega^{-1/m}
\]

\[
C = \begin{bmatrix}
1.082 \times 10^{-10} & -0.197 \times 10^{-10} & -0.006 \times 10^{-10} \\
-0.197 \times 10^{-10} & 1.124 \times 10^{-10} & -0.197 \times 10^{-10} \\
-0.006 \times 10^{-10} & -0.197 \times 10^{-10} & 1.082 \times 10^{-10}
\end{bmatrix} \text{F/m}
\]

The lines in B and C are respectively long 10 and 5 cm, while those in E and F have the same length of 15 cm.

A voltage generator of waveform

\[
e_1(t) = 10 \cdot e^{-\left(\frac{t-2T}{T}\right)^2} \cdot \sin \left[2\pi f (t-2T)\right] V
\]

is applied to terminal \#1, with \( T = 0.11 \text{ns}, f = 2.8 \text{ GHz} \) and characterized by an internal resistive impedance \( R_1 = 10 \Omega \).

The amplitude of the waveform \( e_1(t) \) shown in Fig. 4, has been chosen in order to strongly evidence the nonlinear effects.

Terminals 2, 3, 5, 6, 8, 9, 11, and 12 are connected to 50 \( \Omega \) termination resistances.

Terminal \# 4 is terminated on the series connection of a 15 \( \Omega \) resistance and of a nonlinearity described by the following characteristic

\[
v_4(t) = 10^2 \cdot i_4^2(t).
\]

Terminals 7 and 10 are terminated on series connection between a 20\( \Omega \) resistance and a diode of characteristic

\[
i_{4,7}(t) = I_0 \cdot \left(e^{v_4(t)/v_T} - 1\right)
\]

where \( I_0 = 1 \text{ pA} \) and \( v_T = 0.025865 \text{ V} \).

The system has been simulated by using a basis of 128 Daubechies wavelet with 6 vanishing moments.

The same system has also been simulated with SPICE and the results have been compared with those obtained by the approach here presented.
Two versions of the proposed method have been implemented: the first one which performs the sparsification in order to take advantage of the available numerical routines for treatment of sparse matrices while the other uses the original full matrices.

The comparison has been performed by considering the Jacobian matrices related to comparable deviation of $F(v_{NL})$ from zero in the cases of solution of the sparsified and original system.

Figures 5 and 6, respectively, show the Jacobian used in Newton-Raphson scheme with and without sparsification.

Figures 7 and 8, respectively, show the voltage and the current on the cubic nonlinearity.
The waveforms obtained by the two approaches are practically indistinguishable. In order to appreciate the good agreement between the results a zoom on the voltages is shown in Fig. 9.

Figures 10 and 11 show the voltage and the current on the diodes. As expected the current on the diode is unidirectional and the voltage on the diode does not exceed the value of 0.65 V that approximately represents the direct bias voltage of a silicon diode.

The first procedure has required 23 iterations while the second one 35. The solution of the linear system obtained after the thresholding procedure was about five times faster than that in the other one. The total CPU time (2.8 sec) required to solve the nonlinear problem by using sparse matrices using the standard Newton-Raphson was three times shorter than the modified Newton-Raphson with full matrices (9.3 sec). The used threshold was 5% of the maximum element of each column.

As for the SPICE simulation the CPU time was about 4 seconds.
IV. CONCLUSIONS

A method for the simulation of complex circuits, in presence of nonlinear elements is here proposed. The method is based on wavelet expansion and a special treatment of the nonlinearity. The method allows a fast computation, together with the required accuracy and low memory consumption.

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