Efficient Multilevel Compressed Block Decomposition for Large-Scale Electromagnetic Problems using Asymptotic Phasefront Extraction

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Abstract - A large dense complex linear system can be obtained when solving an electromagnetic scattering problem with the surface integral equation approach. To analyze the large dense complex linear system efficiently, the multilevel compressed block decomposition (MLCBD) is used to accelerate the matrix-vector multiplication operations. Although the MLCBD is efficient compared with the direct method of moments, it is still less efficient for the large-scale electromagnetic problems. Therefore, an efficient version of MLCBD is proposed in this paper. It utilizes the asymptotic phasefront extraction (APE) to reduce the exorbitant dependence on computer storage and solution time in the MLCBD for analyzing the large-scale electromagnetic problems. The numerical results demonstrate that the APE combined with MLCBD is much more efficient than conventional MLCBD for analyzing the large-scale electromagnetic scattering problems.

Index terms - Asymptotic phasefront extraction (APE), electromagnetic scattering, multilevel compressed block decomposition (MLCBD).

I. INTRODUCTION

In electromagnetic wave scattering calculations, a classical problem is to compute the equivalent surface currents induced by a given incident plane wave. Such calculations, relying on the Maxwell equations, are required in the simulation of many industrial processes ranging from antenna design, electromagnetic compatibility, computation of back-scattered fields, and so on. All these simulations require fast and efficient numerical methods to compute an approximate solution of Maxwell’s equations. The method of moments (MoM) [1-2] is one of the most widely used techniques for electromagnetic problems. However, it is basically impractical to solve electric-field integral-equation (EFIE) matrix equations using MoM because its memory requirement and computational complexity are of the orders of $O(N^2)$ and $O(N^3)$, respectively, where $N$ is the number of unknowns.

To alleviate this problem, many fast solution algorithms have been developed. The first kind of algorithms is the fast iterative solution. The most popular fast iterative solution includes the multilevel fast multipole algorithm (MLFMA) [3-4], with $O(N \log N)$ complexity for a given accuracy. Though efficient and accurate, this algorithm is highly technical. It utilizes a large number of tools, such as partial wave expansion, exponential expansion, filtering, and interpolation of spherical harmonics. MDA-SVD is another popular iterative solution used to analyze the scattering/radiation [5-6], which exploits the well known fact that for well separated sub-scatterers, the corresponding sub-matrices are low rank and can be compressed. The second kind of algorithms
is the fast direct solution. The $H$-matrix technique [7] is one of the most popular fast direct solutions, which is based on a data-sparse representation. [8] introduces a popular direct solution, which is based on the adaptive cross approximation (ACA). The MLCBD algorithm [9-11] is another popular direct solution. It is based on a blockwise compression of the impedance matrix, by the same technique as used in the matrix decomposition algorithm (MDA) [12-13]. The numerical complexity of the algorithm is shown to be $O(N^2)$ and the storage requirements scale with $O(N^{1.5})$ [11].

Although both the numerical complexity and the storage requirement of the fast iterative solution are less than that of the fast direct solution, the convergence rate of iterative methods can vary in an unpredictable way. The complexity of the iterative solution method is depending on the matrix condition number. Iterative solvers may be quite satisfactory for only a few right-hand sides (RHS) such as antenna or bistatic problems, but become expensive for monostatic scattering with many required sampling angles. It is well known that the matrix condition number of EFIE for electrically large problem is large [14]. Therefore, the system has poor convergence history.

The aim of this paper is to present a more efficient MLCBD for the large-scale electromagnetic problems. It utilizes the APE method [15-21] to reduce the number of unknowns. Simulation results show that the proposed method is computationally more efficient than for the conventional MLCBD.

The remainder of this paper is organized as follows. Section II gives the theory of APE. Section III describes the theory and implementation of MLCBD in more details. Numerical experiments are presented to demonstrate the efficiency of this proposed method in Section IV. Conclusions are provided in Section V.

I. The Theory of APE

A. The formulation of EFIE

In the work proposed for this paper, the electric field integral equation (EFIE) formulation is adopted [2]. The formulation of EFIE can be expressed as

$$\hat{\mathbf{G}}(\mathbf{r}, \mathbf{r'}) \mathbf{J}(\mathbf{r'}) dS' = \frac{4\pi i}{k\eta} \hat{\mathbf{i}} \mathbf{E}(\mathbf{r}).$$

Here, $\hat{\mathbf{G}}(\mathbf{r}, \mathbf{r'})$ refers to the dyadic Green’s function, $\hat{\mathbf{i}}$ is any unit tangential vector to $S$ at $\mathbf{r}$, $\mathbf{E}(\mathbf{r})$ is the incident excitation plane wave, and $\eta$ and $k$ denote the free space impedance and wave number, respectively. $\mathbf{J}(\mathbf{r'})$ is the induced current density on $S$, which is the unknown of the problem.

B. Asymptotic phasefront extraction

According to the formulation of the conventional MoM [2], the induced current $\mathbf{J}$ is expanded in terms of subsectional basis functions. On the smooth regions $S$ of the object, where the induced surface currents present an asymptotic behavior [22], the current density is expanded in terms of the proposed asymptotic phasefront extraction (APE) basis functions. Since the dominant phase variation is included within the function formulation, the current density in these regions can be efficiently represented using a low number of basis functions. The formulation of the asymptotic phasefront extraction (APE) basis functions is given as follows:

$$f_a(r) = \begin{cases} 
\Lambda_+ e^{-jk_+ (\mathbf{r}_n - \mathbf{r}_n')} & r \in T^+_n \\
\Lambda_- e^{-jk_- (\mathbf{r}_n - \mathbf{r}_n')} & r \in T^-_n \\
0 & \text{otherwise}
\end{cases}.$$
where $\Lambda_n^\pm$ are the RWG vector basis functions [23], defined by

$$\Lambda_n^\pm = \pm \frac{l_n}{2A_n^\pm} \rho_n^\pm, \quad r \in T_n^\pm. \quad (3)$$

The $l_n$ is the length of the common edge to the triangles $T_n^\pm$ conforming the basis function, $A_n^\pm$ is the area of each triangle, $\rho_n^\pm$ is the corresponding vector from the free vertex of $T_n^\pm$ to a point $r$ on the triangle, and $\rho_n^\pm$ is the vector from the free vertex of the triangle $T_n^\pm$ to the midpoint of the common edge $r_n$. Finally, $k_n$ is the vector wavenumber associated to the phase of the current density on the function. Thus, including the incident phase in the RWG basis functions should allow a reduction in the density of the mesh for regions away from discontinuities.

Away from the smooth parts, where the asymptotic representation becomes invalid, the current density can be accurately modeled using a higher density of ordinary RWG basis functions. Then, the impedance matrix which is gained through the EFIE equation can be symbolically rewritten as:

$$ZI = V. \quad (4)$$

To solve the above matrix equation by a direct method, a fast method is needed. In this paper, the multilevel compressed block decomposition (MLCBD) is used.

II. Multilevel Compressed Block Decomposition

A. Block decomposition

When the equation (4) is solved by the fast iterative solution, the convergence rate can vary in an unpredictable way. Therefore, the multilevel compressed block decomposition (MLCBD) is used, which is independent on the matrix condition number. Take three dimensional problems into account; MLCBD is based on the data structure of the binary trees. The binary trees are obtained by subdividing an index set into two subsets recursively. In Fig. 1, the box enclosing the object is subdivided into smaller boxes at multiple levels, in the form of a binary tree. The far interaction boxes [9] are analyzed by MDA-SVD at each level. Consider there exists two boxes at the same level, one is a source box $i$ which contains $m_1$ basis functions and the other is an observation box $j$ which contains $m_2$ test functions. The impedance matrix between two well-separated boxes can be expressed through low rank representations [10-11].

![Fig. 1. Sketch of the binary trees structure.](image)

MDA utilizes this low rank nature of the interaction between two well-separated boxes. In MDA implementation, the impedance matrix of the two well-separated boxes can be expressed as three small matrices [10-11]

$$[Z_{ij}]_{m_1 m_2} = [\tilde{U}_{ij}]_{m_1 r}[\tilde{V}_{ij}]_{r m_2}, \quad (5)$$

where $[Z_{ij}]_{m_1 m_2}$ is the interaction matrix between observation and source boxes. $r$ denotes the number of equivalent RWG sources [10-11], which is much smaller than $m_1$ and $m_2$. 
Since the matrices \( [\tilde{U}]_{m,r} \) and \( [\tilde{V}]_{m,r}^T \) generated by MDA are usually not orthogonal, they may contain redundancies, which can be removed by the following algebraic recompression technique. This method may be regarded as the singular value decomposition optimized for rank-\( k \) matrices. Utilize QR and SVD to reorthonormalize \( [\tilde{U}]_{m,r} \) and \( [\tilde{V}]_{m,r}^T \) and the equation (5) can be obtained as
\[
Z_{ij} = [U_{ij}]_{m,r} [\alpha_{ij}]_{rr} [V_{ij}]_{m,r}^T, \quad (6)
\]
where \( [U_{ij}]_{m,r} \) and \( [V_{ij}]_{m,r}^T \) are both orthogonal. These techniques can reduce the required amount of storage of MDA, while the asymptotic complexity of the approximation remains the same.

At the finest level, blocks (which do not include the self-interaction blocks) representing interactions between adjacent source and observation boxes are compressed by SVD (T) compression. The details of the SVD (T) compression are shown in [23]. The impedance matrix can be expressed as that shown in Fig. 2 (a). The A and B blocks of the impedance matrix are stored in compressed form as the product of three matrices (6).

![Fig. 2. Transformation from blocked impedance matrix to block inverse.](image)

**B. The theory of CBD**

The procedure of the CBD is shown in this part. According to [23], CBD is based on the blockwise compressed matrix which is shown in Fig. 2 (a).

The details of the CBD is shown in following,

**Algorithm CBD (M blocks)**

1) \( \Pi_i = C_0^{-1} \)
2) For \( i = 1 \) to \( M-1 \) do
3) For \( j = 1 \) to \( i \) do
4) \( P_{ji} = \gamma_j^{-1} (B_{ji} - \Sigma_{k=1}^{i-1} \alpha_j \beta_{j,k} B_{jk}) \)
5) \( Q_{ji} = (A_{ji} - \Sigma_{k=1}^{i-1} \alpha_j \beta_{j,k}) \gamma_j^{-1} \)
6) reorthonormalize \( P_{ji} \) and \( Q_{ji} \)
7) For \( k = 1 \) to \( j-1 \) do
8) \( P_{jk} = P_{j,k} - \beta_{j,k} P_{ji} \)
9) \( Q_{jk} = Q_{j,k} - \beta_{j,k} P_{ji} \)
10) reorthonormalize \( P_{jk} \) and \( Q_{jk} \)
11) End(k)
12) End(j)
13) For \( k = 1 \) to \( i \) do; \( \beta_{ik} = P_{ik} ; \alpha_{ik} = Q_{ik} ; \) End(k)
14) \( \gamma_i = (C_i - \Sigma_{j=1}^{i-1} A_j \beta_j)^{-1} \)
15) End(i)

Steps 6) and 10) of the algorithm are explained in [23]. In order to use the decomposed matrix to an independent vector \( X \) to obtain the linear system solution \( y \), the independent vector is subdivided according to the blocks of the impedance matrix, yielding a set of vectors \( X_1, \cdots, X_{M-1} \). Then the algorithm Apply_CBD given in the following is used to compute the solution block by block.

**Algorithm Apply_CBD (X)**

1) For \( i = 0 \) to \( M-1 \) do
2) \( y_i = \gamma_i (X_i - \Sigma_{j=1}^{i} \alpha_j X_j) \)
3) For \( i = 0 \) to \( M-1 \) do
4) \( y_{j-1} = y_{j-1} - \beta_j y_i \)
5) End(j)
6) End(i)
C. The multilevel CBD

When the size of solving problem increases, the computational costs of the step 14) of the algorithm is very large [9]. The remedy is to subdivide again the self-interaction matrix into several smaller submatrices. This procedure is called the multilevel version of CBD [9-11], which is shown in Fig. 3.

![Fig. 3. The procedure of recursive sub-division of the self-interaction matrices.](image)

III. NUMERICAL RESULTS

To validate and demonstrate the accuracy and efficiency of the proposed method, some numerical results are presented in this section. It is because of that the impedance matrix formed by asymptotic phase is not symmetrical; the impedance matrix of the proposed method is not symmetrical. All the computations are carried out on Intel(R) Core(TM) 2 Quad CPU at 2.83 GHz and 8 GB of RAM in double precision and the truncating tolerance of the MDA-SVD is $10^{-3}$ relative to the largest singular value.

A. Sphere

First, we consider the scattering of a perfectly electrically conducting (PEC) sphere with radius $1.6\lambda$ at 300 MHz. The incident direction is $\theta_i = 0^\circ$, $\phi_i = 0^\circ$ and the scattered angles vary from $0^\circ$ to $180^\circ$ in the azimuthal direction when the pitch angle is fixed at $\theta$. The conventional MLCBD solution needs 10107 unknowns with 0.1$\lambda$ patch size, while the proposed method only needs 396 unknowns with 0.5$\lambda$ patch size. The number of the binary trees is $L = 7$. Figure 4 compares the analytical Mie solution with the results of the proposed method. There is a good agreement between them.

Table I summarizes the matrix building and inversion time and the matrix building and inversion memory of the conventional MLCBD and the proposed method. It can be observed that the time and memory of the matrix building and inversion time of the proposed method are much less than that of the conventional MLCBD. The matrix building and inversion time and the matrix building and inversion memory of the proposed method for the sphere at 1 GHz are also shown in the Tab. I. The sphere is discretized with 115707 unknowns with 0.1$\lambda$ patch size by using the conventional MLCBD, while it is discretized with only 1689 unknowns with 0.8$\lambda$ patch size by using the proposed method. The number of the unknowns by using the conventional MLCBD is very large, which can not be analyzed on the computer used in this paper. It can be observed that the memory and time of the proposed are very few for the $10.6\lambda$ sphere.
Table 1: The matrix building and inversion time and the matrix building and inversion memory of the proposed method for the sphere

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Algorithms</th>
<th>Matrix building</th>
<th>Matrix inversion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Memory (MB)</td>
<td>Time (S)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Memory (MB)</td>
<td>Time (S)</td>
</tr>
<tr>
<td>300 MHz</td>
<td>Proposed method</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>MLCBD</td>
<td>222</td>
<td>151</td>
</tr>
<tr>
<td>1 GHz</td>
<td>Proposed method</td>
<td>56</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>MLCBD</td>
<td>~</td>
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</table>

**B. Plane**

The second example is a plane. The dimension of the structure is $4\,m \times 2\,m$. The rotation axis of plane geometry is $z$-axis. The incident and scattered angles are $(\theta_i = 0^\circ, \phi_i = 0^\circ)$ and $(0^\circ \leq \phi_s \leq 180^\circ, \theta_s = 90^\circ)$, respectively.

The frequency is 3 GHz. The conventional MLCBD solution needs 276606 unknowns with $0.1\lambda$ patch size, while the proposed method only needs 5691 unknowns with the middle part of the plane is discretized with $0.9\lambda$. The number of the binary trees is $L = 9$. The bistatic RCS by use of the proposed method is shown in Fig. 5, and is agreed well with that of the MLFMA.

Table 2 shows the matrix building and inversion...
time and the matrix building and inversion memory of the proposed method. It can be found that the proposed method needs much less memory and time for analyzing the $40\lambda$ plane.

![Fig. 5. Bistatic scattering cross section of plane.](image)

Table 2: The matrix building and inversion time and the matrix building and inversion memory of the proposed method for plane

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Matrix building</th>
<th>Matrix inversion</th>
</tr>
</thead>
<tbody>
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<td>Memory (MB)</td>
<td>Time (s)</td>
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<tr>
<td>Proposed method</td>
<td>130</td>
<td>326</td>
</tr>
</tbody>
</table>

C. Airplane

The last example is the airplane model. The dimension of the structure is $10\,m \times 8.5\,m \times 2.75\,m$. The rotation axis of the airplane geometry is the $x$-axis. The incident and scattered angles are $(\theta_i = 90^\circ, \phi_i = 0^\circ)$ and $(0^\circ \leq \phi_s \leq 180^\circ, \theta_s = 90^\circ)$, respectively. The frequency is 600 MHz. The conventional MLCBD solution needs 99795 unknowns with $0.1\lambda$ patch size, while the proposed method only needs 6246 unknowns with the smooth parts of the airplane discretized with $0.8\lambda$. The number of the binary trees is $L = 9$. The bistatic RCS by using the proposed method is shown in Fig. 6, which is compared with the results of the MLFMA. It can be found that the results of the proposed method are well agreed with that of the MLFMA.

The matrix building and inversion time and the matrix building and inversion memory of the proposed method are also given in the Tab. III. It can be seen that the proposed method needs much less memory and time for analyzing the $20\lambda$ airplane.

![Fig. 6. Bistatic scattering cross section of airplane.](image)

IV. CONCLUSIONS

In this paper, an efficient version of MLCBD is introduced. It utilizes the APE method to represent the unknown surface currents in very large scatters using a low number of unknowns. The numerical results demonstrate that the efficient version of the MLCBD is efficient for the EFIE analysis of large-scale electromagnetic scatterings. Compared with the conventional MLCBD, the efficient version of MLCBD can reduce the solution time and memory requirement significantly.
Table 3: The matrix building and inversion time and the matrix building and inversion memory of the proposed method for airplane

<table>
<thead>
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<th>Algorithms</th>
<th>Matrix building</th>
<th>Matrix inversion</th>
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</thead>
<tbody>
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<td>Time (s)</td>
</tr>
<tr>
<td>Proposed method</td>
<td>215</td>
<td>614</td>
</tr>
</tbody>
</table>

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