Efficient Analysis of Object with Fine Structures by Combined MLSSM/MLFMA via Compressed Block Decomposition Preconditioner

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Abstract — A large dense fine mesh is used to model object with fine structures to guarantee good solution accuracy, and this in turn places an inordinately heavy burden on the CPU in terms of both memory requirement and computational complexity. To analyze the large dense complex linear system efficiently, the combined MLSSM/MLFMA is used to accelerate the matrix-vector multiplication. Multilevel fast multipole algorithm (MLFMA) cannot be used to analyze the box’s size of tree structure below 0.2 wavelength, because the “low frequency breakdown” phenomenon would happen. For the large-scale problems, the matrix assembly time of multilevel simply sparse method (MLSSM) is much longer than that of MLFMA. This combined method takes advantage of the virtues of both MLFMA and MLSSM, which is more efficient than either conventional MLFMA or conventional MLSSM. An efficient preconditioning technique based on compressed block decomposition (CBD) is applied to speed up the convergence rate. Numerical results are presented to demonstrate the accuracy and efficiency of the proposed method.

Index Terms — Compressed Block Decomposition (CBD) preconditioner, Multilevel Simply Sparse Method (MLSSM), object with fine structures.

I. INTRODUCTION
The increased power and availability of computational resources and acceleration schemes have enabled solution of problems with very large number of unknowns, varying from few thousands to few millions [1]. Another class of problems arises when analyzing structures which require a high local density of unknowns to accurately capture geometric features. This class of problems is referred to as object with fine structures problems exhibit multiple scales in length. For example, small length scale discretizations are required to capture sharp or fine geometric features that are embedded within large and smooth geometries discretized at a coarser length scale. Generally, the characteristic of an object with fine structures problem is the concentration of large number of unknowns in electrically small domains. All these simulations require fast and efficient numerical methods to compute an approximate solution of Maxwell’s equations. Numerous authors have derived a variety of methods that are used for computing the electromagnetic problems. The method of moments (MoM) [2-4] is one of the most widely used techniques for electromagnetic problems. However, for large-scale problems, a great number of unknowns are required for modeling objects, which always leads to intensive computation and unaffordable CPU time.

The fast algorithms are developed to reduce computational cost. The most popular fast solution include the multilevel fast multipole algorithm (MLFMA) [5-6], has \( O(N \log N) \) (where \( N \) denotes the number of unknowns) complexity for a given accuracy. Though efficient and accurate, this algorithm is highly technical. It utilizes a large number of tools, such as partial wave expansion, exponential expansion, filtering, and interpolation of spherical harmonics. However, MLFMA becomes numerically unstable and inefficient when applied to object with fine structures problems. This is a consequence of the fact that Helmholtz MLFMA does not smoothly transition to Laplace MLFMA as frequency tends to zero. Therefore, when the finest level box’s size is below 0.2 \( \lambda \) ( \( \lambda \) indicates the incident wavelength), MLFMA will suffer from “low frequency breakdown” phenomenon [1]. As a result, it cannot be easily applied to analyze the object with fine structures problems.
In recent years, the matrix decomposition technique has been introduced to analyze the electromagnetic problems, which exploits the well-known fact that for well separated sub-scatterers, the corresponding sub-matrices are low rank and can be compressed [7-8]. In contrast with MLFMA, it is purely algebraic and can be easily interfaced to existing MOM codes. MLSSM is a popular matrix decomposition technique, which has been successfully applied in [9-14] to electromagnetic problems. It has no limit of the box’s size and has a memory requirement of \(O(N)\) and computational complexity is proportional to \(O(N \log N)\). However, for the large-scale problems, the matrix filling time of MLSSM is much longer than that of MLFMA.

In this paper, a hybrid method called combined MLSSM/MLFMA is proposed, which uses the main framework of MLFMA but adopts the MLSSM to deal with the box’s size is below \(0.2 \lambda\). This method takes advantage of both MLFMA and MLSSM and is more efficient than either conventional MLFMA or conventional MLSSM for analyzing the multi-scale problems. For the object with fine structures problems, the matrix condition number is very large due to the mixed discretization. Therefore, the system has poor convergence history and requires urgently a good solver or preconditioner. In this paper, an efficient preconditioning technique based on CBD algorithm [15-17] is applied to improve the property of electric field integral equation (EFIE) formulation.

II. MLSSM ALGORITHM

The impedance matrix filled by MLSSM is carried out based on the same multilevel spatial decomposition of MLFMA. The single level of SSM is presented in [18] and the MLSSM is shown in [9-14]. The structure of the MLSSM representation is given in a multilevel recursion manner [9-14]:

\[
Z_l = \hat{Z}_l + U_l Z_{l+1} V_l^H ,
\]

where \(Z_l\) is the reduced order impedance matrix and consists of only far interactions at level \(l+1\), which will be compressed in the coarser levels recursively up to level-3. There is no level \(L+1\) near interactions at the finest level \(L\) (where \(L\) denotes the number of the levels). Thus, \(Z_l\) is the impedance matrix \(Z\). In (1), \(\hat{Z}_l\) is the sparse matrix containing all near-neighbor interactions at level \(l\) of the oct-tree which were not represented at finer level of the oct-tree. \(U_l\) and \(V_l\) are the new basis and testing function matrices, respectively, which are block diagonal unitary matrices that compress interaction between sources in non-touching groups at level \(l\). The following is the procedure in details. Suppose that the object is decomposed in 4-level oct-tree, the impedance matrix can be expressed as:

\[
Z = \hat{Z}_4 + Z_{4}^{nn} = \hat{Z}_4 + U_4 Z_4 V_4^H ,
\]

where

\[
Z_4 = \hat{Z}_4 + Z_{4}^{nn} = \hat{Z}_4 + U_4 Z_4 V_4^H .
\]

The forms of matrices \(U_l\), \(Z_{l+1}\) and \(V_l^H\) are shown in Figs. 1 and 2 at levels 4 and 3, respectively.

Fig. 1. Level 4 SSM matrices: (a): \(U_4\), (b): \(\hat{Z}_4\), and (c): \(V_4^H\).

Fig. 2. Level 3 SSM matrices: (a): \(U_3\), (b): \(\hat{Z}_2\), and (c): \(V_3^H\).

The major requirements of the MLSSM memory is to store the matrices \(\hat{Z}_l\), \(U_l\), and \(V_l^H\) at all levels. The matrix-vector multiplication of MLSSM is very similar to MLFMA in manner, which has \(O(N \log N)\) complexity for a given accuracy.

III. COMBINED MLSSM/MLFMA ALGORITHM

A. MLSSM/MLFMA algorithm

MLFMA [5-6] has been widely used to solve the electromagnetic scattering of complex object with the surface integral equation approach. When it is applied to analyze the scattering from the multi-scale objects where dense discretization is necessary to capture geometric features accurately, the memory usage of MLFMA is very large. It is because of that the box’s size of tree structure cannot be set to less than \(0.2 \lambda\), the number of unknowns in the near-field boxes is very large. The near-field matrices of MLFMA is filled by direct method. Therefore, the near-field of MLFMA needs large memory requirement for large-scale problems.

In this section, a hybrid method called combined MLSSM/MLFMA algorithm is proposed, which uses the
main framework of MLFMA. The framework of the hybrid method is shown in the Fig. 3, the box's sizes below the dotted line are less than 0.2\( \lambda \) and are filled by MLSSM algorithm, the box's sizes up the dotted line are larger than 0.2\( \lambda \) and are filled by MLFMA.

Fig. 3. The framework of combined MLSSM/MLFMA algorithm.

Using the combined MLSSM/MLFMA for filling the impedance matrix \( Z \), a fast matrix-vector production algorithm (MVP) can be obtained as follows:

Subroutine MVP,
(1) The direct MVP algorithm is used to the near interaction impedance matrix;
Begin
\( l = L : 3 : -1 \),
(2) From \( l = L : L_{\text{MFLMA}} + 1 \),
MLSSM is applied to speed up MVP;
End.
(3) From \( l = L_{\text{MFLMA}} : 3 \),
MLFMA is used to speed up MVP;
End.
End.

This new method takes advantages of the virtues of both MLFMA and MLSSM, which uses MLFMA to reduce the matrix assembly time of MLSSM and utilizes MLSSM to alleviate the near-field pressure of MLFMA. The efficiency of the method is demonstrated by the numerical results.

B. CBD preconditioner

In order to accelerate the convergence rate of the Krylov iteration, the linear system is transformed into an equivalent one:

\[
[M][Z][I] = [M][V],
\]

where \( [M] \) referred as the preconditioner for the impedance matrix \( [Z] \). The product matrix \( [M][Z] \) has much better spectral property than original matrix \( [Z] \), which leads to a greatly reduced number of iterations. Since the sparse near-field matrix is the best available approximation to the coefficient matrix \( [Z] \), it makes sense to use near-field matrix to construct a preconditioner. In this paper, an efficient CBD preconditioner [19] is applied to form the matrix \( [M] \).

IV. NUMERICAL RESULTS

In this section, a number of numerical examples are presented to demonstrate the efficiency of the proposed method for analyzing the object with fine structures. All the computations are carried out on a Core-i5 3350P with 3.1 GHz CPU and 4GB RAM in single precision and the MLSSM truncating tolerance is \( 10^{-3} \) relative to the largest singular value. In the implementation of the combined MLSSM/MLFMA algorithm, the restarted version of GMRES algorithm [20] is used as the iterative method. The iteration process is terminated when the normalized backward error is reduced by \( 10^{-3} \) for all examples.

A. A metal helicopter model

A metal helicopter model with many fine structures is considered in the first example in Fig. 4, which needs large number of unknowns to accurately capture fine structures of the helicopter model. The dimension of the structure is \( 20.48 \times 12.37 \times 6.2 \) m. The incident and scattered angles are \((\theta_i = 0, \phi_i = 0)\) and \((0 \leq \phi_s \leq 180, \theta_s = 0)\), respectively. The maximum dimension of the structure is \(10.24\lambda\) at 150 MHz and the number of unknowns is 74913. The number of the octrees for the combined MLSSM/MLFMA algorithm is \( L = 5 \), and \( L = 4 \) for the MLFMA. The finest level box's sizes of combined MLSSM/MLFMA algorithm and MLFMA are \(0.16\lambda\) and \(0.32\lambda\), respectively. The bistatic RCS of the proposed method is shown in Fig. 5, and is agreed well with that of FEKO. Table 1 shows the memory storages and the MVP times of the MLSSM/MLFMA and MLFMA. The near-field memory of MLSSM/MLFMA is much less than that of MLFMA, while the total memory consumption of MLSSM/MLFMA is half less than that of MLFMA. The MVP time of MLSSM/MLFMA is also much less than that of MLFMA. The convergence history curves of the MLSSM/MLFMA solved with CBD preconditioner are shown in Fig. 6. It can be found that the proposed method has a much better convergence properties by using the CBD preconditioner.

Fig. 4. The configuration of the metal helicopter model.
Fig. 5. Bistatic scattering cross section of metal helicopter model.

Table 1: The memory requirements and the MVP times of the proposed method for the metal helicopter model

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Near-Field (MB)</th>
<th>Total (MB)</th>
<th>MVP Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLFMA</td>
<td>1097</td>
<td>1209</td>
<td>6.07</td>
</tr>
<tr>
<td>MLSSM/MLFMA</td>
<td>273</td>
<td>465</td>
<td>2.96</td>
</tr>
</tbody>
</table>

Fig. 6. Convergence histories of the MLSSM/MLFMA solved with CBD preconditioner.

B. A metal missile model

The second multi-scale example is a complex metal missile structure and is analyzed shown in Fig. 7. The dimension of the structure is $6 \times 2.56 \times 1.32 \, \text{m}$. The incident and scattered angles are $(\theta_i = 0, \phi_i = 0^\circ)$ and $(0^\circ \leq \phi_s \leq 180^\circ, \theta_s = 0^\circ)$, respectively. The maximum dimension of the structure is $6 \, \lambda$ at 300 MHz and the number of unknowns is 67420. The number of the octrees for the MLSSM/MLFMA algorithm is $L = 4$, and $L = 3$ for the MLFMA. The finest level box’s sizes of combined MLSSM/MLFMA algorithm and MLFMA are $0.18 \, \lambda$ and $0.37 \, \lambda$, respectively. The bistatic RCS of the metal missile structure is analyzed by the proposed method shown in Fig. 8. It can be observed that the result of the proposed method is agreed well with that of FEKO. The efficiency of the proposed method is analyzed in this example shown in Table 2. It can be found that the memory storage and the MVP time of the MLSSM/MLFMA are both much less than that of the conventional MLFMA. The convergence rate of the CBD preconditioner is shown in Fig. 9. It can be found that the CBD preconditioner is much more efficient than the unpreconditioned GMRES algorithm.

Table 2: The memory requirements and the MVP times of the proposed method for the metal missile model

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Near-Field (MB)</th>
<th>Total (MB)</th>
<th>MVP Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLFMA</td>
<td>871</td>
<td>1045</td>
<td>4.52</td>
</tr>
<tr>
<td>MLSSM/MLFMA</td>
<td>209</td>
<td>362</td>
<td>1.67</td>
</tr>
</tbody>
</table>

Fig. 7. The configuration of the metal missile model.

Fig. 8. Bistatic scattering cross section of metal missile model.

Fig. 9. Convergence histories of the MLSSM/MLFMA solved with CBD preconditioner.
VI. CONCLUSION

In this paper, a novel combined MLSSM/MLFMA algorithm is proposed to solve the electromagnetic scattering of object with fine structures. It takes advantage of the virtues of both MLFMA and MLSSM, which uses MLFMA to reduce the matrix assembly time of MLSSM and utilizes MLSSM to alleviate the near interaction pressure of MLFMA. Since compression of near interactions, the matrix-vector multiplication of MLSSM/MLFMA is more efficient than that of MLFMA. The CBD preconditioner is used to further speed up the convergence. It can be found that MLSSM/MLFMA combined with CBD preconditioner is very efficient for analyzing the object with fine structures problems.

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REFERENCES


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