Modeling of Anisotropic Magnetic Objects by Volume Integral Equation Methods

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Abstract — This paper presents the modeling of electromagnetic scattering from objects with magnetic anisotropy. We study the solutions of both the volume integral equation (VIE) method and augmented volume integral equation (A-VIE) method. For the VIE method, it is built from the 3D vector wave equation for electric field only. For the A-VIE method, it is built from 3D vector wave equation for both electric and magnetic fields. Numerical results show that the A-VIE method has better accuracy and convergence for magnetic objects compared to the VIE method.

Index Terms — Anisotropic magnetics, augmented volume integral equation (A-VIE), method of moments, volume integral equation (VIE).

I. INTRODUCTION

The solution of electromagnetic wave scattering and propagation problems from penetrable objects has always been an active research area. In early years, approximate methods such as the geometrical theory of diffraction were used [1]. The extended boundary condition method was also investigated as a possible approach to solve such problems [2]. More recently, numerical methods have been adopted to tackle it such as finite difference method [3], finite element method [4], generalized multipole method [5] and method of moments [6-7]. Among them, method of moments equipped with modern computing power and fast algorithms provides an accurate and efficient numerical method for solving the scattering problems. There are two main categories of moment methods for penetrable objects. One is the surface integral equation based method, in which the unknown parameters are defined on the surfaces of the objects [8-9] and it is efficient in solving problems with piecewise homogenous properties. The other is the volume integral equation based method [10-12], which can solve the scattering problems from highly inhomogeneous scatterers.

Early research in analyzing wave scattering and propagation from penetrable objects mainly focuses on objects with isotropic material properties. With the revolution and development of new materials and technologies, modeling of 3D objects with generalized anisotropy has become of great interest in research. The applications of anisotropic materials cover a wide range from electromagnetic and optical design to geophysical exploration. In oil and gas exploration, with the development of highly deviated and horizontal drilling technology, formation anisotropy has become an important concern. Without consideration of the anisotropy effect in the modeling and inverse algorithms, it is difficult to interpret the measurements from modern logging tools. On the other hand, the incorporation of metamaterials and artificial materials in the electromagnetic and optical device design, induces a great need for the accurate and efficient electromagnetic solvers to model generalized anisotropic materials. In recent years, various volume integral equation methods have been proposed to solve scattering problems for anisotropic materials [13-18]. Analytical solutions for simple structures such as anisotropic spheres or spherical shells have been investigated in [20-21]. Most of these methods mainly focus on modeling of uniaxially anisotropic objects and the investigations mainly focus on dielectrics. Investigations for generalized anisotropic materials, especially anisotropic magnetics are still limited.

In this paper, the VIE method and A-VIE method with curl-conforming bases are applied to model the scattered fields of magnetic objects. The contributions of this work are twofold: i) convergence studies of the VIE method and A-VIE method for magnetic objects are presented; ii) it is demonstrated that the A-VIE method has better convergence and accuracy for magnetic objects.
II. VIE AND A-VIE METHODS

Consider a 3D inhomogeneous and anisotropic object in free space with relative permittivity and permeability \( \varepsilon_r (r) \) and \( \mu_r (r) \). The volume of the anisotropic object is denoted as \( V \) and it is enclosed by the surface \( S \). We assume that the object is excited by an incident plane wave characterized by \((E^{inc}(r), H^{inc}(r))\).

To solve the scattered field of the anisotropic object, a VIE method built from the 3D vector wave equation has been introduced in [16]. It is derived from the 3D vector wave equation for the electric field given by:

\[
\nabla \times \bar{\mu}^{-1} \nabla \times \mathbf{E}(\mathbf{r}) - \omega^2 \varepsilon_r (\mathbf{r}) \mu_0 \sigma_0 \mathbf{E}(\mathbf{r}) = i \omega \mu_0 \mathbf{J}(\mathbf{r}). \tag{1}
\]

Here, \( \mathbf{J}(\mathbf{r}) \) is the current that produces the incident field, \( \mathbf{E}(\mathbf{r}) \) is the total electric field, \( \varepsilon_r (\mathbf{r}) \) and \( \mu_r (\mathbf{r}) \) are the relative permittivity and permeability tensors, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space.

From the equation above and the definition for dyadic Green’s function, we can get the volume integral equation:

\[
\mathbf{E}(\mathbf{r}) = E^{inc}(\mathbf{r}) + \nabla \times \int_{V'} \mathbf{g}(\mathbf{r}, \mathbf{r'}) \left[ \mathbf{I} - \bar{\mu}^{-1}(\mathbf{r'}) \right] \nabla \times \mathbf{E}(\mathbf{r'}) d\mathbf{r'} - k_0^2 \int_{V'} \bar{G}(\mathbf{r}, \mathbf{r'}) \cdot \left[ \mathbf{I} - \bar{\varepsilon}(\mathbf{r'}) \right] \mathbf{E}(\mathbf{r'}) d\mathbf{r}'. \tag{2}
\]

Here, \( V' \) represents the volume that is slightly larger than the volume of the object \( V \), \( \bar{G}(\mathbf{r}, \mathbf{r'}) \) is the dyadic Green’s function for the unbounded and homogeneous media. It is a 3 by 3 matrix given by:

\[
\bar{G}(\mathbf{r}, \mathbf{r'}) = \left[ \mathbf{I} + \frac{\nabla \nabla}{k_0^2} \right] \mathbf{g}(\mathbf{r}, \mathbf{r'}). \tag{3}
\]

or

\[
\bar{G}(\mathbf{r}, \mathbf{r'}) = \frac{1}{k_0^2} \left[ \nabla \nabla \times \nabla \mathbf{g}(\mathbf{r}, \mathbf{r'}) - \mathbf{I} \delta(\mathbf{r}, \mathbf{r'}) \right], \tag{4}
\]

where \( \mathbf{g}(\mathbf{r}, \mathbf{r'}) \) is the scalar Green’s function, \( k_0 \) is the wave number in free space.

By substituting the definitions of the dyadic Green’s function into (2), we can get two sets of volume integral equations [16]. Then by discretizing the volume object using a sum of tetrahedra and expanding the total electric field \( \mathbf{E}(\mathbf{r}) \) using the edge basis on each edge of the tetrahedron, the volume integral equation can be converted into the discrete form. Using the Galerkin’s method, we can convert the discretized volume integral equation to a linear matrix equation. By solving this equation using the iterative method, the total electric field in the whole solution domain can be obtained.

The VIE method presented in [16] serves as an efficient method to model the scattering problem of generalized anisotropic materials. However, further study shows that due to the curl operator acting on the electric field in (2), the permeability term is not represented as well as the permittivity term. In order to overcome this, we use the similar idea for the augmented EFIE (A-EFIE) method [19] and apply it to the VIE method. In A-VIE, the magnetic field unknowns are added to the original VIE method that is based on the electric field. Hence, the permeability term is better represented compared to the original VIE method. Next, the A-VIE formulations are introduced [17]. First, by substituting Faraday’s law into (2), we get:

\[
-E^{inc}(\mathbf{r}) = -\mathbf{E}(\mathbf{r}) + ik_0 \nabla \times \int_{V'} \mathbf{g}(\mathbf{r}, \mathbf{r'}) \left[ \bar{\mu}(\mathbf{r'}) - \mathbf{I} \right] \mathbf{H}(\mathbf{r'}) d\mathbf{r'} \tag{5}
\]

\[
-k_0^2 \int_{V'} \bar{G}(\mathbf{r}, \mathbf{r'}) \cdot \left[ \mathbf{I} - \bar{\varepsilon}(\mathbf{r'}) \right] \mathbf{E}(\mathbf{r'}) d\mathbf{r}'. \tag{6}
\]

And using the duality principle, we get a dual equation for (5):

\[
-\eta_0 \mathbf{H}^{inc}(\mathbf{r}) = -\eta_0 \mathbf{H}(\mathbf{r}) - ik_0 \nabla \times \int_{V'} \mathbf{g}(\mathbf{r}, \mathbf{r'}) \left[ \bar{\varepsilon}(\mathbf{r'}) - \mathbf{I} \right] \mathbf{E}(\mathbf{r'}) d\mathbf{r'} \tag{7}
\]

\[
-k_0^2 \int_{V'} \bar{G}(\mathbf{r}, \mathbf{r'}) \cdot \left[ \mathbf{I} - \bar{\mu}(\mathbf{r'}) \right] \eta_0 \mathbf{H}(\mathbf{r'}) d\mathbf{r'}. \tag{8}
\]

We see that in (5) and (6), the curl operator acting on the electric field is removed and replaced by the magnetic field in the solution domain. Here, \( \eta_0 \) is used as a normalization factor for the magnetic field, it is the intrinsic impedance of free space.

To solve the A-VIE in (5) and (6) by the moment method, we need to convert it into a set of linear algebraic equations. First we expand the electric field \( \mathbf{E}(\mathbf{r}) \) and magnetic field \( \mathbf{H}(\mathbf{r}) \) into discretized forms using the edge bases:

\[
\mathbf{E}(\mathbf{r}) = \sum_{i=1}^{N_e} J_i \mathbf{N}_i(\mathbf{r}), \mathbf{r} \in V, \tag{9}
\]

\[
\mathbf{H}(\mathbf{r}) = \frac{1}{\eta_0} \sum_{i=1}^{N_e} \eta_i J_i \mathbf{N}_i(\mathbf{r}), \mathbf{r} \in V. \tag{10}
\]

Here, \( \mathbf{N}_i(\mathbf{r}) \) is the basis function on the \( i \)-th edge, \( I_i \) and \( J_i \) are the expansion coefficients for the electric and magnetic field respectively. \( N_e \) is the total number of the edge bases. \( V \) is the support of the object. The summation in the above includes an assembly process as in the FEM. That is using the fact that tangential \( \mathbf{E} \) and \( \mathbf{H} \) are continuous from element to element, \( I_i \) and \( J_i \) from contiguous elements are the same.

Next by inserting (7) and (8) into (5) and (6), testing them with \( \mathbf{N}_j(\mathbf{r}) \) and integrating over the tetrahedral element that \( \mathbf{N}_j \) is defined on, we can get the matrix representation of the augmented volume integral equation:

\[
\begin{bmatrix}
\mathbf{e}^{inc} \\
\mathbf{h}^{inc}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{Z}_{ee}^{inc} & \mathbf{Z}_{eh}^{inc} \\
\mathbf{Z}_{he}^{inc} & \mathbf{Z}_{hh}^{inc}
\end{bmatrix}
\begin{bmatrix}
\mathbf{I} \\
\mathbf{J}
\end{bmatrix}, \tag{11}
\]
where the matrix element in each block is given by:
\[
(\mathbf{Z}_{el})_{ij} = (\mathbf{Z}_{el})_{ii} + (\mathbf{Z}_{el})_{ji} = -\left\{ \mathbf{N}_j(r), \mathbf{N}_i(r) \right\} \\
-k_0^2 \left\{ \mathbf{N}_j(r), \mathbf{G}(r, r') \left[ \mathbf{I} - \mathbf{\bar{e}}(r') \right], \mathbf{N}_i(r') \right\},
\]
\[
(\mathbf{Z}_{eh})_{ij} = (\mathbf{Z}_{eh})_{ii} + (\mathbf{Z}_{eh})_{ji} = ik_0 \left\{ \mathbf{N}_j(r), \nabla \times \mathbf{g}(r, r') \left[ \mathbf{\bar{B}}(r') - \mathbf{I} \right], \mathbf{N}_i(r') \right\},
\]
\[
(\mathbf{Z}_{he})_{ij} = (\mathbf{Z}_{he})_{ii} + (\mathbf{Z}_{he})_{ji} = -\left\{ \mathbf{N}_j(r), \mathbf{N}_i(r) \right\} \\
-k_0^2 \left\{ \mathbf{N}_j(r), \mathbf{G}(r, r') \left[ \mathbf{I} - \mathbf{\bar{B}}(r') \right], \mathbf{N}_i(r') \right\},
\]
and the \( j \)-th elements of the incident vectors \( \mathbf{e}^{inc} \) and \( \mathbf{h}^{inc} \) are written as:
\[
e^{inc}_j = -\left\{ \mathbf{N}_j(r), \mathbf{E}^{inc}(r) \right\},
\]
\[
h^{inc}_j = -\left\{ \mathbf{N}_j(r), \mathbf{H}^{inc}(r) \right\}.
\]

In the above, \( i = 1, \ldots, N_e \), \( j = 1, \ldots, N_e \), \( \mathbf{I} \) and \( \mathbf{J} \) are the vectors of length \( N_e \) for the expansion coefficients \( I_i \) and \( J_i \) respectively.

### III. NUMERICAL EXAMPLES

The first example is used to show the convergence performance of the original VIE method for the permittivity term. We calculate the RCS of a sphere with radius of 0.2\( \lambda \) and material properties of \( \varepsilon_r = 2.2, \mu_r = 1.0 \) using different mesh densities. The sphere is placed in free space and illuminated by a \( \theta \)-polarized plane wave. The observation points are at \( \theta = [0^\circ, 180^\circ] \) and \( \phi = 0^\circ \). The iterative method GMRES (generalized minimum residue method) is applied to solve the final matrix equation. Figure 1 (a) shows the RCS plots with different mesh densities compared to the Mie analytical result. It is seen that the RCS converges to the analytical result as the mesh density increases. Figure 1 (b) shows the convergence of the RCS error. As the mesh density increases, the error of RCS decreases fast. Hence, the original VIE method has good convergence performance for the \( \varepsilon_r \) term.

The second example is to show the convergence performance of the original VIE method for the permeability term. We calculate the RCS for a sphere with material properties of \( \mu_r = 2.2, \varepsilon_r = 1.0 \). The radius of the sphere is 0.15\( \lambda \). It is excited by a \( \phi \)-polarized plane wave in free space. We calculate the RCS results using the same mesh densities as those in the first case. Figure 2 shows the RCS and convergence results. We can see that the RCS result converges to the analytical value slowly as the mesh density increases compared to the first example. Next, we show the convergence performance of the A-VIE method for the permeability term. We calculate the RCS results by the A-VIE method using different meshes for the same sphere as in the second example. It is shown in Fig. 3 that the RCS results by the A-VIE method have better accuracy and convergence performance than those by the VIE method.

![Fig. 1. Convergence of RCS for different mesh densities for the sphere of \( \varepsilon_r = 2.2, \mu_r = 1.0 \) and \( r = 0.2\lambda \) by original VIE method.](image-url)
The third example is the scattered problem of an anisotropic spherical shell using the A-VIE method. The electric dimension of the inner and outer spherical surfaces are $k_0a_1 = 0.6\pi$ and $k_0a_2 = 1.2\pi$, where $k_0$ is the wave number in the free space, $a_1$ is the inner radius of the spherical shell and $a_2$ is the outer radius of the spherical shell. It is placed in the free space and the incident $E$-field is $\hat{x}$ polarized propagating in $+\hat{z}$ direction. In order to test the accuracy of the $\mu_r$ term for the general anisotropic case, we consider the permeability of the sphere as a 3 by 3 matrix:

$$
\begin{pmatrix}
0 & 2.5 & 0 \\
2.5 & i & 0 \\
0 & 0 & 1.5
\end{pmatrix}
$$

$\epsilon_r$ is an identity. The mesh includes 3,354 tetrahedra and 4,824 edge elements. Figure 4 shows the RCS result of the spherical shell in the H-plane. We can see that it agrees well with the result of the duality case shown in Ref. [21] by the analytical method. Figure 5 shows the error convergence of GMRES method. We see that the matrix solution takes 24 steps to converge to $10^{-3}$ by GMRES method.
Finally, we show an example for a coated magnetic sphere using the A-VIE method. The dimensions of the inner and outer spheres are 0.9 m and 1 m as in Fig. 6. It is placed in the free space and the incident wave is \( \phi \)-polarized. The frequency of incident plane wave is 0.02 GHz. We consider the permeability of the inner and outer sphere are 3.0 and 10.0 respectively. The mesh includes 2,283 tetrahedrons and 3,798 edges. Figure 7 shows the RCS result compared to Mie series result. It can be seen the result from the A-VIE method agrees well with that of Mie series.

**Fig. 6. Coated sphere.**

**Fig. 7. RCS of the A-VIE method for the coated magnetic sphere with inner and outer radii of \( r_1 = 0.9 \text{ m} \) and \( r_2 = 1.0 \text{ m}, \mu_{r_1} = 3.0 \) and \( \mu_{r_2} = 10.0 \).**

### IV. CONCLUSION

A convergence study for magnetic objects by the VIE and A-VIE methods is discussed. The VIE method originally proposed in [16] is based on vector wave equation for electric field, and the A-VIE method is based on vector wave equations for both electric and magnetic fields. Compared to the VIE method, the A-VIE method has improved accuracy and convergence for the permeability term by removing the differential operations of the electric field in the original formulation. Numerical results show the accuracy of the RCS results for anisotropic magnetics illuminated by the plane waves.

### REFERENCES


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