A Vector Parabolic Equation Method Combined with MLFMM for Scattering from a Cavity

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Abstract — The three-dimensional vector parabolic equation (PE) method was used for the fast analysis of the electromagnetic scattering with reasonable accuracy. The algorithm was marched from plane to plane along some preferred directions so that the computational load could be reduced significantly. However, large errors will be introduced for cavities with the multiple scattering. In this paper, the vector parabolic equation method combined with the multilevel fast multipole method (MLFMM) is presented for the analysis of the electromagnetic scattering from a cavity. Numerical results demonstrate that the proposed technique can efficiently give reasonably accurate results for the cavities when compared with the conventional PE method.

Index Terms — Cavity, electromagnetic scattering, MLFMM, vector parabolic equation method.

I. INTRODUCTION

The parabolic equation (PE) method provides an approximate solution for the wave equation with the energy propagating along the paraxial direction [1]. The calculation of the PE is taken to march gradually from one plane to another plane along the paraxial direction so that the 3D problem can be converted into a series of 2D problems to be solved by the PE method. Therefore, the electromagnetic scattering from electrically large objects can be analyzed efficiently. However, the PE is applicable only to the object which does not have large changes in direction, because large errors will be introduced for both nonconvex objects and cavities with the multiple scattering. The electromagnetic scattering from cavities has been paid more and more attention due to its wide range of applications in both the industry and the military. Many numerical methods, including the method of moment (MoM) [2-3], the finite difference method (FDM) [4], and the finite element method (FEM) [5-6] have been used to analyze the electromagnetic scattering from cavities. The error of each solution is large unless fine discretization is used for the scatterers. But a fine mesh will bring a large system of equations which may be computationally prohibitive. Therefore, fast and accurate calculation of the electromagnetic scattering from cavities becomes significant problem to be solved.

The MoM accelerated by the multilevel fast multipole method (MLFMM) can be used to analyze the electromagnetic scattering from cavities accurately [2]. However, a large number of computations are required for the very electrically large three-dimensional PEC objects. In this paper, the PE method is combined with MLFMM for the analysis of electromagnetic scattering from electrically large PEC objects with cavities. Firstly, the scattered electric fields in the aperture can be obtained by calculating the current density on the inner sides of the cavity with the MLFMM. Then, the PE method can be applied to analyze the scattered electric fields of other parts with the corresponding boundary conditions on the PEC objects. By this way, both the accuracy and the efficiency for electromagnetic scattering from electrically large PEC cavities can be assured.

This paper is organized as follows. Section 2 gives a brief introduction to the traditional vector parabolic equation method. At the same time, the vector parabolic equation method combined with the multilevel fast multipole method is explained in detail. A slant cavity embedded in a block is analyzed as the numerical example to show the
validity and the efficiency of the proposed method in Section 3. Section 4 gives some conclusions and comments.

II. THEORY AND FORMULATIONS
A. Vector parabolic equation method for electromagnetic scattering problems
The three-dimensional vector wave equation can be used as an efficient tool to analyze the electromagnetic scattering problems. The scattered field components $E_x, E_y, E_z$ satisfy scalar wave equation in the Cartesian coordinate as follows:
\[
\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0
\]
\[
\xi = x, y, z,
\]
where $k$ is the wave number.

The reduced scattered fields $u_x, u_y, u_z$ are defined as:
\[
u_x'(x, y, z) = e^{-i\omega t} E_x'(x, y, z)
\]
\[
\xi = x, y, z,
\]

Substitute equation (1) with equation (2), the vector parabolic equations can be gotten in air domain:
\[
\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} = \frac{1}{\omega} \nabla \times (\nabla \times u_z)
\]
\[
\xi = x, y, z.
\]

It should be noted that the first order Taylor expansions of the square root and the exponential are used in this paper. And it can be seen from the equation (3) that the solutions at $(x + \Delta x)$ plane can be calculated from those at $x$ plane. As shown in Fig. 1, the computation can start in the plane before the object and stop in the plane beyond the object.

When the FD scheme of the Crank-Nicolson type is applied to the equation (3), the computational format for the vector parabolic equations can be written as follows:
\[
\frac{\Delta x}{2jk(\Delta y)} u_x'(x + \Delta x, y, y + \Delta y, z) + \frac{\Delta x}{2jk(\Delta z)} u_x'(x + \Delta x, y, z + \Delta z, z) + (1 - \frac{\Delta x}{jk(\Delta y)} - \frac{\Delta x}{jk(\Delta z)}) u_x'(x + \Delta x, y, z, z) + \frac{\Delta x}{2jk(\Delta y)} u_x'(x + \Delta x, y - \Delta y, z) + \frac{\Delta x}{2jk(\Delta z)} u_x'(x + \Delta x, y, z - \Delta z, z) = u_x'(x, y, z) \]
\[
\xi = x, y, z.
\]

The incident field is taken into account by the boundary conditions on the scatterer in each transverse plane. Perfect matching layers (PML) are placed around the object to truncate an infinite space to a finite computation domain. For PML domain, the following coordinate transformation is introduced [7-9]:
\[
\hat{y} = y - i\frac{\sigma}{\Delta y} d\xi
\]
\[
\hat{z} = z - i\frac{\sigma}{\Delta z} d\xi
\]
\[
in equation (5), \sigma(\xi) = \frac{3}{2\delta} \frac{1}{\eta} \log\left(\frac{1}{10}\right) \left(\frac{\xi}{\delta}\right)^2,
\]
\[
\delta is the thickness of the PML and $\eta$ is the wave impedance. As in air domain, the FD scheme of the Crank-Nicolson type is used. Therefore, the equation (6) can be replaced with equation (4) for the computation in the PML domain:
\[
\Delta x e_{i,j+1/2} \frac{i\sigma}{\Delta z} u_z'(x + \Delta x, y, z, z) + \Delta x e_{i,j-1/2} \frac{i\sigma}{\Delta y} u_y'(x + \Delta x, y, z, z) + \Delta x e_{i+1/2} \frac{i\sigma}{\Delta y} u_z'(x + \Delta x, y, z, z) + \Delta x e_{i-1/2} \frac{i\sigma}{\Delta y} u_z'(x + \Delta x, y, z, z) = \frac{1}{\Delta z} \int_{\xi_{i-1/2}}^{\xi_{i+1/2}} \sigma(\xi) d\xi
\]
\[
\xi = x, y, z,
\]
where
\[
e_i = \frac{1}{1 - i\sigma(y)},
\]
\[
\sigma(y) = \frac{1}{\Delta z} \int_{\xi_{i-1/2}}^{\xi_{i+1/2}} \sigma(\xi) d\xi
\]
\[
e_j = \frac{1}{1 - i\sigma(z)},
\]
\[
\sigma(z) = \frac{1}{\Delta z} \int_{\xi_{j-1/2}}^{\xi_{j+1/2}} \sigma(\xi) d\xi
\]

The narrow-angle approximation is only accurate when energy does not have large changes along the paraxial direction. As a result, the traditional parabolic equation method cannot be used to analyze the scattering from cavities.
B. Vector parabolic equation method combined with the MLFMM for scattering from a cavity

As shown in Fig. 2, a PEC object with cavity in free space is illuminated by a plane wave \( e^{j\omega t} \) propagating along \(-x\) direction with wave number \( k \).

The scattering by 3-D arbitrary conducting objects can be formulated by the electrical-field integral equation (EFIE). As shown in Fig. 2, the current density on the inner sides of the cavity can be calculated with the equation (11) by the MLFMM

\[
\begin{align*}
-ikn(r) \times \int_{S_{\text{inner}}} G(r, r') \mathbf{J}(r') dS' \\
+ \frac{1}{ik} \nabla \int_{S_{\text{inner}}} G(r, r') \nabla \mathbf{J}(r') dS' \\
= -n(r) \times \mathbf{E}^i(r) & \quad \forall r \in S_{\text{inner}},
\end{align*}
\]

where \( S_{\text{inner}} \) denotes the surface of the inner sides in the cavity, \( \mu_0 \) and \( \varepsilon_0 \) are the free-space permittivity and permeability, \( k = \omega \sqrt{\mu_0 \varepsilon_0} \) is the wave number, \( \mathbf{J} \) is the current density in the inner sides of the cavity, \( \mathbf{E}^i(r) \) is the incident plane wave, \( n \) denotes the surface outward pointing unit normal and \( G(r, r') \) is the Green’s function in free space.

For the aperture, the scattered electric fields on \( S_2 \) can be obtained by the integration of the electric current on the inner sides \( S_{\text{inner}} \) of the cavity, as written in equation (12).

\[
\mathbf{E}'(r) = -\frac{i\omega \mu_0}{4\pi} \int_{S_{\text{inner}}} \left( I - \frac{1}{k^2} \nabla \nabla' \right) G(r, r') \cdot \mathbf{J}(r') dS' \quad \forall r \in S_2.
\]

To efficiently analyze the electromagnetic scattering from PEC objects with cavities, the vector PE method is introduced to calculate the scattered electric fields from the surface \( S_1 \) and the aperture surface \( S_2 \) with equation (3).

The FD scheme of the Crank-Nicolson type is used to the equation (3), as shown in equation (4). Therefore, the \( u_z^i(x+\Delta x, y, z) \) at \( (x+\Delta x) \) plane can be calculated from \( u_z^i(x, y, z) \) at \( x \) plane [1]. The tangential electric field must be zero on the scatterer for a PEC object. To guarantee the unicity of the solution, the reduced scattered fields in equation (3) are coupled through both the boundary conditions on the surface \( S_1 \) and the divergence-free condition [1], as shown in equation (13):

\[
\begin{align*}
& \left\{ n_x u_z^i(P) - n_y u_y^i(P) = -e^{-ikx} \left( n_x E_x^i(P) - n_y E_y^i(P) \right) \\
& \quad n_x u_z^i(P) - n_y u_y^i(P) = -e^{-ikx} \left( n_x E_x^i(P) - n_y E_y^i(P) \right) \\
& \quad n_x u_z^i(P) - n_y u_y^i(P) = -e^{-ikx} \left( n_x E_x^i(P) - n_y E_y^i(P) \right) \\
& \quad \frac{i k u_y^i(P) + \frac{\partial u_y^i(P)}{\partial y} + \frac{\partial u_z^i(P)}{\partial z}}{2k} = 0
\end{align*}
\]

where \( P \) is a point on the surface of the scatterer, \( (E_x^i, E_y^i, E_z^i) \) is the field components of the incident wave and \( (n_x, n_y, n_z) \) is the outer normal to the surface at \( P \).

For the last transverse \((y-z)\) plane, the boundary
conditions for the equation (3) on both the PEC surface \( S_1 \) and the aperture \( S_2 \) are obtained by equation (13) and (12), respectively. The computation for equation (3) is taken from one plane to another, which begins just before the PEC scattering object and stops beyond it. Once the scattered electric fields are obtained in the last transverse plane, the RCS can be calculated by Fourier transform of them [1].

![PEC object with cavity in free space](image1)

**Fig. 2. A PEC object with cavity in free space.**

### III. NUMERICAL RESULTS

As shown in Fig. 3, a slant cavity nested in a block is presented to demonstrate the efficiency of the proposed method. The incident field is a plane wave, vertical polarized and propagating along \(-x\) direction. The RCS curves are compared between the proposed method and the MLFMM in Fig. 4 for a wide band from 200 to 600 MHz. It can be observed that the proposed method is in good agreement with the MLFMM. As shown in Fig. 5, the bi-static RCS results at the frequency of 300 MHz are compared among the proposed method, the conventional PE method and the MLFMM. Moreover, the amplitude and the phase of the scattering electric fields for the cavity’s middle line paralleled to the \( y \) axis in the last transverse plane are given in Fig. 6. It can be found that the proposed method is more accurate than the conventional PE method when compared with the MLFMM. As listed in Table 1, both the CPU time and the memory requirement are compared between the proposed method and the MLFMM at the frequency of 300 MHz. It can be found that memory requirement and CPU time can be reduced 71.9% and 43.6% for the proposed method, respectively.

![Model of the slant cavity nested in a block](image2)

**Fig. 3. The model of the slant cavity nested in a block: (a) stereogram, and (b) side view.**

![RCS from 200 MHz to 600 MHz](image3)

**Fig. 4. The RCS from 200 MHz to 600 MHz.**
Fig. 5. The bi-static RCS of the cavity at 300 MHz.

Table 1: Comparison for the CPU time and the memory requirement between the proposed method and the MLFMM at the frequency of 300 MHz

<table>
<thead>
<tr>
<th></th>
<th>CPU Time (s)</th>
<th>Memory Requirement (MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLFMM</td>
<td>1184</td>
<td>448</td>
</tr>
<tr>
<td>The proposed method</td>
<td>668</td>
<td>126</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

In this paper, a vector parabolic equation method combined with the MLFMM has been proposed to analyze the electromagnetic scattering from a PEC cavity. The MLFMM is introduced to compute the scattered currents on the inner side of the cavity and the electric fields in the aperture induced by these scattered currents are obtained. The scattered fields from the cavity are calculated by the vector PE method with the corresponding boundary conditions. It can be found that both the CPU time and the memory requirement are significantly reduced when compared with the MLFMM.

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