Optimization of Electromagnetics Problems Using an Improved Teaching-Learning-Based-Optimization Technique

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Abstract — Teaching-learning-based optimization (TLBO) is a rising star technique among metaheuristic techniques with highly competitive performance. This technique, which has been recently introduced, is based on the effect of influence of a teacher on learners and learners on their colleagues. This paper intends to apply an improved version of TLBO in the field of electromagnetics. To demonstrate its effectiveness in this area, the proposed technique is applied to two benchmarks related to brushless direct current wheel motor problem and testing electromagnetic analysis methods problem number 22. The quality of the results presented shows that the proposed technique is very competitive with other well-known optimization techniques; hence, it is a promising alternative technique for optimization in the field of electromagnetics.

Index Terms — electromagnetics, metaheuristics, optimization, teaching-learning-based-optimization.

I. INTRODUCTION

Optimization is a commonly encountered mathematical problem in all engineering disciplines where the efficient and effective design of products and systems is crucial [1]. It is even so in the field of electromagnetics, where the designers are faced with the challenge of optimizing ever more complex components, devices and systems [2], [3], [4]. Many electromagnetic problems require optimization; some examples are electrical machines design, antenna design, target image reconstruction, anti-reflective coating design for low radar cross section and microstrip filters design.

Recently, a new metaheuristic that is the teaching-learning-based optimization (TLBO) technique has been introduced. This technique has not yet received much attention by the electromagnetic optimization community. This technique is mainly based on the effect of influence of a teacher on learners and the interaction between learners themselves.

The main contribution of this paper is developing and applying an improved version of the TLBO technique to solve electromagnetics optimization problems (this will be referred to as ITLBO). Furthermore, in order to assess the performance of the developed technique, it is compared with that of other techniques provided in the literature.

The remainder of this paper is organized as follows. Section 2 introduces the concept and main features of the TLBO technique and the improvements that have been made. In Section 3, the brushless DC wheel motor (BLDC) and testing electromagnetic analysis methods (TEAM) number 22 benchmarks are presented. In Section 4, ITLBO is tested on the considered benchmarks and the results detailed. Finally, the paper conclusions are drawn in Section 5.

II. TEACHING-LEARNING-BASED-OPTIMIZATION

Design optimization process comprises three elements: objective functions, feasible solutions and optimization methods [5]. The optimization method searches for the optimal design among all available feasible designs. Generally, nature-inspired heuristic optimization methods seem to work better than traditional (deterministic) methods, and hence, are widely used [5]. Among all nature-inspired techniques, genetic algorithm (GA) is the most widely used which provides a near optimal solution for a complex design problem with large number of variables and constraints [5]. However, the algorithm performance is affected by its specific control parameters [6]. This triggers the need for parameter-free optimization techniques where no algorithm parameters are required.

TLBO is a parameter-less metaheuristic technique introduced recently by Rao and colleagues [5]. In contrast with the other techniques, TLBO only requires such
controlling parameters as population size and maximum number of iterations for its operation [6]. Moreover, TLBO outperforms some other widespread metaheuristics with regard to constrained benchmark functions, constrained mechanical design, and continuous non-linear numerical optimization problems [7].

TLBO is a population-based optimization technique that uses a population of solutions to advance to the global solution [5], [8]. The technique is based on the principle of sharing knowledge by a teacher with his students in a classroom environment (i.e., Teacher Phase) and then sharing knowledge by learners with their classmates (i.e., Learner Phase) [7]. Therefore, TLBO works on the influence of a teacher on learners and influence of learners on their colleagues. The influence is usually manifested by the learners’ results or outcomes. Better results of a class are typically represented by the students’ mean grade. In general, the teacher attempts to distribute knowledge among learners to increase their knowledge level and help them enhance their grades. Consequently, the teacher will increase the mean grade of the class according to his capability. On the other hand, students will not only gain knowledge based on teaching quality, but also on the quality of students sitting in the class. Quality of the students is assessed through the mean value of the population. Moreover, the teacher puts effort to increase the mean of students to a higher level, at which students will require another teacher of better quality to teach them [5]. The TLBO algorithm is given in Algorithm 1.

Algorithm 1: TLBO pseudocode.
1 n: dimension of the problem
2 m: population size
3 MAXITER: maximum number of iterations
4 Initialization()
5 while ITER<MAXITER
6     Elite ← SelectBest(P,Elite)
7     for i=1:m
8         TF=round(1+rand)
9         X_mean ← mean(X_i)
10        X_teacher ← best(X_i)
11        X_new,i = X_i + rand ∙ (X_teacher - X_mean)
12        if f(X_new,i) < f(X_i)
13            X_i ← X_new,i
14        end if
15        j ← randi(m)
16        if j≠i
17            if f(X_i) < f(X_j)
18                X_new,j = X_j + rand ∙ (X_j - X_i)
19            else
20                X_new,j = X_i + rand ∙ (X_j - X_i)
21            end if
22        end if
23        if f(X_new,i) < f(X_i)
24            X_i ← X_new,i
25        end if
26    end for
27    P ← ReplaceWorstWithElite(P,Elite)
28    P ← RemoveDuplicateIndividuals(P)
29    ITER = ITER +1
30 end while

As aforesaid, the process of TLBO is divided into two phases namely: the ‘Teacher Phase’ and the ‘Learner Phase’.

In the teacher phase, consider $M_i$ as the mean and $T_i$ as the teacher at any iteration $i$, $T_i$ will try to move mean $M_i$ towards its own (new) level denoted by $M_{new}$. The solution is modified according to the difference between the existing and the new mean given by:
\[
\text{Difference}_\text{Mean}_i = r_i(M_{new} - T_i M_i),
\]
(1)
where $T_F$ is a teaching factor that decides the value of mean to be changed, and $r_i$ is a random number in the range $[0,1]$. $T_F$ has the value of either 1 or 2, which is a heuristic step and decided randomly with equal probability as $T_F = \text{round}(1 + \text{rand}(0, 1))[2 - 1]$. This difference updates the existing solution using the following expression:
\[
X_{new,i} = X_{old,i} + \text{Difference}_\text{Mean}_i.
\]
(2)

In the learner phase, a learner interacts randomly with other learners through group discussions, presentations, formal communications, etc. A learner increases his knowledge if the other learner is more knowledgeable than him. To express the learner modification, if $X_i$ and $X_j$ are two different learners ($i ≠ j$), and $X_i$ is more knowledgeable than $X_j$, then:
\[
X_{new,i} = X_{old,i} + r_i(X_j - X_i).
\]
(3)

On the contrary, if $X_i$ is more knowledgeable than $X_j$, then:
\[
X_{new,i} = X_{old,i} + r_i(X_i - X_j).
\]
(4)

In order to implement the TLBO for optimization problem, the following five steps are required: 1) Define the optimization problem and initialize the optimization parameters; 2) Initialize the population; 3) Apply the teacher phase; 4) Apply the learner phase; and 5) Apply the termination criterion.

In the first step, the population size ($m$), the maximum number of iterations (MAXITER), the number of design variables ($n$) need to be initialized, and design variables limits defined ($U_i, L_i$). Moreover, the optimization problem should be defined as: minimize $f(X)$, where $f(X)$ is the objective function, $X$ is a vector for design variables such that $L_i ≤ x_i ≤ U_i$, and $X_i \in X_i = 1, 2, \ldots, n$.

In the second step, a random population is generated according to the population size and number of design variables.

In the third step, mean of the population is calculated to give the mean for the particular subject as:
\[
M_\beta = [m_1, m_2, \ldots, m_n].
\]
(5)
For each iteration, the best solution acts as a teacher; that is:

\[ X_{\text{teacher}} = X_i^*(x) = \min \]  

The teacher will try to move the mean value towards \( X_{\text{teacher}} \), which will act as a new mean for the iteration: see (1).

The value of \( T_i \) is selected as 1 or 2. The obtained difference is added to the current solution to update its values using the relation in (2). \( X_{\text{new}} \) is then accepted if it gives better function value, and so on.

In the fourth step, learners increase their knowledge through their interaction with their colleagues: see (3) and (4).

In the fifth step, if the stopping criterion is achieved, for example the maximum number of iterations, the whole process is stopped; otherwise, it will be repeated from the third step, and so on.

### III. THE IMPROVED TEACHING-LEARNING-BASED-OPTIMIZATION

In the improved version of the TLBO (i.e., ITLBO), the lines between 16 – 25 in Algorithm 1 are replaced by the lines between 16 – 31 in Algorithm 2. The objective of introducing this improvement is to test the new vector \( X_{\text{new}} \) after every update in the \( k \) dimension of this vector \( (k = 1: n) \). This improvement is inspired, somehow, from the differential evolution algorithm where only one dimension (not all the vectors) is updated at a time.

Algorithm 2: ITLBO pseudocode.

1. \( n \): dimension of the problem
2. \( m \): population size
3. MAXITER: maximum number of iterations
4. Initialization()
5. while ITER<MAXITER
6. \( \text{Elite} \leftarrow \text{SelectBest(P,Elite)} \)
7. for \( i=1:m \)
8. \( T_i=\text{round}(1+\text{rand}) \)
9. \( X_{\text{mean}} \leftarrow \text{mean}(X_i) \)
10. \( X_{\text{teacher}} \leftarrow \text{best}(X_i) \)
11. \( X_{\text{new},i} = X_i + \text{rand} \cdot (X_{\text{teacher}} - X_{\text{mean}}) \)
12. if \( f(X_{\text{new},i})<f(X_i) \)
13. \( X_i \leftarrow X_{\text{new},i} \)
14. end if
15. \( j \leftarrow \text{randi}(m) \)
16. if \( j \neq i \)
17. if \( f(X_i)<f(X_j) \)
18. for \( k=1:n \)
19. \( X_{\text{new},i} = X_i(k) + \text{rand} \cdot (X(k) - X_i(k)) \)
20. if \( f(X_{\text{new},i})<f(X_i) \)
21. \( X_i(k) \leftarrow X_{\text{new},i}(k) \)
22. end if
23. end for
24. for \( k=1:n \)
25. \( X_{\text{new}}(k) = X_i(k) + \text{rand} \cdot (X(k) - X_i(k)) \)
26. if \( f(X_{\text{new}}(k))<f(X_i) \)
27. \( X_i(k) \leftarrow X_{\text{new}}(k) \)
28. end if
29. end for
30. end if
31. end if
32. end for
33. \( P \leftarrow \text{RemoveWorstWithElite}(P,\text{Elite}) \)
34. \( P \leftarrow \text{RemoveDuplicateIndividuals}(P) \)
35. ITER = ITER +1
36. end while

### IV. APPLICATIONS

#### A. BLDC benchmark

The application of TLBO in electromagnetics optimization is first illustrated on the BLDC benchmark designed for a race solar vehicle. This benchmark is presented in [9]. The authors proposed a benchmark with five design variables and one objective function to be maximized, that is the efficiency (which is equivalent to minimizing the motor losses) [10]. Figure 1 shows the prototype of the motor and Table 1 summarizes the five design variables with mapping ranges used in this study. In addition to the constraints imposed on the design variables, the problem is subject to six inequality constraints. These last constraints are related to technological and operational considerations regarding the specific wheel motor [10]. Thus, the optimization problem can be formulated as follows:

\[ \text{OF} = -\eta. \]  

Subject to:

\[ M_{\text{tot}} \leq 15 \text{ [kg]}, \]  

\[ D_{\text{ext}} \leq 340 \text{ [mm]}, \]  

\[ D_{\text{int}} \geq 76 \text{ [mm]}, \]  

\[ I_{\text{max}} \geq 125 \text{ [A]}, \]  

\[ \text{discr}(D_s, \delta, B_s, B_t) \geq 0, \]

where, \( \text{OF} \) is the objective function (the minus sign is for transforming the minimization problem to a maximization one), \( \eta \) is the efficiency, \( M_{\text{tot}} \) is the total mass of the active parts, \( D_{\text{ext}} \) is the outer diameter, \( D_{\text{int}} \) is the inner diameter, \( I_{\text{max}} \) is the current in the phases and \( \text{discr}(D_s, \delta, B_s, B_t) \) is the determinant used for the calculation of the slot height. One of the main advantages of the BLDC benchmark is the availability of the source code to compute the objective function and constraints. Thus, the comparison of optimization results for different techniques is independent of differences in the calculation. These features make this benchmark ideal for comparing the performance of different techniques.
Fig. 1. Prototype of the wheel motor. The inner stator is visible with the coils rolled up around the teeth [9].

Table 1: Design variables and their ranges for the BLDC benchmark

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ds [mm]</td>
<td>Bore (stator) diameter</td>
<td>150</td>
<td>330</td>
</tr>
<tr>
<td>Bg [T]</td>
<td>Air gap induction</td>
<td>0.50</td>
<td>0.76</td>
</tr>
<tr>
<td>(\delta) [A/mm²]</td>
<td>Conductor current density</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Bd [T]</td>
<td>Teeth magnetic induction</td>
<td>0.9</td>
<td>1.8</td>
</tr>
<tr>
<td>Bcs [T]</td>
<td>Stator back iron induction</td>
<td>0.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

B. TEAM22 benchmark

The TEAM Workshop Problem 22 or TEAM22 concerns the optimal design of a superconducting magnetic energy storage (SMES) device (Fig. 2). The goal of the optimization is to find the SMES configurations that offer an energy stored as close as possible to a defined reference value and a value for the stray field, as small as possible compared to a reference value [14], [15], [16].

There are two formulations of TEAM22 benchmark, based on the number of design variables of the optimization problem. The problem studied in this paper consists in the continuous, constrained, eight-parameter problem, shown in Fig. 2. The design variables are given in Table 2. Moreover, the system has two constraints: the first one is a design constraint where the solenoids should not overlap each other, and the second one is that the superconducting material should not violate the quench condition that links together the value of the current density and the maximum value of magnetic flux density. Thus, the optimization problem can be formulated as follows [14], [15]:

\[
\text{OF} = \frac{B_{\text{norm}}^2}{B_{\text{norm}}^2} + w \left( \frac{E - E_{\text{ref}}}{E_{\text{ref}}} \right),
\]

Subject to:

\[
R_1 + \frac{d_1}{2} < R_2 - \frac{d_2}{2},
\]

where, \(E_{\text{ref}}\) is the reference value of the energy and it is equal to 180MJ, \(B_{\text{norm}}\) is the reference value of the stray field and it is equal to 200\(\mu\)T, \(B_{\text{max}}\) represent the maximum values of the magnetic induction, \(w\) is a penalty factor with value equal to 100 (this factor has been introduced in [15] in order to make the two terms of the objective function, i.e., the stray field and energy terms error, of roughly the same magnitude) and the stray field \(B_{\text{stray}}\) (evaluated along 22 equidistant points along line a and line b in Fig. 2) is defined as:

\[
B_{\text{stray}}^2 = \frac{\sum_{i=1}^{22} B_{\text{stray},i}^2}{22}.
\]
Table 2: Design variables and their ranges for TEAM22 benchmark

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$ [m]</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$R_2$ [m]</td>
<td>1.8</td>
<td>5</td>
</tr>
<tr>
<td>$h_{12}$ [m]</td>
<td>0.2</td>
<td>3.6</td>
</tr>
<tr>
<td>$h_{22}$ [m]</td>
<td>0.2</td>
<td>3.6</td>
</tr>
<tr>
<td>$d_1$ [m]</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>$d_2$ [m]</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>$J_1$ [MA/m²]</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>$J_2$ [MA/m²]</td>
<td>-30</td>
<td>-10</td>
</tr>
</tbody>
</table>

V. OPTIMIZATION RESULTS

A. BLDC benchmark

In order to show the robustness and effectiveness of both the TLBO and ITLBO techniques, they have been applied to the BLDC benchmark, where 100 independent trials have been performed for four cases corresponding to different population sizes. The results of this investigation are shown in Table 3. We can notice that the best, the mean, the median and the worst values of the objective function for the four cases after 100 trials are very close. This is also shown by the low values of standard deviations calculated.

A small comparison of results obtained using the TLBO and the ITLBO techniques, shows the effect of the improvements that have been introduced. We can see clearly that the ITLBO technique outperforms the standard TLBO technique. Furthermore, this investigation reveals the effectiveness of the ITLBO technique and its ability to reach either the optimum value or very near to it in every trial and with different sizes of population. The results obtained here using the TLBO and ITLBO techniques are compared to some other well-known techniques reported in the literature, i.e., sequential quadratic programming (SQP) genetic algorithm (GA), ant colony optimization (ACO), particle swarm optimization (PSO) and modified imperialist competitive algorithm (MICA).

The results of this comparison are reported in Table 4. It appears from this table that the results obtained using the proposed ITLBO technique correspond to the optimal motor configuration as reported in the literature.

Table 3: Simulation results of the BLDC benchmark in 100 trials

<table>
<thead>
<tr>
<th>Population Size</th>
<th>Method</th>
<th>Best</th>
<th>Mean</th>
<th>Median</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>TLBO</td>
<td>94.92</td>
<td>95.29</td>
<td>95.31</td>
<td>95.32</td>
</tr>
<tr>
<td></td>
<td>ITLBO</td>
<td>95.11</td>
<td>95.30</td>
<td>95.31</td>
<td>95.32</td>
</tr>
<tr>
<td>20</td>
<td>TLBO</td>
<td>95.30</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
<tr>
<td></td>
<td>ITLBO</td>
<td>95.31</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
<tr>
<td>30</td>
<td>TLBO</td>
<td>95.31</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
<tr>
<td></td>
<td>ITLBO</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
<tr>
<td>40</td>
<td>TLBO</td>
<td>95.31</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
<tr>
<td></td>
<td>ITLBO</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
<tr>
<td>50</td>
<td>TLBO</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
<tr>
<td></td>
<td>ITLBO</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
<td>95.32</td>
</tr>
</tbody>
</table>

Table 4: Comparison of results using different optimization techniques

<table>
<thead>
<tr>
<th>Method</th>
<th>$D_s$ [mm]</th>
<th>$B_s$ [T]</th>
<th>$\delta$ [A/mm²]</th>
<th>$B_d$ [T]</th>
<th>$B_{cs}$ [T]</th>
<th>$\eta$ [%]</th>
<th>$M_{rot}$ [kg]</th>
<th>$I_{max}$ [A]</th>
<th>$D_{rot}$ [mm]</th>
<th>$D_{ext}$ [mm]</th>
<th>$T_a$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITLBO</td>
<td>201.37</td>
<td>0.6481</td>
<td>2.051</td>
<td>1.8</td>
<td>0.8</td>
<td>95.32</td>
<td>15</td>
<td>125</td>
<td>76.5</td>
<td>239.1</td>
<td>95.26</td>
</tr>
<tr>
<td>TLBO</td>
<td>201.24</td>
<td>0.6482</td>
<td>2.044</td>
<td>1.8</td>
<td>0.8963</td>
<td>95.32</td>
<td>15</td>
<td>125</td>
<td>76.0</td>
<td>238.9</td>
<td>95.35</td>
</tr>
<tr>
<td>SQP [11]</td>
<td>201.20</td>
<td>0.6481</td>
<td>2.044</td>
<td>1.8</td>
<td>0.8959</td>
<td>95.32</td>
<td>15</td>
<td>125</td>
<td>76</td>
<td>238.9</td>
<td>95.35</td>
</tr>
<tr>
<td>GA [12]</td>
<td>201.50</td>
<td>0.648</td>
<td>2.060</td>
<td>1.8</td>
<td>0.8817</td>
<td>95.31</td>
<td>15</td>
<td>125</td>
<td>76.9</td>
<td>239.2</td>
<td>95.21</td>
</tr>
<tr>
<td>GA &amp; SQP [12]</td>
<td>201.20</td>
<td>0.6481</td>
<td>2.062</td>
<td>1.8</td>
<td>0.87</td>
<td>95.31</td>
<td>15</td>
<td>125</td>
<td>76</td>
<td>238.9</td>
<td>95.31</td>
</tr>
<tr>
<td>ACO [13]</td>
<td>201.20</td>
<td>0.6481</td>
<td>2.044</td>
<td>1.8</td>
<td>0.8959</td>
<td>95.32</td>
<td>15</td>
<td>125</td>
<td>76</td>
<td>238.9</td>
<td>95.35</td>
</tr>
<tr>
<td>PSO [13]</td>
<td>202.10</td>
<td>0.6476</td>
<td>2.042</td>
<td>1.8</td>
<td>0.9298</td>
<td>95.32</td>
<td>15</td>
<td>125</td>
<td>79.2</td>
<td>239.8</td>
<td>94.98</td>
</tr>
<tr>
<td>MICA [10]</td>
<td>201.20</td>
<td>0.6481</td>
<td>2.044</td>
<td>1.8</td>
<td>0.8959</td>
<td>95.32</td>
<td>15</td>
<td>125</td>
<td>76</td>
<td>238.9</td>
<td>95.35</td>
</tr>
</tbody>
</table>

B. TEAM22 benchmark

The second example studied in this paper is the TEAM 22 benchmark. The numerical experiments were conducted for 30 independent trials. The parameters of the optimal configurations found using the TLBO and the ITLBO techniques are tabulated in Table 5. Further,
the magnetic flux equipotentials and the magnetic flux density of the optimal configuration found using the ITLBO are represented in Fig. 3 and Fig. 4, respectively.

In Table 6, the ITLBO technique is compared with the TLBO technique and with some other optimizations techniques that are GA, PSO, differential evolution (DE), league championship algorithm (LCA) and electromagnetism-like mechanism (EM). From this comparison, it is worth to mention the superiority of the proposed ITLBO technique over some well-known optimization techniques. In addition, it is to be noted that, the ITLBO performances are better than the standard TLBO. In other words, the ITLBO is more efficient and more robust than the TLBO.

Table 5: Optimal configuration of TEAM22 benchmark obtained using ITLBO

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TLBO</th>
<th>ITLBO</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁ [m]</td>
<td>1.27</td>
<td>1.20</td>
</tr>
<tr>
<td>R₂ [m]</td>
<td>1.96</td>
<td>1.95</td>
</tr>
<tr>
<td>h₁₂ [m]</td>
<td>2.09</td>
<td>2.80</td>
</tr>
<tr>
<td>h₂₂ [m]</td>
<td>2.97</td>
<td>3.60</td>
</tr>
<tr>
<td>d₁ [m]</td>
<td>0.61</td>
<td>0.80</td>
</tr>
<tr>
<td>d₂ [m]</td>
<td>0.10</td>
<td>0.11</td>
</tr>
<tr>
<td>J₁ [MA/m²]</td>
<td>14.85</td>
<td>10.29</td>
</tr>
<tr>
<td>J₂ [MA/m²]</td>
<td>-27.65</td>
<td>-23.35</td>
</tr>
<tr>
<td>OF</td>
<td>0.40</td>
<td>0.18</td>
</tr>
<tr>
<td>E [MJ]</td>
<td>179.55</td>
<td>179.96</td>
</tr>
<tr>
<td>Bstray [mT]</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 6: Comparison of results using different optimization techniques after 30 trials

<table>
<thead>
<tr>
<th>Method</th>
<th>Worst</th>
<th>Mean</th>
<th>Median</th>
<th>Best</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITLBO</td>
<td>2.08</td>
<td>0.66</td>
<td>0.54</td>
<td>0.18</td>
<td>0.46</td>
</tr>
<tr>
<td>TLBO</td>
<td>11.59</td>
<td>3.06</td>
<td>1.51</td>
<td>0.40</td>
<td>2.71</td>
</tr>
<tr>
<td>GA</td>
<td>32.71</td>
<td>15.95</td>
<td>14.16</td>
<td>5.93</td>
<td>7.33</td>
</tr>
<tr>
<td>PSO</td>
<td>101.88</td>
<td>26.79</td>
<td>10.82</td>
<td>0.03</td>
<td>32.29</td>
</tr>
<tr>
<td>DE</td>
<td>57.11</td>
<td>6.33</td>
<td>3.52</td>
<td>0.01</td>
<td>10.38</td>
</tr>
<tr>
<td>LCA</td>
<td>88.44</td>
<td>41.76</td>
<td>43.48</td>
<td>1.65</td>
<td>20.97</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

This paper starts by describing the TLBO technique which is a powerful yet easy technique for optimization of various design problems including electromagnetic ones. The TLBO has the advantage of being a parameterless optimization technique, i.e., its algorithm has no specific control parameters to tune.

In this study, an improved version of the TLBO technique was developed and applied to the BLDC and TEAM22 benchmarks. Considering the quality of the obtained results, it is possible to conclude that the ITLBO constitutes an efficient and robust technique for optimization in electromagnetics area. Further benchmarks for other common electromagnetic problems are currently under investigation. Also, implementing multi-objective ITLBO is a possible extension of the current work in the area concerned with in this study.

REFERENCES


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