Drastic-Variation Condition of Electric Field and Its Applications in Microwave Heating

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Abstract — Microwave heating usually leads to non-uniform temperatures due to the existence of drastic-variation of electric field. In this paper, a numerical method combined with integral equation and spectrum analysis is proposed to find its critical condition. Results show that the electric field is generally unstable, and a tiny shift in microwave frequency, permittivity of dielectric object or cavity geometrical parameters will produce a drastic variation in electric field distribution, moreover, the smallest shift of parameter can be obtained by the reverse search. FEM method is used to verify the conclusions. Finally, some supplements for the interest regarding practical applications are presented and analyzed.

Index Terms — Drastic-variation condition, electric field, integral equation, microwave heating.

I. INTRODUCTION

Microwave heating has been widely used for domestic, scientific and industrial applications due to its convenience and high efficiency [1-3]. However, in a multimode cavity, microwave heating usually leads to non-uniform temperatures [4], which will restrict the quality of a product. Since heating uniformity is mainly dependent on and affected by the distribution of electric field [5], and in a multimode cavity the electric field is prone to be unstable, it is necessary to study the detailed characteristics of electric field.

The drastic-variation in electric field refers to the effects on either the distribution or amplitude, or both, because of the tiny shift in system parameters. The drastic-variation of electric field usually causes non-uniform heating and results in hot spots (huge temperature gradient in certain areas) and thermal runaway (uncontrollable temperature rise due to strong dielectric loss – temperature positive feedback of the heating object), which restrict heating efficiency and even lead to some serious safety issues [6]. The instability of the electric field, especially its critical condition, is of great importance to the prevention of hot spots, thermal runaway and enhancement of heating uniformity.

To determine the condition of a drastic-variation in electric field, it is important to find the relevant parameters. Several researches are published to investigate the characteristics of electric field, such as the study of improving electric field uniformity in reverberation chambers via mode stirs [7-8]. These might be considered as positive applications of the instability of electric field. Moreover, Hill studied the instability of electric field by examining the mode density in cavity [9]. He gave a relatively rough consideration, such as his research didn’t involve the discussion of permittivity. Whereas, for microwave heating, permittivity is an important parameter that may affect the interaction between microwave and the heating object. More recently, Budko et al. investigated the electric field resonance by the spatial spectrum of electric field volume integral equation [10-11]. Although it can provide us with a useful guidance, it is not specifically aimed at the case of microwave heating.

This study presents a numerical method to determine the drastic-variation condition of electric field in multimode microwave heating system. Integral equation and spectrum analysis are used to find the electric field highly dependent parameters. On the bases of such analysis the relevant parameters are obtained, and then the critical condition for a drastic variation in electric field is analyzed. Results show that even if a tiny shift in microwave frequency, the permittivity of dielectric

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object or cavity geometrical parameters may produce a drastic variation in electric field, and the smallest shift of parameter can be obtained by the reverse search. Because the critical condition plays an important role in preventing hot spots and improving heating uniformity, two examples are given in this paper, which provide some supplement for practical application.

II. METHODOLOGY

The multimode microwave heating system analyzed is shown in Fig. 1. It consists of a rectangular metallic cavity connected to microwave source via a rectangular waveguide operating in the TE_{10} mode. The dimensions of the cavity and rectangular waveguide are \( l_x \times l_y \times l_z \) and \( a \times b \times c \), respectively. In addition, the area of access port is denoted as \( S_a \). At the center of the bottom, there is a single cylindrical object, the radius, height and volume of which are \( r \), \( h \) and \( V_a \), respectively. Inside the cavity the permittivity can be shown as:

\[
\varepsilon(r) = \begin{cases} 
\varepsilon_0 \varepsilon_i(r) - \frac{i \sigma(r)}{\omega}, & r \in V_0, \\
\varepsilon_0, & r \notin V_0.
\end{cases}
\]

where, \( \varepsilon_0 \) is relative permittivity of dielectric object, \( \varepsilon_0 \) is permittivity of free space, and \( \sigma(r) \) is conductivity.

![Fig. 1. Schematic of microwave heating.](image)

The electromagnetic analysis of a microwave heating cavity containing dielectric objects and excited by a rectangular waveguide on the external surface can be performed using the Green’s function of the empty structure. Moreover, the electric field can be represented by the sum of impressed electric field on the external surface and the polarization electric field inside the dielectric object [12]. The impressed electric field can be shown as [13]:

\[
\mathbf{E}^{\text{inc}}(r) = \int_{V_a} \mathbf{E}_{\text{inc}}(r') \cdot \nabla \times \mathbf{G}_e(r, r') \, dS_a,
\]

where \( r' \) and \( r \) represent the source point and field point, respectively. \( \mathbf{E}_{\text{inc}}(r') \) is a tangential impressed field over the cross area and \( \mathbf{G}_e(r, r') \) is dyadic Green’s function of the empty cavity [14-15]. When adding the polarization electric field, the total field can be expressed as:

\[
\mathbf{E}(r) = \mathbf{E}^{\text{inc}}(r) + j \omega \mu_0 \int_{V_a} \mathbf{G}_e(r, r') \cdot \mathbf{J} \, dS_a.
\]

Take into account the singularity arising when the field point coincides with the source point [16], Equation (2) becomes [17]:

\[
\mathbf{E}^{\text{inc}}(r) = \left[ \mathbf{I} + \frac{1}{3} \chi(r) \right] \mathbf{E}(r) - \lim_{\delta \to 0} j \omega \mu_0 \int_{r-v_r} \mathbf{G}_e(r, r') \cdot \mathbf{J} \, dS_a.
\]

Equation (3) is a typical volume scattering equation, the instability of which can be analyzed by its numerical characteristics. Since the eigenvalues of the electric field equation is linked to its resonant modes, the electromagnetic problem can be studied by the characteristics of this equation. By taking similar strategy in previous studies [10], the instability of electric field is characterized by its discrete eigenvalues, and the discrete eigenvalues are bounded by:

\[
\sigma(r) - \sigma(r) \text{Re} \lambda + \omega \text{Im} \lambda \leq 0, r \in V_a,
\]

where, \( \text{Re} \lambda \) and \( \text{Im} \lambda \) are the real part and imaginary part, respectively. In Equation (7), when \( \text{Re} \lambda \to 0 \), the following equation can be obtained:

\[
\text{Im} \lambda = \frac{\sigma(r)}{\omega} \left( \frac{\varepsilon(r) - \varepsilon_0}{\varepsilon(r) - \varepsilon_0} \right), r \in V_a.
\]

It is not difficult to see the distributions of eigenvalues are affected by microwave frequency, the relative permittivity, and the cavity geometrical parameters, hence, the instability of the electric field is affected by these parameters. Without loss of generality, the average rate of variation of electric field is defined to quantify the instability of electric field:

\[
V_{av} = \frac{1}{n-1} \sum_{i=1}^{n} (E_i - E_0)^2,
\]

where \( n \) denotes the number of sampling points, \( E_{10} \) and \( E_i \) denote the electric field amplitude of a single point before and after parameter shifting, respectively. Obviously, \( V_{av} \) can reflect the degree of electric field change of the whole system and the smallest shift of parameter, can be obtained by the reverse search of the critical value of \( V_{av} \).
III. NUMERICAL RESULTS AND ANALYSIS

FEM method is used and performed by Comsol Multiphysics software 5.3b to calculate the electric field regarding the model shown in Fig. 1. The detailed parameters of the system are listed in Table 1, moreover, the relative permittivity, the microwave frequency and microwave power are assumed as 10-j*0.1, 2.45 GHz, and 1W, respectively.

A. Dependent parameters

Figure 2 shows the detailed distribution of electric field for the case of 1% decrease in $l_x$, and 1% increase in $l_y$, respectively. For simplicity, others are summarized in Table 2. Although the variation of electric field caused by the tiny shift of permittivity is relatively small, as can be seen, only a tiny shift of $l_x$ or $f$ can result in a drastic variation of electric field. The case of permittivity differs from the others mainly because it is also dependent on the volume ratio of dielectric body and cavity, as is demonstrated in [19]. A drastic variation of electric field will also be produced when the volume ratio of dielectric body and cavity big enough. With the increase in the dimension of dielectric body, the volume ratio of dielectric body and cavity becomes 0.04, the variation of electric field also turns out to be conspicuous, as can be seen in Fig. 3. In such a case, with only 1% decrease in $\varepsilon_x$ or 1% increase in $\varepsilon_x$, the variation of electric field is 44.68% and 103.44%, respectively.

Fig. 2. Distribution of electric field after the shift of $l_x$.

Fig. 3. Distribution of electric field after the shift of $\varepsilon_x$.

B. Critical values

The critical value of $V_{av}$ is assumed as $V_c$, which means if the calculated $V_{av}$ is larger than $V_c$, a drastic variation of electric field has been produced, and the reverse means the electric field hasn’t made major change. Here, $V_c = 5\%$ is assumed as a critical value, the reverse parameter search with a step of $\frac{1}{8000}$ is implemented. The smallest shift of parameters are then obtained as is shown in Table 3. For example, it shows that the electric field will have a drastic variation with a decrease of $\frac{1}{3200}$ or an increase of $\frac{3}{8000}$ in microwave frequency, respectively.

Table 1: Parameters of the microwave heating system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension and Value/(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity</td>
<td>$l_x \times l_y \times l_z$</td>
</tr>
<tr>
<td>Waveguide</td>
<td>$a \times b \times c$</td>
</tr>
<tr>
<td>Radius</td>
<td>$r$</td>
</tr>
<tr>
<td>Height</td>
<td>$h$</td>
</tr>
</tbody>
</table>

Table 2: Results of $V_{av}$ because of only 1% shift of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average Rate of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{av-}$</td>
<td>58.16%</td>
</tr>
<tr>
<td>$V_{av+}$</td>
<td>126.17%</td>
</tr>
</tbody>
</table>

Table 3: The smallest shift of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shift of Parameters/Average Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$-\frac{1}{3200}$ /5.42%</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>$-\frac{3}{4000}$ /5.12%</td>
</tr>
<tr>
<td>$l_x$</td>
<td>$-\frac{19}{4000}$ /5.09%</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>$+\frac{9}{16000}$ /4.59%</td>
</tr>
</tbody>
</table>

IV. TYPICAL APPLICATION OF ELECTRIC FIELD CHARACTERISTICS

As is mentioned above, in a multimode microwave heating system, the electric field is sensitive to parameters such as microwave frequency, geometrical parameters, permittivity and so on. The information of the drastic-variation condition can help us in making better use of microwave energy.

A. System design

Usually, the price of a system component increases with its accuracy. Choosing the appropriate component
according to the requirements is both economical and effective for microwave heating. For example, magnetron, as the core part of the microwave oven, can be selected according to the critical conditions obtained. If for an ordinary microwave heating with low precision requirements, the magnetron with the frequency of 2450MHz ± 50MHz is enough, there is no need to spend more money on the one with the accuracy of ± 10MHz. On the contrary, if a system requires high precision, such as in microwave chemistry, the high performance of the magnetron and the fine technics of the system should be ensured. Moreover, the influence of the attached material on the dielectric properties of the heated sample must be small enough. Therefore, in such a case, acquiring the drastic-variation condition of electric field is both important and necessary.

B. Improving heating uniformity

The instability of electric field has many possible uses in microwave engineering and has been investigated as an important factor to improve the reliability of EMC test. Similarly, new methods to improve the heating uniformity can be developed on the basis of mastering the characteristics of electric field, especially its drastic-variation condition.

One way to improve the heating uniformity is to imitate the method used in EMC test. For example, mode stirrer and turntable rotation were used to improve the uniformity of microwave heating [1]. Here is an example of the direct use of the instability of electric field to improve heating uniformity. In the system shown in Fig. 4, assume the length of \( l_x \) has a shift range of ± 1cm, which means the wall of the cavity containing \( l_x \) and \( l_y \) is moveable. In addition, we assume the wall can move back and forth with a speed of 0.5cm/s within a range of 2 cm. Other parameters are assumed the same as those of the system shown in Table 1. As is demonstrated in [20], the variation of \( l_x \) we assumed is sufficient to cause large changes in the distribution of electric field in the system we designed. Therefore, the continuous movement of the wall will stimulate different standing waves with plenty of resonant modes. On account of the heating patterns associated with resonant modes begin to overlap, a time-averaged heating results and temperature rise in the heating object tends to be more uniform in the end. Since many heated objects contain large amounts of water, and the dielectric properties of which will vary with the temperature at different locations in the objects, it can be treated as the most common inhomogeneous load during the heating process. Hence, we calculate the temperature deviation of water due to small parameter changes.

Microwave heating is a process involves multiple physics, electromagnetic in the cavity and heated samples, as well as mass and heat transport [21]. The basic equations describing the electromagnetic field distribution inside the microwave cavity is the Maxwell equation. After this equation being solved, the power dissipated inside the heated object can then be obtained by the following equation:

\[
P_d = \frac{1}{2} \varepsilon_o \omega \varepsilon''(\omega) |E|^2,
\]

where \( \varepsilon''(\omega) \) is the imaginary part of the complex permittivity including the total loss. Finally, the temperature rise can be expressed as:

\[
\rho C_p \frac{\partial T}{\partial t} - \nabla \cdot (k T \nabla T) = P_d,
\]

where \( \rho \) is density of the dielectric, \( C_p \) is the specific heat capacity, \( k_T \) is the thermal conductivity, and \( T \) is the real-time temperature.

![Fig. 4. Schematic of a moveable wall microwave heating.](image)

Comsol Multiphysics software 5.3 is also used to perform the calculation of the temperature distribution of the dielectric object heated with the fixed wall cavity shown in Fig. 1 and the moveable wall cavity shown in Fig. 4. To solve the problem regarding the dynamics of the cavity wall we use this software’s moving mesh function. In addition, we have assumed \( \rho = 1000 \text{kg/m}^3 \), \( C_p = 4180 \text{J/kg·K} \), and \( k_T = 4180 \text{W/(m·K)} \), respectively. Besides, the temperature-dependent permittivity of water is specified as [22- 23]:

\[
\varepsilon_i = \frac{3\varepsilon_{\infty} T + A (\varepsilon_{\infty} + 2)^2}{12T} + \sqrt{\left[\frac{3\varepsilon_{\infty} T + A (\varepsilon_{\infty} + 2)^2}{12T}\right]^2 + 72\varepsilon_{\infty} T^2},
\]

with

\[
A = 1186.78 \exp\left(\frac{2.88 \times 10^{21}}{kT}\right),
\]

where \( \varepsilon_{\infty} \) is the infinite frequency relative permittivity and assumed \( \varepsilon_{\infty} = 5.5 \), \( k \) is the Boltzmann’s constant. After 10s of microwave heating, their temperature distribution, coefficient of variation (COV) and temperature rise histories [5] of the central cross section are compared in Fig. 5 and Fig. 6, respectively.
Fig. 5. The temperature distribution of central cross section after 10s of microwave heating with: (a) fixed wall and (b) moveable wall.

Fig. 6. Comparison of COV and temperature rise histories, with fixed wall and moveable wall: (a) COV, and (b) temperature rise histories.

The COV in Fig. 6 is defined as [5]:

\[
COV = \sqrt{\frac{\sum (T_i - \bar{T})^2}{n \Delta T}},
\]

where \(n\) are points considered in the ample, \(\Delta T = \bar{T} - 20^\circ C\). It is well known that COV can effectively quantify the heating uniformity, and the smaller the COV is, the more uniform the heating sample is. As can be seen in Fig. 5, the temperature distribution of the object inside the moveable wall cavity is much more uniform than that of the fixed one. Due to the fact that the heating uniformity is highly dependent on the stability of electric field, Fig. 6 (a) also demonstrates that with the movement of the cavity wall, although it forms a more chaotic distribution at first, the heating uniformity can be significantly improved in the end. Moreover, Fig. 6 (b) shows that the temperature rise in the case of moveable wall is much faster than that in the case of fixed wall. This example also proves that it is feasible to make use of the instability of electric field to improve heating uniformity.

V. CONCLUSIONS

In this paper, we proposed a numerical method to study on the drastic-variation condition of electric field in a multimode microwave heating system. This method combined with integral equation and spectrum analysis. The characteristic of electric field and its dependent parameters were discussed. For microwave heating, the electric field is usually unstable, and a tiny shift in microwave frequency, permittivity of dielectric object or cavity geometrical parameters will produce a drastic variation in electric field distribution. Moreover, the smallest shift of parameter that results in a drastic variation of electric field can be obtained by the reverse search of the critical value. To make some supplements for the interest regarding practical applications, two examples of the drastic-variation condition in making better use of microwave energy were given.

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REFERENCES


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